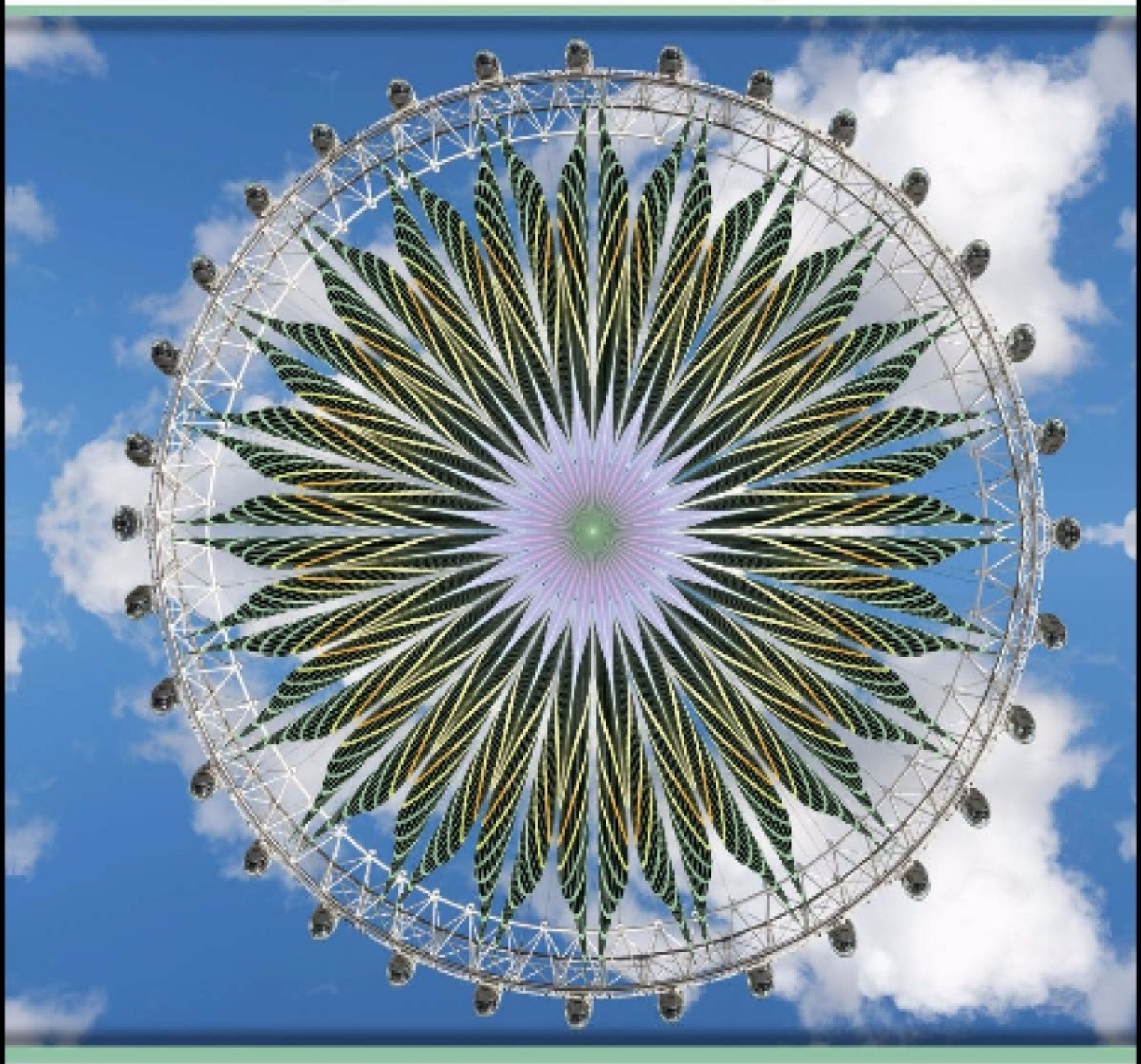
# Solutions Manual for Chapters 5-8

# Trigonometry 8e



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# Chapter 5
# Identities and Formulas

## 5.1 Proving Identities

**EVEN SOLUTIONS**

**2.** To prove, or verify, and identity, we start with one side of the equation and transform it until it is identical to the other side of the equation.

**4.** If nothing else comes to mind, try changing everything in the expression into sines and cosines and then simplify.

**6.** To prove that an equation is not an identity, find a value of the variable for which each expression is defined, but which makes the statement false. This value is called a counterexample.

**8. a.** Factoring: $1-y^2=(1+y)(1-y)$

**b.** Factoring: $1-\cos^2\theta=(1+\cos\theta)(1-\cos\theta)$

**10. a.** Factoring: $x^4-y^4=\left(x^2+y^2\right)\left(x^2-y^2\right)=\left(x^2+y^2\right)(x+y)(x-y)$

**b.** Factoring: $\sin^4\theta-\cos^4\theta=\left(\sin^2\theta+\cos^2\theta\right)\left(\sin^2\theta-\cos^2\theta\right)=(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)$

Note that we used the identity $\sin^2\theta+\cos^2\theta=1$ in the last step.

**12. a.** Multiplying by the conjugate: $\dfrac{2}{1-\sqrt{3}}\cdot\dfrac{1+\sqrt{3}}{1+\sqrt{3}}=\dfrac{2\left(1+\sqrt{3}\right)}{1^2-\left(\sqrt{3}\right)^2}=\dfrac{2\left(1+\sqrt{3}\right)}{1-3}=\dfrac{2\left(1+\sqrt{3}\right)}{-2}=-1-\sqrt{3}$

**b.** Multiplying by the conjugate: $\dfrac{\cos x}{1-\sin x}\cdot\dfrac{1+\sin x}{1+\sin x}=\dfrac{\cos x(1+\sin x)}{1-\sin^2 x}=\dfrac{\cos x(1+\sin x)}{\cos^2 x}=\dfrac{1+\sin x}{\cos x}$

**14. a.** Multiplying by the conjugate: $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\cdot\dfrac{\sqrt{3}+1}{\sqrt{3}+1}=\dfrac{3+2\sqrt{3}+1}{\left(\sqrt{3}\right)^2-(1)^2}=\dfrac{4+2\sqrt{3}}{3-1}=\dfrac{2\left(2+\sqrt{3}\right)}{2}=2+\sqrt{3}$

**b.** Multiplying by the conjugate: $\dfrac{\csc x+1}{\csc x-1}\cdot\dfrac{\csc x+1}{\csc x+1}=\dfrac{(\csc x+1)^2}{\csc^2 x-1}=\dfrac{(\csc x+1)^2}{\cot^2 x}$

**16.** Working from the left side: $\cos\theta\tan\theta=\cos\theta\cdot\dfrac{\sin\theta}{\cos\theta}=\sin\theta$

**18.** Working from the left side: $\dfrac{\cot A}{\csc A}=\dfrac{\dfrac{\cos A}{\sin A}}{\dfrac{1}{\sin A}}=\dfrac{\cos A}{\sin A}\cdot\dfrac{\sin A}{1}=\cos A$

**20.** Working from the left side: $\sin x(\sec x+\csc x)=\sin x\left(\dfrac{1}{\cos x}+\dfrac{1}{\sin x}\right)=\dfrac{\sin x}{\cos x}+\dfrac{\sin x}{\sin x}=\tan x+1$

**22.** Working from the left side: $\tan\theta\csc\theta\cos\theta=\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\sin\theta}\cdot\cos\theta=1$



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**24**. Working from the left side: $(1-\cos x)(1+\cos x) = 1+\cos x-\cos x-\cos^2 x = 1-\cos^2 x = \sin^2 x$

**26**. Working from the left side:

$$\begin{aligned}\frac{\sin^4 t-\cos^4 t}{\sin^2 t\cos^2 t} &= \frac{\left(\sin^2 t+\cos^2 t\right)\left(\sin^2 t-\cos^2 t\right)}{\sin^2 t\cos^2 t}\\ &= \frac{\sin^2 t-\cos^2 t}{\sin^2 t\cos^2 t}\\ &= \frac{\sin^2 t}{\sin^2 t\cos^2 t}-\frac{\cos^2 t}{\sin^2 t\cos^2 t}\\ &= \frac{1}{\cos^2 t}-\frac{1}{\sin^2 t}\\ &= \sec^2 t-\csc^2 t\end{aligned}$$

**28**. Working from the right side: $\dfrac{\cos^2\theta}{1+\sin\theta} = \dfrac{\cos^2\theta}{1+\sin\theta}\cdot\dfrac{1-\sin\theta}{1-\sin\theta} = \dfrac{\cos^2\theta(1-\sin\theta)}{1-\sin^2\theta} = \dfrac{\cos^2\theta(1-\sin\theta)}{\cos^2\theta} = 1-\sin\theta$

**30**. Working from the left side: $\dfrac{1-\cos^4\theta}{1+\cos^2\theta} = \dfrac{\left(1+\cos^2\theta\right)\left(1-\cos^2\theta\right)}{1+\cos^2\theta} = 1-\cos^2\theta = \sin^2\theta$

**32**. Working from the left side: $\csc^2\theta-\cot^2\theta = \dfrac{1}{\sin^2\theta}-\dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1-\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta}{\sin^2\theta} = 1$

**34**. Working from the left side (and using the result from Problem 32):

$$\csc^4\theta-\cot^4\theta = \left(\csc^2\theta-\cot^2\theta\right)\left(\csc^2\theta+\cot^2\theta\right) = 1\cdot\left(\frac{1}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}\right) = \frac{1+\cos^2\theta}{\sin^2\theta}$$

**36**. Working from the left side: $\sec\theta-\csc\theta = \dfrac{1}{\cos\theta}-\dfrac{1}{\sin\theta} = \dfrac{\sin\theta}{\sin\theta\cos\theta}-\dfrac{\cos\theta}{\sin\theta\cos\theta} = \dfrac{\sin\theta-\cos\theta}{\sin\theta\cos\theta}$

Note that this could also be proved working from the right side:

$$\frac{\sin\theta-\cos\theta}{\sin\theta\cos\theta} = \frac{\sin\theta}{\sin\theta\cos\theta}-\frac{\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\cos\theta}-\frac{1}{\sin\theta} = \sec\theta-\csc\theta$$

**38**. Working from the left side: $\sec B-\cos B = \dfrac{1}{\cos B}-\cos B = \dfrac{1-\cos^2 B}{\cos B} = \dfrac{\sin^2 B}{\cos B} = \dfrac{\sin B}{\cos B}\cdot\sin B = \tan B\sin B$

**40**. Working from the left side: $\tan\theta\sin\theta+\cos\theta = \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta+\cos\theta = \dfrac{\sin^2\theta}{\cos\theta}+\dfrac{\cos^2\theta}{\cos\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

**42**. Working from the left side: $\dfrac{\cos x}{1+\sin x}-\dfrac{1-\sin x}{\cos x} = \dfrac{\cos^2 x}{\cos x(1+\sin x)}-\dfrac{1-\sin^2 x}{\cos x(1+\sin x)} = \dfrac{\cos^2 x}{\cos x(1+\sin x)}-\dfrac{\cos^2 x}{\cos x(1+\sin x)} = 0$

**44**. Working from the left side: $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x} = \dfrac{1+\sin x}{1-\sin^2 x}+\dfrac{1-\sin x}{1-\sin^2 x} = \dfrac{2}{1-\sin^2 x} = \dfrac{2}{\cos^2 x} = 2\sec^2 x$

**46**. Working from the left side: $\dfrac{\csc x-1}{\csc x+1} = \dfrac{\dfrac{1}{\sin x}-1}{\dfrac{1}{\sin x}+1}\cdot\dfrac{\sin x}{\sin x} = \dfrac{1-\sin x}{1+\sin x}$

**48**. Working from the left side: $\dfrac{\cos t}{1+\sin t} = \dfrac{\cos t}{1+\sin t}\cdot\dfrac{1-\sin t}{1-\sin t} = \dfrac{\cos t(1-\sin t)}{1-\sin^2 t} = \dfrac{\cos t(1-\sin t)}{\cos^2 t} = \dfrac{1-\sin t}{\cos t}$

Note that this could also be proved working from the right side:

$$\frac{1-\sin t}{\cos t} = \frac{1-\sin t}{\cos t}\cdot\frac{1+\sin t}{1+\sin t} = \frac{1-\sin^2 t}{\cos t(1+\sin t)} = \frac{\cos^2 t}{\cos t(1+\sin t)} = \frac{\cos t}{1+\sin t}$$

**50**. Working from the left side: $\dfrac{\sin^2 t}{(1-\cos t)^2} = \dfrac{1-\cos^2 t}{(1-\cos t)^2} = \dfrac{(1+\cos t)(1-\cos t)}{(1-\cos t)^2} = \dfrac{1+\cos t}{1-\cos t}$



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**52.** Working from the left side: $\dfrac{\csc\theta-1}{\cot\theta}=\dfrac{\csc\theta-1}{\cot\theta}\bullet\dfrac{\csc\theta+1}{\csc\theta+1}=\dfrac{\csc^2\theta-1}{\cot\theta(\csc\theta+1)}=\dfrac{\cot^2\theta}{\cot\theta(\csc\theta+1)}=\dfrac{\cot\theta}{\csc\theta+1}$

**54.** Working from the right side:

$$(\sec x-\tan x)^2=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2=\frac{(1-\sin x)^2}{\cos^2 x}=\frac{(1-\sin x)^2}{1-\sin^2 x}=\frac{(1-\sin x)^2}{(1+\sin x)(1-\sin x)}=\frac{1-\sin x}{1+\sin x}$$

**56.** Working from the left side:

$$\frac{1}{\csc x-\cot x}=\frac{1}{\csc x-\cot x}\bullet\frac{\csc x+\cot x}{\csc x+\cot x}=\frac{\csc x+\cot x}{\csc^2 x-\cot^2 x}=\frac{\csc x+\cot x}{1}=\csc x+\cot x$$

**58.** Working from the left side: $\dfrac{\cos x+1}{\cot x}=\dfrac{\cos x+1}{\dfrac{\cos x}{\sin x}}=\dfrac{\sin x\cos x+\sin x}{\cos x}=\sin x+\dfrac{\sin x}{\cos x}=\sin x+\tan x$

**60.** Working from the left side: $\cos^4 A-\sin^4 A=\left(\cos^2 A+\sin^2 A\right)\left(\cos^2 A-\sin^2 A\right)=1\bullet\left(1-\sin^2 A-\sin^2 A\right)=1-2\sin^2 A$

**62.** Working from the left side:

$$\frac{\sin^2 B-\tan^2 B}{1-\sec^2 B}=\frac{\sin^2 B-\dfrac{\sin^2 B}{\cos^2 B}}{1-\dfrac{1}{\cos^2 B}}\bullet\frac{\cos^2 B}{\cos^2 B}=\frac{\sin^2 B\cos^2 B-\sin^2 B}{\cos^2 B-1}=\frac{\sin^2 B\left(\cos^2 B-1\right)}{\cos^2 B-1}=\sin^2 B$$

**64.** Working from the left side: $\dfrac{\csc^2 y+\cot^2 y}{\csc^4 y-\cot^4 y}=\dfrac{\csc^2 y+\cot^2 y}{\left(\csc^2 y+\cot^2 y\right)\left(\csc^2 y-\cot^2 y\right)}=\dfrac{1}{\csc^2 y-\cot^2 y}=\dfrac{1}{1}=1$

**66.** Working from the left side (factor the numerator as a difference of cubes):

$$\frac{1-\tan^3 t}{1-\tan t}=\frac{(1-\tan t)\left(1+\tan t+\tan^2 t\right)}{1-\tan t}=1+\tan t+\tan^2 t=\sec^2 t+\tan t$$

**68.** Working from the left side:

$$\frac{\cot^2 x}{\sin x+\cos x}=\frac{\cos^2 x}{\sin^2 x(\sin x+\cos x)}\bullet\frac{\sin x-\cos x}{\sin x-\cos x}$$

$$=\frac{\cos^2 x\sin x-\cos^3 x}{\sin^2 x\left(\sin^2 x-\cos^2 x\right)}$$

$$=\frac{\cos^2 x\sin x-\cos^3 x}{\sin^2 x\left(\sin^2 x-1+\sin^2 x\right)}$$

$$=\frac{\cos^2 x\sin x-\cos^3 x}{\sin^2 x\left(2\sin^2 x-1\right)}$$

$$=\frac{\cos^2 x\sin x-\cos^3 x}{2\sin^4 x-\sin^2 x}$$

**70.** Substituting $\theta=\dfrac{\pi}{6}$: $\sin\dfrac{\pi}{6}+\cos\dfrac{\pi}{6}=\frac{1}{2}+\dfrac{\sqrt{3}}{2}=\dfrac{1+\sqrt{3}}{2}\neq 1$

**72.** Substituting $\theta=\dfrac{\pi}{4}$: $\tan^2\dfrac{\pi}{4}+\cot^2\dfrac{\pi}{4}=1^2+1^2=2\neq 1$

**74.** Substituting $\theta=0$: $\sin 0\cos 0=0\bullet 1=0\neq 1$



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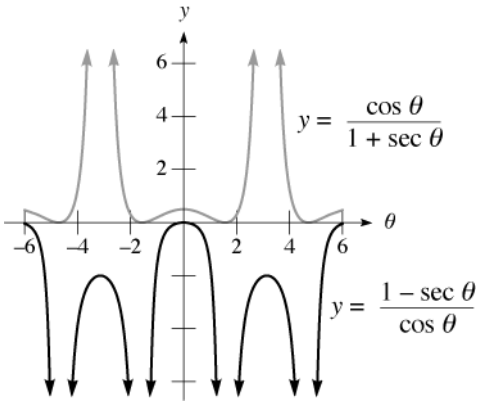
**76**. Graphing both curves:

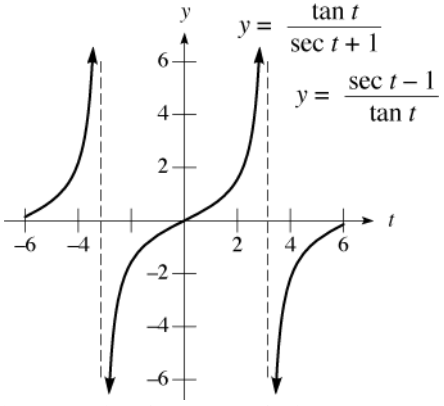



The equation does not appear to be an identity. Evaluating each side when $\theta = 0$ :

$$\frac{1-\sec 0}{\cos 0} = \frac{1-1}{1} = 0 \qquad \frac{\cos 0}{1+\sec 0} = \frac{1}{1+1} = \frac{1}{2}$$

The two expressions are not equal when $\theta = 0$ .

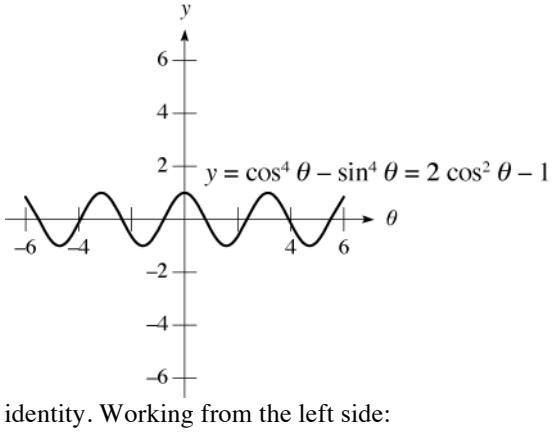
**78**. Graphing both curves:




The equation appears to be an identity. Working from the left side:

$$\frac{\tan t}{\sec t+1} \cdot \frac{\sec t-1}{\sec t-1} = \frac{(\tan t)(\sec t-1)}{\sec^2 t-1} = \frac{(\tan t)(\sec t-1)}{\tan^2 t} = \frac{\sec t-1}{\tan t}$$

**80**. Graphing both curves:




The equation appears to be an identity. Working from the left side:

$$\cos^4\theta - \sin^4\theta = \left(\cos^2\theta + \sin^2\theta\right)\left(\cos^2\theta - \sin^2\theta\right) = (1)\left(\cos^2\theta - \left(1-\cos^2\theta\right)\right) = 2\cos^2\theta - 1$$



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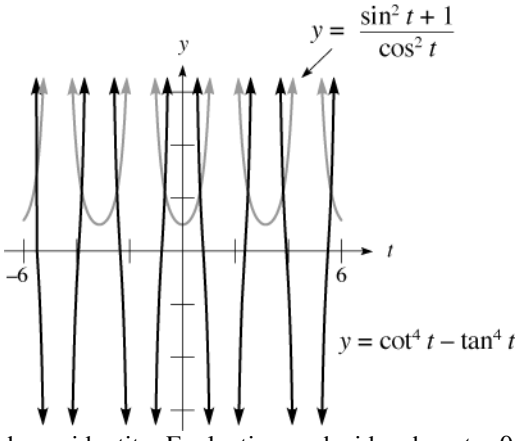
**82**. Graphing both curves:




The equation does not appear to be an identity. Evaluating each side when $t = 0$:

$\cot^4 0 - \tan^4 0$ is undefined $\qquad \dfrac{\sin^2 0 + 1}{\cos^2 0} = \dfrac{0+1}{1} = 1$

The two expressions are not equal when $t = 0$.

**84**. Substituting $x = 30°$ into both expressions:

$$\sin 2x = \sin(2 \cdot 30°) = \sin 60° = \frac{\sqrt{3}}{2}$$

$$2\sin x = 2\sin 30° = 2 \cdot \frac{1}{2} = 1$$

So $\sin 2x \neq 2\sin x$ for all values of $x$.

**86**. Working from the left side:

$$\frac{\tan^2 \psi + 2}{1 + \tan^2 \psi} = \frac{\dfrac{\sin^2 \psi}{\cos^2 \psi} + 2}{1 + \dfrac{\sin^2 \psi}{\cos^2 \psi}} \cdot \frac{\cos^2 \psi}{\cos^2 \psi}$$

$$= \frac{\sin^2 \psi + 2\cos^2 \psi}{\cos^2 \psi + \sin^2 \psi}$$

$$= \sin^2 \psi + 2\cos^2 \psi$$

$$= \left(\sin^2 \psi + \cos^2 \psi\right) + \cos^2 \psi$$

$$= 1 + \cos^2 \psi$$



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**88**. Working from the left side:

$$\frac{\cos\beta}{1-\tan\beta}+\frac{\sin\beta}{1-\cot\beta}=\frac{\cos\beta}{1-\dfrac{\sin\beta}{\cos\beta}}+\frac{\sin\beta}{1-\dfrac{\cos\beta}{\sin\beta}}$$

$$=\frac{\cos\beta}{1-\dfrac{\sin\beta}{\cos\beta}}\cdot\frac{\cos\beta}{\cos\beta}+\frac{\sin\beta}{1-\dfrac{\cos\beta}{\sin\beta}}\cdot\frac{\sin\beta}{\sin\beta}$$

$$=\frac{\cos^2\beta}{\cos\beta-\sin\beta}+\frac{\sin^2\beta}{\sin\beta-\cos\beta}$$

$$=\frac{\cos^2\beta}{\cos\beta-\sin\beta}-\frac{\sin^2\beta}{\cos\beta-\sin\beta}$$

$$=\frac{\cos^2\beta-\sin^2\beta}{\cos\beta-\sin\beta}$$

$$=\frac{(\cos\beta+\sin\beta)(\cos\beta-\sin\beta)}{\cos\beta-\sin\beta}$$

$$=\sin\beta+\cos\beta$$

**90**. Since $B$ terminates in quadrant III, $\sin B<0$, thus:

$$\sin B=-\sqrt{1-\cos^2 B}=-\sqrt{1-\left(-\frac{5}{13}\right)^2}=-\sqrt{1-\frac{25}{169}}=-\sqrt{\frac{144}{169}}=-\frac{12}{13}$$

Therefore: $\tan B=\dfrac{\sin B}{\cos B}=\dfrac{-12/13}{-5/13}=\dfrac{12}{5}$

**92**. The exact value is given by: $\cos\dfrac{\pi}{4}=\dfrac{\sqrt{2}}{2}$

**94**. The exact value is given by: $\sin\dfrac{\pi}{2}=1$

**96**. Converting to degrees: $\dfrac{5\pi}{9}$ radians $=\dfrac{5\pi}{9}\cdot\dfrac{180^\circ}{\pi}=100^\circ$

**98**. Working from the left side: $\sec\theta-\cos(-\theta)=\dfrac{1}{\cos\theta}-\cos\theta\cdot\dfrac{\cos\theta}{\cos\theta}=\dfrac{1-\cos^2\theta}{\cos\theta}=\dfrac{\sin^2\theta}{\cos\theta}$

**100**. Computing each side with $x=\dfrac{3\pi}{4}$:

$$\frac{1+\cos x}{\cot x}=\frac{1+\cos\dfrac{3\pi}{4}}{\cot\dfrac{3\pi}{4}}=\frac{1-\dfrac{\sqrt{2}}{2}}{-1}=-1+\frac{\sqrt{2}}{2}$$

$$\cot x=\cot\frac{3\pi}{4}=-1$$

The correct answer is a. Each of the other choices results in an undefined value.



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**ODD SOLUTIONS**

**1.** equal, defined

**3.** complicated

**5.** identical

**7.** **a.** $x(x-y)$ **b.** $\sin\theta(\sin\theta-\cos\theta)$

**9.** **a.** $(x+y)(x-y)$ **b.** $(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)$

**11.** **a.** $\dfrac{1}{1+\sqrt{3}}\cdot\dfrac{1-\sqrt{3}}{1-\sqrt{3}}=\dfrac{1-\sqrt{3}}{1-3}=\dfrac{1-\sqrt{3}}{-2}=\dfrac{\sqrt{3}-1}{2}$

**b.** $\dfrac{1}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x}=\dfrac{1-\cos x}{1-\cos^2 x}=\dfrac{1-\cos x}{\sin^2 x}$

**13.** **a.** $\dfrac{1-\sqrt{2}}{1+\sqrt{2}}\cdot\dfrac{1-\sqrt{2}}{1-\sqrt{2}}=\dfrac{1-2\sqrt{2}+2}{1-2}=\dfrac{3-2\sqrt{2}}{-1}=-3+2\sqrt{2}$

**b.** $\dfrac{1-\sin x}{1+\sin x}\cdot\dfrac{1-\sin x}{1-\sin x}=\dfrac{(1-\sin x)^2}{1-\sin^2 x}=\dfrac{(1-\sin x)^2}{\cos^2 x}$

**15.**
$$\csc\theta\tan\theta=\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$$ Ratio and reciprocal identities
$$=\frac{1}{\cos\theta}$$ Reduce
$$=\sec\theta$$ Reciprocal identity

**17.**
$$\frac{\tan A}{\sec A}=\frac{\dfrac{\sin A}{\cos A}}{\dfrac{1}{\cos A}}$$ Ratio and reciprocal identities
$$=\frac{\sin A\cos A}{\cos A}$$ Divide
$$=\sin A$$ Reduce

**19.**
$$\cos x(\csc x+\tan x)=\cos x\csc x+\cos x\tan x$$ Distributive property
$$=\cos x\cdot\frac{1}{\sin x}+\cos x\cdot\frac{\sin x}{\cos x}$$ Reciprocal and ratio identities
$$=\frac{\cos x}{\sin x}+\frac{\cos x\sin x}{\cos x}$$ Multiply
$$=\cot x+\sin x$$ Ratio identity and reduce second fraction

**21.**
$$\sec\theta\cot\theta\sin\theta=\frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1}$$ Reciprocal and ratio identities
$$=1$$ Multiply and reduce

**23.**
$$\sin^2 x(\cot^2 x+1)=\sin^2 x(\csc^2 x)$$ Pythagorean identity
$$=\sin^2 x\left(\frac{1}{\sin^2 x}\right)$$ Reciprocal identity
$$=1$$ Multiply and reduce

**25.**
$$\frac{\cos^4 t-\sin^4 t}{\sin^2 t}=\frac{(\cos^2 t+\sin^2 t)(\cos^2 t-\sin^2 t)}{\sin^2 t}$$ Factor
$$=\frac{1(\cos^2 t-\sin^2 t)}{\sin^2 t}$$ Pythagorean identity
$$=\frac{\cos^2 t}{\sin^2 t}-\frac{\sin^2 t}{\sin^2 t}$$ Separate into 2 fractions
$$=\cot^2 t-1$$ Ratio identity and reduce second fraction



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**27.**
$$\frac{\cos^2\theta}{1-\sin\theta} = \frac{1-\sin^2\theta}{1-\sin\theta}$$ Pythagorean identity
$$= \frac{(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta}$$ Factor numerator
$$= 1+\sin\theta$$ Reduce

**29.**
$$\frac{1-\sin^4\theta}{1+\sin^2\theta} = \frac{(1-\sin^2\theta)(1+\sin^2\theta)}{1+\sin^2\theta}$$ Factor
$$= 1-\sin^2\theta$$ Reduce
$$= \cos^2\theta$$ Pythagorean identity

**31.**
$$\sec^2\theta - \tan^2\theta = (\tan^2\theta + 1) - \tan^2\theta$$ Pythagorean identity
$$= 1$$ Combine

**33.**
$$\sec^4 - \tan^4\theta = (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$$ Factor
$$= 1(\sec^2\theta + \tan^2\theta)$$ Pythagorean identity
$$= \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$ Reciprocal and ratio identities
$$= \frac{1+\sin^2\theta}{\cos^2\theta}$$ Add fractions

**35.**
$$\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin^2\theta}{\sin\theta\cos\theta} - \frac{\cos^2\theta}{\sin\theta\cos\theta}$$ Separate into 2 fractions
$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$ Reduce
$$= \tan\theta - \cot\theta$$ Ratio identities

**37.**
$$\csc B - \sin B = \frac{1}{\sin B} - \sin B$$ Reciprocal identity
$$= \frac{1}{\sin B} - \sin B \cdot \frac{\sin B}{\sin B}$$ LCD is sin$B$
$$= \frac{1-\sin^2 B}{\sin B}$$ Subtract fractions
$$= \frac{\cos^2 B}{\sin B}$$ Pythagorean identity
$$= \frac{\cos B}{\sin B} \cdot \cos B$$ Separate fraction
$$= \cot B \cos B$$ Ratio identity

**39.**
$$\cot\theta\cos\theta + \sin\theta = \frac{\cos\theta}{\sin\theta} \cdot \cos\theta + \sin\theta$$ Ratio identity
$$= \frac{\cos^2\theta}{\sin\theta} + \sin\theta \cdot \frac{\sin\theta}{\sin\theta}$$ LCD is $\sin\theta$
$$= \frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta}$$ Multiply
$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$ Add fractions
$$= \frac{1}{\sin\theta}$$ Pythagorean identity
$$= \csc\theta$$ Reciprocal identity



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**41.** $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}=\dfrac{\cos x}{1+\sin x}\cdot\dfrac{\cos x}{\cos x}+\dfrac{1+\sin x}{\cos x}\cdot\dfrac{1+\sin x}{1+\sin x}$ LCD

$=\dfrac{\cos^2 x}{\cos x(1+\sin x)}+\dfrac{1+2\sin x+\sin^2 x}{\cos x(1+\sin x)}$ Multiply fractions

$=\dfrac{(\cos^2 x+\sin^2 x)+1+2\sin x}{\cos x(1+\sin x)}$ Add fractions and regroup terms

$=\dfrac{1+1+2\sin x}{\cos x(1+\sin x)}$ Pythagorean identity

$=\dfrac{2+2\sin x}{\cos x(1+\sin x)}$ Combine

$=\dfrac{2(1+\sin x)}{\cos x(1+\sin x)}$ Factor out a 2

$=\dfrac{2}{\cos x}$ Reduce

$=2\sec x$ Reciprocal identity

**43.** $\dfrac{1}{1+\cos x}+\dfrac{1}{1-\cos x}=\dfrac{1}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x}+\dfrac{1}{1-\cos x}\cdot\dfrac{1+\cos x}{1+\cos x}$ LCD

$=\dfrac{1-\cos x}{1-\cos^2 x}+\dfrac{1+\cos x}{1-\cos^2 x}$ Multiply

$=\dfrac{1-\cos x+1+\cos x}{1-\cos^2 x}$ Add fractions

$=\dfrac{2}{1-\cos^2 x}$ Combine numerator

$=\dfrac{2}{\sin^2 x}$ Pythagorean identity

$=2\csc^2 x$ Reciprocal identity

**45.** $\dfrac{1-\sec x}{1+\sec x}=\dfrac{1-\dfrac{1}{\cos x}}{1+\dfrac{1}{\cos x}}$ Reciprocal identity

$=\dfrac{\cos x\left(1-\dfrac{1}{\cos x}\right)}{\cos x\left(1+\dfrac{1}{\cos x}\right)}$ Multiply numerator and denominator by LCD

$=\dfrac{\cos x-1}{\cos x+1}$ Distributive property

**47.** $\dfrac{\sin t}{1+\cos t}=\dfrac{\sin t}{1+\cos t}\cdot\dfrac{1-\cos t}{1-\cos t}$ Multiply numerator and denominator by $1-\cos t$

$=\dfrac{\sin t(1-\cos t)}{1-\cos^2 t}$ Multiply fractions

$=\dfrac{\sin t(1-\cos t)}{\sin^2 t}$ Pythagorean identity

$=\dfrac{1-\cos t}{\sin t}$ Reduce



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**49.** $\dfrac{1-\sin t}{1+\sin t} = \dfrac{1-\sin t}{1+\sin t}\cdot\dfrac{1-\sin t}{1-\sin t}$ Multiply numerator and denominator by $1-\sin t$

$= \dfrac{(1-\sin t)^2}{1-\sin^2 t}$ Multiply fractions

$= \dfrac{(1-\sin t)^2}{\cos^2 t}$ Pythagorean identity

**51.** $\dfrac{\sec\theta+1}{\tan\theta} = \dfrac{\sec\theta+1}{\tan\theta}\cdot\dfrac{\sec\theta-1}{\sec\theta-1}$ Multiply numerator and denominator by $\sec\theta-1$

$= \dfrac{\sec^2\theta-1}{\tan\theta(\sec\theta-1)}$ Multiply fractions

$= \dfrac{\tan^2\theta}{\tan\theta(\sec\theta-1)}$ Pythagorean identity

$= \dfrac{\tan\theta}{\sec\theta-1}$ Reduce

**53.** $(\csc x+\cot x)^2 = \left(\dfrac{1}{\sin x}+\dfrac{\cos x}{\sin x}\right)^2$ Reciprocal and ratio identities

$= \left(\dfrac{1+\cos x}{\sin x}\right)^2$ Subtract fractions

$= \dfrac{(1+\cos x)^2}{\sin^2 x}$ Property of exponents

$= \dfrac{(1+\cos x)^2}{1-\cos^2 x}$ Pythagorean identity

$= \dfrac{(1+\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x)}$ Factor

$= \dfrac{1+\cos x}{1-\cos x}$ Reduce

**55.** $\sec x+\tan x = \dfrac{\sec x+\tan x}{1}\cdot\dfrac{\sec x-\tan x}{\sec x-\tan x}$ Multiply numerator and denominator by $\sec x-\tan x$

$= \dfrac{\sec^2 x-\tan^2 x}{\sec x-\tan x}$ Multiply fractions

$= \dfrac{1}{\sec x-\tan x}$ Pythagorean identity

**57.** $\dfrac{\sin x+1}{\cos x+\cot x} = \dfrac{\sin x+1}{\cos x+\dfrac{\cos x}{\sin x}}$ Ratio identity

$= \dfrac{\sin x}{\sin x}\cdot\dfrac{(\sin x+1)}{\left(\cos x+\dfrac{\cos x}{\sin x}\right)}$ Multiply numerator and denominator by LCD

$= \dfrac{\sin x(\sin x+1)}{\sin x\cos x+\cos x}$ Distributive property

$= \dfrac{\sin x(\sin x+1)}{\cos x(\sin x+1)}$ Factor

$= \dfrac{\sin x}{\cos x}$ Reduce

$= \tan x$ Ratio identity



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**59.** $\sin^4 A - \cos^4 A = \left(\sin^2 A + \cos^2 A\right)\left(\sin^2 A - \cos^2 A\right)$ Factor

$= 1\left(\sin^2 A - \cos^2 A\right)$ Pythagorean identity

$= 1 - \cos^2 A - \cos^2 A$ Pythagorean identity

$= 1 - 2\cos^2 A$ Combine

**61.** $\dfrac{\cot^2 B - \cos^2 B}{\csc^2 B - 1} = \dfrac{\dfrac{\cos^2 B}{\sin^2 B} - \cos^2 B}{\dfrac{1}{\sin^2 B} - 1}$ Ratio identity and reciprocal identity

$= \dfrac{\cos^2 B\left(\dfrac{1}{\sin^2 B} - 1\right)}{\left(\dfrac{1}{\sin^2 B} - 1\right)}$ Factor $\cos^2 x$ from numerator

$= \cos^2 B$ Divide common factor

**63.** $\dfrac{\sec^4 y - \tan^4 y}{\sec^2 y + \tan^2 y} = \dfrac{\left(\sec^2 y - \tan^2 y\right)\left(\sec^2 y + \tan^2 y\right)}{\sec^2 y + \tan^2 y}$ Factor

$= \sec^2 y - \tan^2 y$ Reduce

$= 1$ Pythagorean identity

**65.** $\dfrac{\sin^3 A - 8}{\sin A - 2} = \dfrac{\left(\sin A - 2\right)\left(\sin^2 A + 2\sin A + 4\right)}{\sin A - 2}$ Factor as difference of 2 cubes

$= \sin^2 A + 2\sin A + 4$ Reduce

**67.** $\dfrac{\sin^2 x + \sin x \cos x}{\cos x - 2\cos^3 x} = \dfrac{\sin x\left(\sin x + \cos x\right)}{\cos x\left(1 - 2\cos^2 x\right)}$ Factor

$= \dfrac{\sin x\left(\sin x + \cos x\right)}{\cos x\left[\left(\sin^2 x + \cos^2 x\right) - 2\cos^2 x\right]}$ Pythagorean identity

$= \dfrac{\sin x}{\cos x} \cdot \dfrac{\sin x + \cos x}{\sin^2 x - \cos^2 x}$ Separate fractions and simplify

$= \tan x \cdot \dfrac{\sin x + \cos x}{\left(\sin x + \cos x\right)\left(\sin x - \cos x\right)}$ Ratio identity and factor

$= \dfrac{\tan x}{\sin x - \cos x}$ Reduce common factor

**69.** If $\theta = 210^\circ$, $\sin 210^\circ \neq \sqrt{1 - \cos^2\left(210^\circ\right)}$

$-\dfrac{1}{2} \neq \sqrt{1 - \left(-\sqrt{3}/2\right)^2}$

$-\dfrac{1}{2} \neq \sqrt{1 - \dfrac{3}{4}}$

$-\dfrac{1}{2} \neq \sqrt{\dfrac{1}{4}}$

$-\dfrac{1}{2} \neq \dfrac{1}{2}$

**71.** If $\theta = 30^\circ$, then $\sin 30^\circ \neq \dfrac{1}{\cos 30^\circ}$

$\dfrac{1}{2} \neq \dfrac{1}{\sqrt{3}/2}$

$\dfrac{1}{2} \neq \dfrac{2\sqrt{3}}{3}$



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**73.** If $\theta = 30^\circ$, $\sqrt{\sin^2(30^\circ)+\cos^2(30^\circ)} = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$$= \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = 1$$

Also, $\sin 30^\circ + \cos 30^\circ = \frac{1}{2}+\frac{\sqrt{3}}{2}$

$$= \frac{1+\sqrt{3}}{2}$$

Since $1 \neq \frac{1+\sqrt{3}}{2}$, this statement is false.

**75.**

| | |
|---|---|
| $(\sec B - 1)(\sec B + 1) = \sec^2 B - 1$ | Multiply |
| $= (\tan^2 B + 1) - 1$ | Pythagorean identity |
| $= \tan^2 B$ | Combine |

**77.** Let $x = 0$. Then $\sec 0 + \cos 0 \neq \tan 0 \sin 0$

$$1 + 1 \neq 0 \cdot 0$$
$$2 \neq 0$$

**79.** Let $A = \frac{3\pi}{4}$. Then $\sec\frac{3\pi}{4} - \csc\frac{3\pi}{4} \neq \frac{\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4}}{\cos\frac{3\pi}{4}\sin\frac{3\pi}{4}}$

$$-\sqrt{2} - \sqrt{2} \neq \frac{-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{-\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}}$$

$$-2\sqrt{2} \neq \frac{-2\sqrt{2}/2}{-1/2}$$

$$-2\sqrt{2} \neq 2\sqrt{2}$$

**81.**

| | |
|---|---|
| $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = \frac{(1+\sin x)}{(1+\sin x)(1-\sin x)} + \frac{(1-\sin x)}{(1-\sin x)(1+\sin x)}$ | Multiply num and denom by a fraction equal to 1 |
| $= \frac{1+\sin x + 1 - \sin x}{1-\sin^2 x}$ | Combine |
| $= \frac{2}{\cos^2 x}$ | Pythagorean identity |
| $= 2\sec^2 x$ | Reciprocal identity |

**83.** $\sin(30^\circ + 60^\circ) = \sin 90^\circ$ and $\sin 30^\circ + \sin 60^\circ = \frac{1}{2}+\frac{\sqrt{3}}{2}$

$$= 1$$

$$= \frac{1+\sqrt{3}}{2}$$

Since $1 \neq \frac{1+\sqrt{3}}{2}$, this statement is false.



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**89.** $\cos A = \sqrt{1-\sin^2 A}$ $\qquad$ $\tan A = \dfrac{\sin A}{\cos A}$

$= \sqrt{1-\left(\dfrac{3}{4}\right)^2}$ $\qquad$ $= \dfrac{3/4}{\sqrt{7}/4}$

$= \sqrt{\dfrac{7}{16}}$ $\qquad$ $= \dfrac{3}{\sqrt{7}} = \dfrac{3\sqrt{7}}{7}$

$= \dfrac{\sqrt{7}}{4}$

**91.** $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

**93.** $\cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

**95.** $\dfrac{\pi}{12} = \dfrac{\pi}{12}\cdot\dfrac{180}{\pi}$

$= 15^\circ$

**97.** $\csc\theta + \sin(-\theta) = \csc\theta - \sin\theta$ $\qquad$ Sine is an odd function

$= \dfrac{1}{\sin\theta} - \sin\theta$ $\qquad$ Reciprocal identity

$= \dfrac{1}{\sin\theta} - \sin\theta\cdot\dfrac{\sin\theta}{\sin\theta}$ $\qquad$ L.C.D. is $\sin\theta$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$ $\qquad$ Multiply fractions

$= \dfrac{1-\sin^2\theta}{\sin\theta}$ $\qquad$ Subtract fractions

$= \dfrac{\cos^2\theta}{\sin\theta}$ $\qquad$ Pythagorean identity

## 5.2 Sum and Difference Formulas

**EVEN SOLUTIONS**

**2.** The identity is: $\sin(x-y) = \sin x\cos y - \cos x\sin y$

**4.** The identity is: $\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

**6.** The identity is: $\tan(C-D) = \dfrac{\tan C - \tan D}{1+\tan C\tan D}$

**8.** This statement is false.

**10.** Using the difference formula for cosine:

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ\cos 30^\circ + \sin 45^\circ\sin 30^\circ = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

**12.** Using the sum formula for tangent:

$$\tan 75^\circ = \tan(30^\circ + 45^\circ) = \frac{\tan 30^\circ + \tan 45^\circ}{1-\tan 30^\circ\tan 45^\circ} = \frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}\cdot 1}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

**14.** Using the difference formula for cosine:

$$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$



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**16**. Using the sum formula for sine:

$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

**18**. Using the difference formula for cosine: $\cos(x - 2\pi) = \cos x \cos 2\pi + \sin x \sin 2\pi = \cos x \cdot 1 + \sin x \cdot 0 = \cos x$

**20**. Using the difference formula for sine: $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2} = \sin x \cdot 0 - \cos x \cdot 1 = -\cos x$

**22**. Using the difference formula for sine: $\sin(180^\circ - \theta) = \sin 180^\circ \cos\theta - \cos 180^\circ \sin\theta = 0 \cdot \cos\theta - (-1) \cdot \sin\theta = \sin\theta$

**24**. Using the sum formula for cosine: $\cos(90^\circ + \theta) = \cos 90^\circ \cos\theta - \sin 90^\circ \sin\theta = 0 \cdot \cos\theta - 1 \cdot \sin\theta = -\sin\theta$

**26**. Using the difference formula for tangent: $\tan\left(x - \frac{\pi}{4}\right) = \dfrac{\tan x - \tan\frac{\pi}{4}}{1 + \tan x \tan\frac{\pi}{4}} = \dfrac{\tan x - 1}{1 + \tan x \cdot 1} = \dfrac{\tan x - 1}{\tan x + 1}$

**28**. Using the difference formula for cosine: $\cos\left(x - \frac{3\pi}{2}\right) = \cos x \cos\frac{3\pi}{2} + \sin x \sin\frac{3\pi}{2} = \cos x \cdot 0 + \sin x \cdot (-1) = -\sin x$

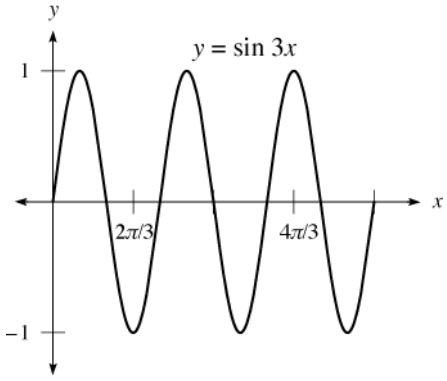
**30**. Using the difference formula for cosine: $\cos 7x \cos 4x + \sin 7x \sin 4x = \cos(7x - 4x) = \cos 3x$

**32**. Using the difference formula for sine: $\sin 8x \cos 7x - \cos 8x \sin 7x = \sin(8x - 7x) = \sin x$

**34**. Using the difference formula for cosine: $\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ = \sin(15^\circ + 75^\circ) = \sin 90^\circ = 1$

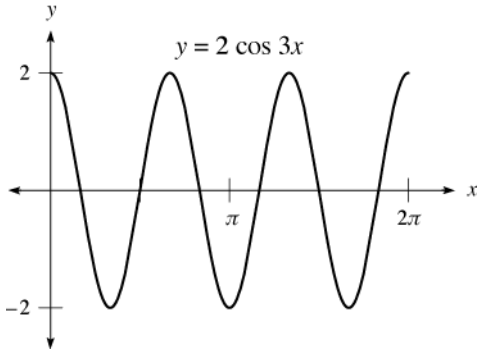
**36**. First use the sum formula for sine: $y = \sin x \cos 2x + \cos x \sin 2x = \sin(x + 2x) = \sin 3x$

The amplitude is 1 and the period is $\frac{2\pi}{3}$. Graphing the curve:




**38**. First use the difference formula for cosine: $y = 2(\cos 4x \cos x + \sin 4x \sin x) = 2\cos(4x - x) = 2\cos 3x$

The amplitude is 2 and the period is $\frac{2\pi}{3}$. Graphing the curve:



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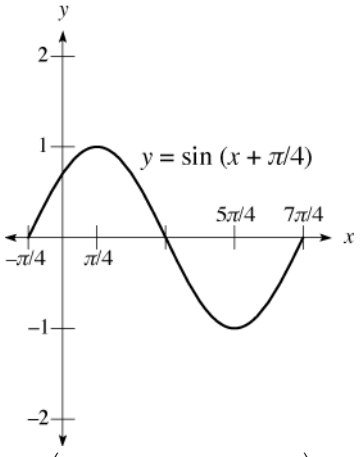
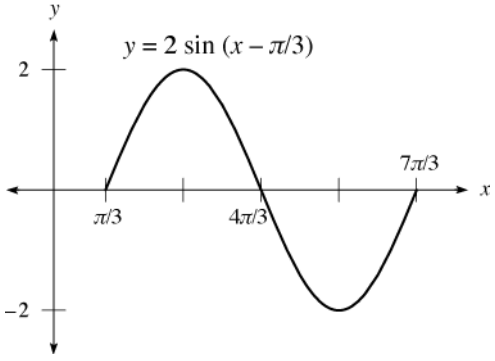
**40**. Using the sum formula for sine: $y = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = \sin\left(x + \frac{\pi}{4}\right)$

The amplitude is 1, the period is $2\pi$, and the horizontal shift is $\frac{\pi}{6}$. Graphing the curve:




**42**. Using the difference formula for sine: $y = 2\left(\sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3}\right) = 2\sin\left(x - \frac{\pi}{3}\right)$

The amplitude is 2, the period is $2\pi$, and the horizontal shift is $\frac{\pi}{3}$. Graphing the curve:




**44**. Since $\cos A = -\frac{5}{13}$ and $A$ terminates in quadrant III, $\sin A < 0$ and thus:

$$\sin A = -\sqrt{1-\cos^2 A} = -\sqrt{1-\left(-\frac{5}{13}\right)^2} = -\sqrt{1-\frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

Since $\sin B = \frac{3}{5}$ and $B$ terminates in quadrant I, $\cos B > 0$ and thus:

$$\cos B = \sqrt{1-\sin^2 B} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Now use the difference formulas for sine and cosine:

$$\sin(A-B) = \sin A\cos B - \cos A\sin B = \left(-\frac{12}{13}\right)\left(\frac{4}{5}\right) - \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right) = -\frac{48}{65} + \frac{15}{65} = -\frac{33}{65}$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B = \left(-\frac{5}{13}\right)\left(\frac{4}{5}\right) + \left(-\frac{12}{13}\right)\left(\frac{3}{5}\right) = -\frac{20}{65} - \frac{36}{65} = -\frac{56}{65}$$



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Now note that $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{-12/13}{-5/13} = \dfrac{12}{5}$ and $\tan B = \dfrac{\sin B}{\cos B} = \dfrac{3/5}{4/5} = \dfrac{3}{4}$, therefore:

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B} = \frac{\frac{12}{5} - \frac{3}{4}}{1+\left(\frac{12}{5}\right)\left(\frac{3}{4}\right)} = \frac{\frac{33}{20}}{1+\frac{36}{20}} = \frac{\frac{33}{20}}{\frac{56}{20}} = \frac{33}{56}$$

Note that we could have also found $\tan(A-B)$ directly: $\tan(A-B) = \dfrac{\sin(A-B)}{\cos(A-B)} = \dfrac{-33/65}{-56/65} = \dfrac{33}{56}$

Since $\sin(A-B) < 0$ and $\cos(A-B) < 0$, $A-B$ must terminate in quadrant III.

**46**. Since $\sec A = \sqrt{5}$, $\cos A = \dfrac{1}{\sqrt{5}}$. Since $A$ terminates in quadrant I, $\sin A > 0$ and thus:

$$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{1-\frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

Since $\sec B = \sqrt{10}$, $\cos B = \dfrac{1}{\sqrt{10}}$. Since $B$ terminates in quadrant I, $\sin B > 0$ and thus:

$$\sin B = \sqrt{1-\cos^2 B} = \sqrt{1-\left(\frac{1}{\sqrt{10}}\right)^2} = \sqrt{1-\frac{1}{10}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

Now use the difference formula for cosine:

$$\cos(A-B) = \cos A \cos B + \sin A \sin B = \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right) + \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) = \frac{1}{\sqrt{50}} + \frac{6}{\sqrt{50}} = \frac{7}{\sqrt{50}} = \frac{7}{5\sqrt{2}}$$

Therefore: $\sec(A-B) = \dfrac{1}{\cos(A-B)} = \dfrac{1}{7/5\sqrt{2}} = \dfrac{5\sqrt{2}}{7}$

**48**. Using the identity $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}$:

$$\begin{aligned}
6 &= \frac{\tan A + \frac{1}{3}}{1 - \tan A \cdot \frac{1}{3}} \\
6 &= \frac{3\tan A + 1}{3 - \tan A} \\
6(3 - \tan A) &= 3\tan A + 1 \\
18 - 6\tan A &= 3\tan A + 1 \\
17 &= 9\tan A \\
\tan A &= \frac{17}{9}
\end{aligned}$$

**50**. Using the addition formula for cosine: $\sin 2x = \sin(x+x) = \sin x \cos x + \cos x \sin x = 2\sin x \cos x$

**52**. Working from the left side:

$$\begin{aligned}
\sin(90° + x) - \sin(90° - x) &= (\sin 90° \cos x + \cos 90° \sin x) - (\sin 90° \cos x - \cos 90° \sin x) \\
&= (1 \cdot \cos x + 0 \cdot \sin x) - (1 \cdot \cos x - 0 \cdot \sin x) \\
&= \cos x - \cos x \\
&= 0
\end{aligned}$$



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**54**. Working from the left side:

$$\begin{aligned}\cos(x+90°)+\cos(x-90°) &= (\cos x\cos 90° - \sin x\sin 90°)+(\cos x\cos 90° + \sin x\sin 90°)\\ &= (\cos x\bullet 0 - \sin x\bullet 1)+(\cos x\bullet 0 + \sin x\bullet 1)\\ &= -\sin x+\sin x\\ &= 0\end{aligned}$$

**56**. Working from the left side:

$$\begin{aligned}\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right) &= \left(\cos\frac{\pi}{3}\cos x-\sin\frac{\pi}{3}\sin x\right)+\left(\cos\frac{\pi}{3}\cos x+\sin\frac{\pi}{3}\sin x\right)\\ &= \left(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x\right)+\left(\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x\right)\\ &= \frac{1}{2}\cos x+\frac{1}{2}\cos x\\ &= \cos x\end{aligned}$$

**58**. Working from the left side:

$$\begin{aligned}\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right) &= \left(\sin\frac{\pi}{4}\cos x+\cos\frac{\pi}{4}\sin x\right)+\left(\sin\frac{\pi}{4}\cos x-\cos\frac{\pi}{4}\sin x\right)\\ &= \left(\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x\right)+\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)\\ &= \frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\cos x\\ &= \sqrt{2}\cos x\end{aligned}$$

**60**. Working from the left side:

$$\begin{aligned}\sin\left(\frac{3\pi}{2}+x\right)+\sin\left(\frac{3\pi}{2}-x\right) &= \left(\sin\frac{3\pi}{2}\cos x+\cos\frac{3\pi}{2}\sin x\right)+\left(\sin\frac{3\pi}{2}\cos x-\cos\frac{3\pi}{2}\sin x\right)\\ &= (-1\bullet\cos x+0\bullet\sin x)+(-1\bullet\cos x-0\bullet\sin x)\\ &= -\cos x-\cos x\\ &= -2\cos x\end{aligned}$$

**62**. Working from the left side:

$$\begin{aligned}\cos(A+B)+\cos(A-B) &= (\cos A\cos B-\sin A\sin B)+(\cos A\cos B+\sin A\sin B)\\ &= \cos A\cos B+\cos A\cos B\\ &= 2\cos A\cos B\end{aligned}$$

**64**. Working from the left side:

$$\frac{\cos(A+B)}{\sin A\cos B}=\frac{\cos A\cos B-\sin A\sin B}{\sin A\cos B}=\frac{\cos A\cos B}{\sin A\cos B}-\frac{\sin A\sin B}{\sin A\cos B}=\frac{\cos A}{\sin A}-\frac{\sin B}{\cos B}=\cot A-\tan B$$



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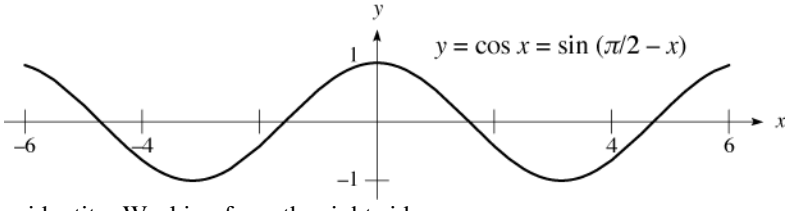
**66**. Working from the left side:

$$\begin{aligned}\sec(A-B) &= \frac{1}{\cos(A-B)} \\ &= \frac{1}{\cos A\cos B+\sin A\sin B} \\ &= \frac{1}{\cos A\cos B+\sin A\sin B}\cdot\frac{\cos A\cos B-\sin A\sin B}{\cos A\cos B-\sin A\sin B} \\ &= \frac{\cos A\cos B-\sin A\sin B}{\cos^2 A\cos^2 B-\sin^2 A\sin^2 B} \\ &= \frac{\cos(A+B)}{\cos^2 A(1-\sin^2 B)-(1-\cos^2 A)\sin^2 B} \\ &= \frac{\cos(A+B)}{\cos^2 A-\cos^2 A\sin^2 B-\sin^2 B+\cos^2 A\sin^2 B} \\ &= \frac{\cos(A+B)}{\cos^2 A-\sin^2 B}\end{aligned}$$

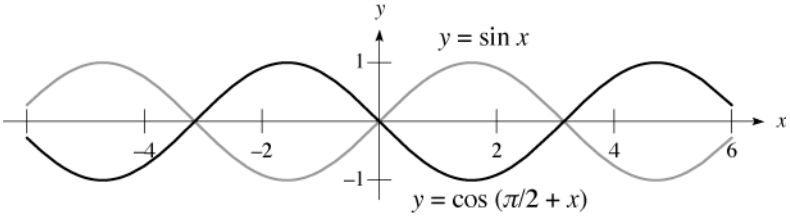
**68**. Graphing each side:




This appears to be an identity. Working from the right side:

$$\sin\left(\frac{\pi}{2}-x\right)=\sin\frac{\pi}{2}\cos x-\cos\frac{\pi}{2}\sin x=1\cdot\cos x-0\cdot\sin x=\cos x$$

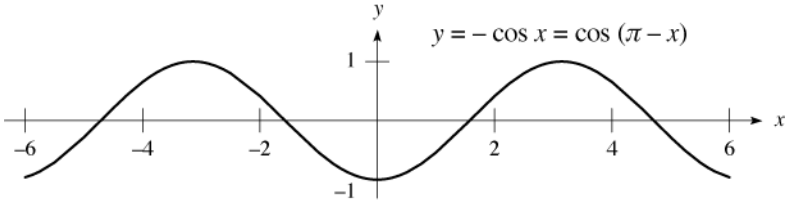
**70**. Graphing each side:




The two graphs are different, so it does not appear the equation is an identity. Evaluating each side when $x=\frac{\pi}{2}$:

$$\sin\frac{\pi}{2}=1 \qquad\qquad \cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)=\cos\pi=-1$$

**72**. Graphing each side:




This appears to be an identity. Working from the right side:

$$\cos(\pi-x)=\cos\pi\cos x+\sin\pi\sin x=-1\cdot\cos x+0\cdot\sin x=-\cos x$$



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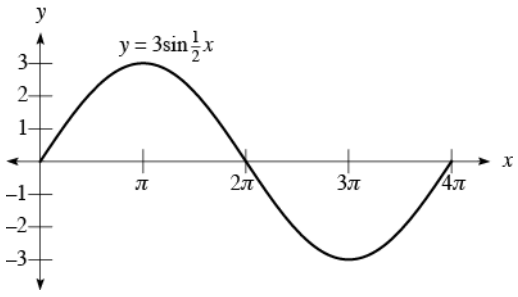
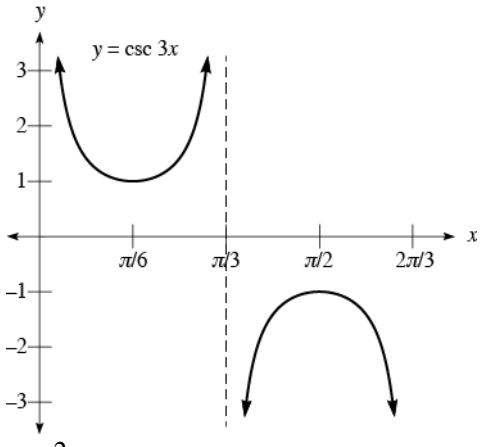
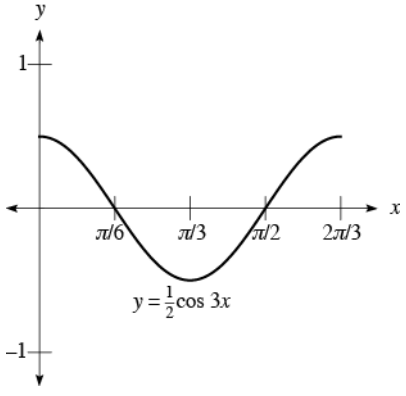
**74**. The amplitude is 3 and the period is $\dfrac{2\pi}{1/2} = 4\pi$ . Sketching the graph:




**76**. The period is $\dfrac{2\pi}{3}$ . Sketching the graph:




**78**. The amplitude is $\dfrac{1}{2}$ and the period is $\dfrac{2\pi}{3}$ . Sketching the graph:




**80**. Using the addition formula for cosine:

$$\cos 165^\circ = \cos(120^\circ + 45^\circ) = \cos 120^\circ \cos 45^\circ - \sin 120^\circ \sin 45^\circ = -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}+\sqrt{6}}{4}$$

The correct answer is a.

**82**. Using the subtraction formula for cosine: $\cos(270^\circ - x) = \cos 270^\circ \cos x + \sin 270^\circ \sin x = 0 \cdot \cos x - 1 \cdot \sin x = -\sin x$

The correct answer is c.



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**ODD SOLUTIONS**

**1.** $\sin x\cos y+\cos x\sin y$

**3.** $\cos\theta\cos\phi-\sin\theta\sin\phi$

**5.** $\dfrac{\tan C+\tan D}{1-\tan C\tan D}$

**7.** False (See formula for $\sin(A+B)$)

**9.** $\sin 15^\circ=\sin\left(45^\circ-30^\circ\right)$

$=\sin 45^\circ\cos 30^\circ-\cos 45^\circ\sin 30^\circ$

$=\left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)-\left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right)$

$=\dfrac{\sqrt{6}}{4}-\dfrac{\sqrt{2}}{4}$

$=\dfrac{\sqrt{6}-\sqrt{2}}{4}$

**11.** $\tan 15^\circ=\tan\left(45^\circ-30^\circ\right)$

$=\dfrac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}$

$=\dfrac{1-\dfrac{1}{\sqrt{3}}}{1+1\left(\dfrac{1}{\sqrt{3}}\right)}$

$=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ Multiply numerator and denominator by $\sqrt{3}$

$=\dfrac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}=\dfrac{3-2\sqrt{3}+1}{3-1}$

$=\dfrac{4-2\sqrt{3}}{2}=2-\sqrt{3}$

**13.** $\cos\dfrac{7\pi}{12}=\cos\left(\dfrac{3\pi}{12}+\dfrac{4\pi}{12}\right)$

$=\cos\left(\dfrac{\pi}{4}+\dfrac{\pi}{3}\right)$

$=\cos\dfrac{\pi}{4}\cos\dfrac{\pi}{3}-\sin\dfrac{\pi}{4}\sin\dfrac{\pi}{3}$

$=\left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right)-\left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$=\dfrac{\sqrt{2}}{4}-\dfrac{\sqrt{6}}{4}$

$=\dfrac{\sqrt{2}-\sqrt{6}}{4}$

**15.** $\cos 105^\circ=\cos\left(60^\circ+45^\circ\right)$

$=\cos 60^\circ\cos 45^\circ-\sin 60^\circ\sin 45^\circ$

$=\dfrac{1}{2}\left(\dfrac{\sqrt{2}}{2}\right)-\dfrac{\sqrt{3}}{2}\left(\dfrac{\sqrt{2}}{2}\right)$

$=\dfrac{\sqrt{2}}{4}-\dfrac{\sqrt{6}}{4}$

$=\dfrac{\sqrt{2}-\sqrt{6}}{4}$

**17.** $\sin\left(x+2\pi\right)=\sin x\cos 2\pi+\cos x\sin 2\pi$

$=\left(\sin x\right)(1)+\left(\cos x\right)(0)$

$=\sin x$

**19.** $\cos\left(x-\dfrac{\pi}{2}\right)=\cos x\cos\dfrac{\pi}{2}+\sin x\sin\dfrac{\pi}{2}$

$=\cos x(0)+\sin x(1)$

$=\sin x$

**21.** $\cos\left(180^\circ-\theta\right)=\cos 180^\circ\cos\theta+\sin 180^\circ\sin\theta$

$=-1\left(\cos\theta\right)+0\left(\sin\theta\right)$

$=-\cos\theta$

**23.** $\sin\left(90^\circ+\theta\right)=\sin 90^\circ\cos\theta+\cos 90^\circ\sin\theta$

$=1\left(\cos\theta\right)+0\left(\sin\theta\right)$

$=\cos\theta$



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**25.** $\tan\left(x+\frac{\pi}{4}\right)=\frac{\tan x+\tan\frac{\pi}{4}}{1-\tan x\tan\frac{\pi}{4}}$

$=\frac{\tan x+1}{1-(\tan x)(1)}$

$=\frac{1+\tan x}{1-\tan x}$

**27.** $\sin\left(\frac{3\pi}{2}-x\right)=\sin\frac{3\pi}{2}\cos x-\cos\frac{3\pi}{2}\sin x$

$=-1(\cos x)-0(\sin x)$

$=-\cos x$

**29.** $\sin 3x\cos 2x+\cos 3x\sin 2x=\sin(3x+2x)=\sin 5x$

**31.** $\cos 5x\cos x-\sin 5x\sin x=\cos(5x+x)=\cos 6x$

**33.** $\cos 15^\circ\cos 75^\circ-\sin 15^\circ\sin 75^\circ=\cos(15^\circ+75^\circ)=\cos 90^\circ=0$

**35.** $y=\sin 5x\cos 3x-\cos 5x\sin 3x$

$y=\sin(5x-3x)=\sin 2x$

The graph is a sine curve with amplitude of 1 and period of $\frac{2\pi}{2}=\pi$.

**37.** $y=3\cos 7x\cos 5x+3\sin 7x\sin 5x$

$=3[\cos 7x\cos 5x+\sin 7x\sin 5x]$

$y=3\cos(7x-5x)=3\cos 2x$

The graph is a cosine curve with an amplitude of 3 and a period of $\frac{2\pi}{2}=\pi$.

**39.** $y=\sin x\cos\frac{\pi}{6}-\cos x\sin\frac{\pi}{6}$

$y=\sin\left(x-\frac{\pi}{6}\right)$

The graph is a sine curve with: Amplitude = 1 Period $=2\pi$

Horizontal shift $=\frac{\pi}{6}$ Spacing $=\frac{1}{4}(2\pi)=\frac{\pi}{2}$

The 5 points on the $x$-axis we use are: $\frac{\pi}{6},\frac{2\pi}{3},\frac{7\pi}{6},\frac{5\pi}{3},\frac{13\pi}{6}$.

The 2 points on the $y$-axis we use are: $-1$ and $1$.

**41.** $y=2\left(\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}\right)$

$y=2\sin\left(x+\frac{\pi}{3}\right)$

The graph is a sine curve with: Amplitude = 2 Period $=2\pi$

Horizontal shift $=-\frac{\pi}{3}$ Spacing $=\frac{1}{4}(2\pi)=\frac{\pi}{2}$

The 5 points on the $x$-axis we use are: $-\frac{\pi}{3},\frac{\pi}{6},\frac{2\pi}{3},\frac{7\pi}{6},\frac{5\pi}{3}$.

The 2 points on the $y$-axis we use are: $2$ and $-2$.



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**43.** If $\sin A = \frac{3}{5}$ with $A$ in Q1I, then

$$\cos A = -\sqrt{1-\sin^2 A}$$

$$= -\sqrt{1-\frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$= \frac{3/5}{-4/5} = -\frac{3}{4}$$

Also, if $\sin B = -\frac{5}{13}$ $B$ in QIII, then

$$\cos B = -\sqrt{1-\frac{25}{169}}$$

$$= -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

$$\tan B = \frac{\sin B}{\cos B}$$

$$= \frac{-5/13}{-12/13} = \frac{5}{12}$$

$$\sin(A+B) = \sin A\cos B + \cos A \sin B$$

$$= \frac{3}{5}\left(-\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)$$

$$= -\frac{36}{65} + \frac{20}{65} = -\frac{16}{65}$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$= -\frac{4}{5}\left(-\frac{12}{13}\right) - \frac{3}{5}\left(-\frac{5}{13}\right)$$

$$= \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{-\frac{3}{4}+\frac{5}{12}}{1-\left(-\frac{3}{4}\right)\left(\frac{5}{12}\right)} = \frac{-\frac{1}{3}}{1+\frac{5}{16}} = \frac{-\frac{1}{3}}{\frac{21}{16}} = -\frac{16}{63}$$

The angle $(A + B)$ must terminate in QIV because the cosine is positive and the sine and tangent are negative.

**45.** If $\sin A = \frac{\sqrt{5}}{5}$ with $A$ in QI, then

$$\cos A = \sqrt{1-\sin^2 A}$$

$$= \sqrt{1-\frac{1}{5}}$$

$$= \sqrt{\frac{4}{5}}$$

$$= \frac{2\sqrt{5}}{5}$$

Also, $\tan A = \frac{\sin A}{\cos A}$

$$= \frac{\sqrt{5}/5}{2\sqrt{5}/5}$$

$$= \frac{1}{2}$$

We have $\tan A = \frac{1}{2}$ and $\tan B = \frac{3}{4}$

Therefore, $\tan(A+B) = \frac{\frac{1}{2}+\frac{3}{4}}{1-\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)}$

$$= \frac{5/4}{5/8} = 2$$

$$\cot(A+B) = \frac{1}{\tan(A+B)}$$

$$= \frac{1}{2}$$

The angle $(A + B)$ terminates in QI because its tangent is positive. (If its tangent were negative, it would terminate in QII.)



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**47.**
$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$
$$3=\frac{\tan A+\frac{1}{2}}{1-(\tan A)\left(\frac{1}{2}\right)}$$
$$\frac{3}{1}=\frac{2\tan A+1}{2-\tan A}$$
$$6-3\tan A=2\tan A+1$$
$$-5\tan A=-5$$
$$\tan A=1$$

**49.**
$$\cos 2x=\cos(x+x)$$
$$=\cos x\cos x-\sin x\sin x$$
$$=\cos^2 x-\sin^2 x$$

**51.**
$$\sin(90^\circ+x)+\sin(90^\circ-x)$$
$$=(\sin 90^\circ\cos x+\cos 90^\circ\sin x)+(\sin 90^\circ\cos x-\cos 90^\circ\sin x)$$ Sum and difference formulas
$$=2\sin 90^\circ\cos x$$ Combine
$$=2(1)\cos x$$ Substitute exact value
$$=2\cos x$$ Multiply

**53.**
$$\cos(x-90^\circ)-\cos(x+90^\circ)$$
$$=[\cos x\cos 90^\circ+\sin x\sin 90^\circ]-[\cos x\cos 90^\circ-\sin x\sin 90^\circ]$$ Sum and difference formulas
$$=[(\cos x)(0)+(\sin x)(1)]-[(\cos x)(0)-(\sin x)(1)]$$ Substitute exact values
$$=\sin x-(-\sin x)$$ Multiply
$$=2\sin x$$ Subtract

**55.**
$$\sin\left(\frac{\pi}{6}+x\right)+\sin\left(\frac{\pi}{6}-x\right)$$
$$=\left(\sin\frac{\pi}{6}\cos x+\cos\frac{\pi}{6}\sin x\right)+\left(\sin\frac{\pi}{6}\cos x-\cos\frac{\pi}{6}\sin x\right)$$ Sum and difference formulas
$$=2\sin\frac{\pi}{6}\cos x$$ Combine
$$=2\left(\frac{1}{2}\right)\cos x$$ Substitute exact values
$$=\cos x$$ Multiply

**57.**
$$\cos\left(x+\frac{\pi}{4}\right)+\cos\left(x-\frac{\pi}{4}\right)$$
$$=\left(\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}\right)+\left(\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}\right)$$ Sum and difference formulas
$$=(\cos x)\left(\frac{\sqrt{2}}{2}\right)-(\sin x)\left(\frac{\sqrt{2}}{2}\right)+(\cos x)\left(\frac{\sqrt{2}}{2}\right)+(\sin x)\left(\frac{\sqrt{2}}{2}\right)$$ Substitute exact values
$$=\sqrt{2}\cos x$$ Combine



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**59.** $\cos\left(x+\frac{3\pi}{2}\right)+\cos\left(x-\frac{3\pi}{2}\right)$

$=\left(\cos x\cos\frac{3\pi}{2}-\sin x\sin\frac{3\pi}{2}\right)+\left(\cos x\cos\frac{3\pi}{2}+\sin x\sin\frac{3\pi}{2}\right)$ Sum and difference formulas

$=\left(\cos x(0)-\sin x(-1)\right)+\left(\cos x(0)+\sin x(-1)\right)$ Substitute exact values

$=\sin x-\sin x=0$ Simplify

**61.** $\sin(A+B)+\sin(A-B)$

$=(\sin A\cos B+\cos A\sin B)+(\sin A\cos B-\cos A\sin B)$ Sum and difference formulas

$=2\sin A\cos B$ Combine

**63.** $\frac{\sin(A-B)}{\cos A\cos B}=\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}$ Difference formula

$=\frac{\sin A\cos B}{\cos A\cos B}-\frac{\cos A\sin B}{\cos A\cos B}$ Separate into 2 fractions

$=\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}$ Reduce

$=\tan A-\tan B$ Ratio identity

**65.** $\sec(A+B)$

$=\frac{1}{\cos(A+B)}$ Reciprocal identity

$=\frac{1}{\cos(A+B)}\cdot\frac{\cos(A-B)}{\cos(A-B)}$ Multiply numerator and denominator by $\cos(A-B)$

$=\frac{\cos(A-B)}{(\cos A\cos B-\sin A\sin B)(\cos A\cos B+\sin A\sin B)}$ Sum and differences formulas

$=\frac{\cos(A-B)}{\cos^2 A\cos^2 B-\sin^2 A\sin^2 B}$ Multiply

$=\frac{\cos(A-B)}{\cos^2 A\cos^2 B+\sin^2 A\cos^2 B-\sin^2 A\sin^2 B-\sin^2 A\cos^2 B}$ Add and subtract $\sin^2 A\cos^2 B$ in denominator

$=\frac{\cos(A-B)}{\cos^2 B(\cos^2 A+\sin^2 A)-\sin^2 A(\sin^2 B+\cos^2 B)}$ Factor by grouping

$=\frac{\cos(A-B)}{\cos^2 B(1)-\sin^2 A(1)}$ Pythagorean identity

$=\frac{\cos(A-B)}{\cos^2 B-\sin^2 A}$ Multiply

**67.** $\cos\left(\frac{\pi}{2}-x\right)=\cos\frac{\pi}{2}\cos x+\sin\frac{\pi}{2}\sin x$ Difference formula

$=0\cdot\cos x+1\cdot\sin x$ Evaluate functions

$=\sin x$ Multiply

**69.** Let $x=0$. Then $-\cos 0\neq\sin\left(\frac{\pi}{2}+0\right)$

$-1\neq 1$

**71.** $\cos\left(\frac{\pi}{2}+x\right)=\cos\frac{\pi}{2}\cos x-\sin\frac{\pi}{2}\sin x$ Sum formula

$=0\cdot\cos x-1\cdot\sin x$ Evaluate functions

$=-\sin x$ Multiply



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**75.** Amplitude = 2

Period $= \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

**77.** Period $= \dfrac{2\pi}{3}$

Asymptotes occur where $y = \cos 3x$ is zero, or at $\dfrac{\pi}{6}$ and $\dfrac{\pi}{2}$.

We use the graph of $y = \cos 3x$ and the asymptotes to sketch the graph of $y = \sec 3x$.

**79.** Amplitude = 2

Period $= \dfrac{2\pi}{\pi/2} = 4$

## 5.3 Double-Angle Formulas

**EVEN SOLUTIONS**

**2.** The identity is: $\cos 2\theta = \cos^2\theta - \sin^2\theta$. Other forms are $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos 2\theta = 1 - 2\sin^2\theta$.

**4.** The statement is false.

**6.** The statement is false.

**8.** Since $A$ terminates in quadrant III, $\cos A < 0$ and thus: $\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\left(-\dfrac{3}{5}\right)^2} = -\sqrt{1-\dfrac{9}{25}} = -\sqrt{\dfrac{16}{25}} = -\dfrac{4}{5}$

Using the double-angle formula for cosine: $\cos 2A = \cos^2 A - \sin^2 A = \left(-\dfrac{4}{5}\right)^2 - \left(-\dfrac{3}{5}\right)^2 = \dfrac{16}{25} - \dfrac{9}{25} = \dfrac{7}{25}$

**10.** Since $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{-3/5}{-4/5} = \dfrac{3}{4}$, use the double-angle formula for tangent:

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{3/2}{7/16} = \frac{3}{2}\cdot\frac{16}{7} = \frac{24}{7}$$

Therefore $\cot 2A = \dfrac{1}{\tan 2A} = \dfrac{1}{24/7} = \dfrac{7}{24}$.

**12.** Since $x$ terminates in quadrant IV, $\sin x < 0$ and thus:

$$\sin x = -\sqrt{1-\cos^2 x} = -\sqrt{1-\left(\frac{3}{4}\right)^2} = -\sqrt{1-\frac{9}{16}} = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4}$$

Using the double-angle formula for sine: $\sin 2x = 2\sin x\cos x = 2\left(-\dfrac{\sqrt{7}}{4}\right)\left(\dfrac{3}{4}\right) = -\dfrac{3\sqrt{7}}{8}$

**14.** Since $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{-\sqrt{7}/4}{3/4} = -\dfrac{\sqrt{7}}{3}$, use the double-angle formula for tangent:

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot\left(-\frac{\sqrt{7}}{3}\right)}{1-\left(-\frac{\sqrt{7}}{3}\right)^2} = \frac{-\frac{2\sqrt{7}}{3}}{1-\frac{7}{9}} = \frac{-\frac{2\sqrt{7}}{3}}{\frac{2}{9}} = -3\sqrt{7}$$



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**16**. Since $\theta$ terminates in quadrant I, $\sec\theta > 0$ and thus: $\sec\theta = \sqrt{\tan^2\theta + 1} = \sqrt{3^2 + 1} = \sqrt{9+1} = \sqrt{10}$

So $\cos\theta = \dfrac{1}{\sqrt{10}}$, therefore: $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\dfrac{1}{\sqrt{10}}\right)^2} = \sqrt{1-\dfrac{1}{10}} = \sqrt{\dfrac{9}{10}} = \dfrac{3}{\sqrt{10}}$

Using the double-angle formula for cosine: $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\dfrac{1}{\sqrt{10}}\right)^2 - \left(\dfrac{3}{\sqrt{10}}\right)^2 = \dfrac{1}{10} - \dfrac{9}{10} = -\dfrac{8}{10} = -\dfrac{4}{5}$

**18**. Since, from Problem 16, $\cos 2\theta = -\dfrac{4}{5}$: $\sin 2\theta = \sqrt{1-\cos^2 2\theta} = \sqrt{1-\left(-\dfrac{4}{5}\right)^2} = \sqrt{1-\dfrac{16}{25}} = \sqrt{\dfrac{9}{25}} = \dfrac{3}{5}$

Therefore: $\csc 2\theta = \dfrac{1}{\sin 2\theta} = \dfrac{1}{3/5} = \dfrac{5}{3}$

**20**. Since $\csc t = \sqrt{5}$, $\sin t = \dfrac{1}{\sqrt{5}}$. Since $t$ terminates in quadrant II, $\cos t < 0$ and thus:

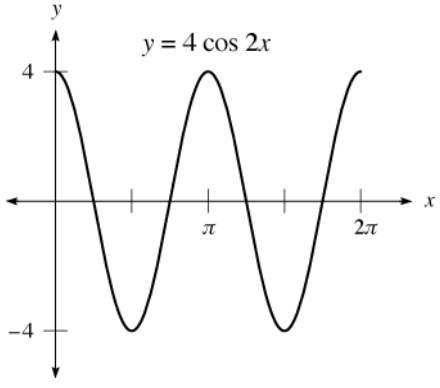
$$\cos t = -\sqrt{1-\sin^2 t} = -\sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2} = -\sqrt{1-\frac{1}{5}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}}$$

Using the double-angle formula for sine: $\sin 2t = 2\sin t\cos t = 2\left(\dfrac{1}{\sqrt{5}}\right)\left(-\dfrac{2}{\sqrt{5}}\right) = -\dfrac{4}{5}$

**22**. Since, from Problem 20, $\sin 2t = -\dfrac{4}{5}$, $\csc 2t = \dfrac{1}{\sin 2t} = \dfrac{1}{-4/5} = -\dfrac{5}{4}$.

**24**. Using the double-angle formula for cosine: $y = 4 - 8\sin^2 x = 4\left(1 - 2\sin^2 x\right) = 4\cos 2x$

The amplitude is 4 and the period is $\dfrac{2\pi}{2} = \pi$. Sketching the graph:



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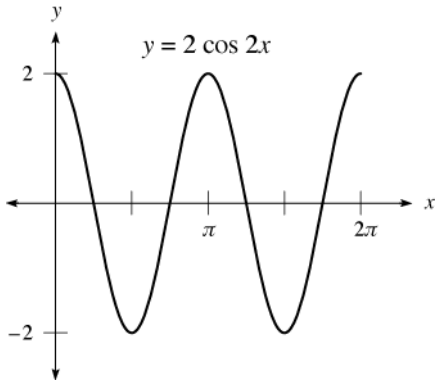
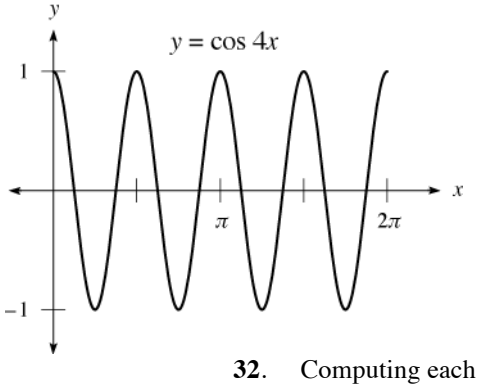
**26**. Using the double-angle formula for cosine: $y = 4\cos^2 x - 2 = 2\left(2\cos^2 x - 1\right) = 2\cos 2x$

The amplitude is 2 and the period is $\frac{2\pi}{2} = \pi$. Sketching the graph:




**28**. Using the double-angle formula for cosine: $y = 2\cos^2 2x - 1 = \cos 4x$

The amplitude is 1 and the period is $\frac{2\pi}{4} = \frac{\pi}{2}$. Sketching the graph:




**30**. Computing each side:

$\cos 90° = 0$

$1 - 2\sin^2 45° = 1 - 2\left(\frac{\sqrt{2}}{2}\right)^2 = 1 - 2 \cdot \frac{1}{2} = 1 - 2 = 0$

Therefore $\cos 90° = 1 - 2\sin^2 45°$.

**32**. Computing each side:

$\sin 300° = -\frac{\sqrt{3}}{2}$

$2\sin 150°\cos 150° = 2\left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$

Therefore $\sin 300° = 2\sin 150°\cos 150°$.

**34**. Using the double-angle formula for tangent: $\tan 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{2\left(\frac{5}{12}\right)}{1 - \left(\frac{5}{12}\right)^2} = \frac{\frac{5}{6}}{1 - \frac{25}{144}} = \frac{5/6}{119/144} = \frac{5}{6} \cdot \frac{144}{119} = \frac{120}{119}$

**36**. Using the double-angle formula for cosine: $\cos^2 165° - \sin^2 165° = \cos\left(2 \cdot 165°\right) = \cos 330° = \frac{\sqrt{3}}{2}$

**38**. Using the double-angle formula for cosine: $2\cos^2 105° - 1 = \cos\left(2 \cdot 105°\right) = \cos 210° = -\frac{\sqrt{3}}{2}$

**40**. Using the double-angle formula for sine: $\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \frac{1}{2}\sin\left(2 \cdot \frac{\pi}{8}\right) = \frac{1}{2}\sin\frac{\pi}{4} = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$

**42**. Using the double-angle formula for tangent: $\frac{\tan 112.5°}{1 - \tan^2 112.5°} = \frac{1}{2}\tan\left(2 \cdot 112.5°\right) = \frac{1}{2}\tan 225° = \frac{1}{2}(1) = \frac{1}{2}$



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**44**. Working from the left side:

$$(\cos x - \sin x)(\cos x + \sin x) = \cos^2 x - \sin x \cos x + \sin x \cos x - \sin^2 x = \cos^2 x - \sin^2 x = \cos 2x$$

**46**. Working from the right side: $\dfrac{1-\cos 2\theta}{2} = \dfrac{1-\left(1-2\sin^2\theta\right)}{2} = \dfrac{1-1+2\sin^2\theta}{2} = \dfrac{2\sin^2\theta}{2} = \sin^2\theta$

**48**. Working from the right side: $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}} = \dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta} = \dfrac{\cos 2\theta}{1} = \cos 2\theta$

**50**. Working from the right side: $\cot x - \tan x = \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x} = \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \dfrac{\cos 2x}{\frac{1}{2}\sin 2x} = 2 \bullet \dfrac{\cos 2x}{\sin 2x} = 2\cot 2x$

**52**. Working from the left side:

$$\begin{aligned}
\cos 3\theta &= \cos(\theta + 2\theta) \\
&= \cos\theta\cos 2\theta - \sin\theta\sin 2\theta \\
&= \cos\theta\left(2\cos^2\theta - 1\right) - \sin\theta \bullet 2\sin\theta\cos\theta \\
&= 2\cos^3\theta - \cos\theta - 2\cos\theta\left(1-\cos^2\theta\right) \\
&= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \\
&= 4\cos^3\theta - 3\cos\theta
\end{aligned}$$

**54**. Working from the left side: $2\sin^4 x + 2\sin^2 x\cos^2 x = 2\sin^2 x\left(\sin^2 x + \cos^2 x\right) = 2\sin^2 x$

Working from the right side: $1-\cos 2x = 1-\left(1-2\sin^2 x\right) = 1-1+2\sin^2 x = 2\sin^2 x$

Since both sides simplify to the same expression, they are equal.

**56**. Working from the left side: $\csc\theta - 2\sin\theta = \dfrac{1}{\sin\theta} - 2\sin\theta = \dfrac{1-2\sin^2\theta}{\sin\theta} = \dfrac{\cos 2\theta}{\sin\theta}$

**58**. Working from the left side:

$$\begin{aligned}
\cos 4A &= \cos(2A + 2A) \\
&= \cos 2A\cos 2A - \sin 2A\sin 2A \\
&= \left(\cos^2 A - \sin^2 A\right)\left(\cos^2 A - \sin^2 A\right) - (2\sin A\cos A)(2\sin A\cos A) \\
&= \cos^4 A - 2\sin^2 A\cos^2 A + \sin^4 A - 4\sin^2 A\cos^2 A \\
&= \cos^4 A - 6\sin^2 A\cos^2 A + \sin^4 A
\end{aligned}$$



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**60**. Working from the right side:

$$\sec x \csc x - \cot x + \tan x = \frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$$

$$= \frac{1}{\sin x \cos x} - \frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\sin x \cos x}$$

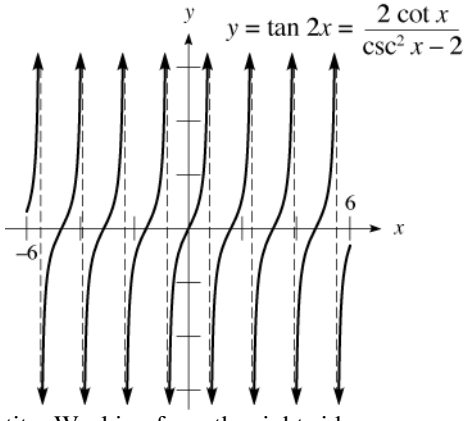
$$= \frac{1 - \cos^2 x + \sin^2 x}{\sin x \cos x}$$

$$= \frac{1 - \left(\cos^2 x - \sin^2 x\right)}{\sin x \cos x}$$

$$= \frac{1 - \cos 2x}{\frac{1}{2}\sin 2x}$$

$$= \frac{2 - 2\cos 2x}{\sin 2x}$$

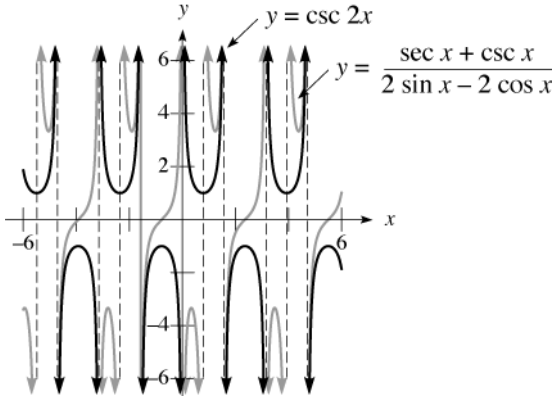
**62**. Graphing the two curves:




The equation appears to be an identity. Working from the right side:

$$\frac{2\cot x}{\csc^2 x - 2} = \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x} - 2} \cdot \frac{\sin^2 x}{\sin^2 x} = \frac{2\sin x \cos x}{1 - 2\sin^2 x} = \frac{\sin 2x}{\cos 2x} = \tan 2x$$

**64**. Graphing the two curves:



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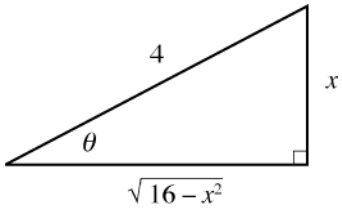
The equation does not appear to be an identity. Substituting $x = \frac{\pi}{6}$ into each expression:

$$\csc\left(2 \cdot \frac{\pi}{6}\right) = \csc\frac{\pi}{3} = \frac{2}{\sqrt{3}} \qquad \frac{\sec\frac{\pi}{6} + \csc\frac{\pi}{6}}{2\sin\frac{\pi}{6} - 2\cos\frac{\pi}{6}} = \frac{\frac{2}{\sqrt{3}} + 2}{2 \cdot \frac{1}{2} - 2 \cdot \frac{\sqrt{3}}{2}} = \frac{\frac{2}{\sqrt{3}} + 2}{1 - \sqrt{3}} = \frac{2 + 2\sqrt{3}}{\sqrt{3} - 3}$$

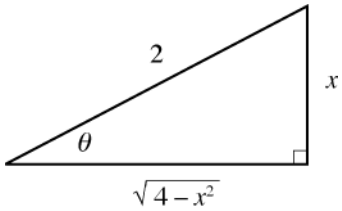
**66**. Since $x = 4\sin\theta$, $\sin\theta = \frac{x}{4}$ and thus $\theta = \sin^{-1}\frac{x}{4}$. Draw the triangle:




So $\cos\theta = \frac{\sqrt{16 - x^2}}{4}$. Thus the expression becomes:

$$8\theta + 4\sin 2\theta = 8\theta + 4\left(2\sin\theta\cos\theta\right) = 8\theta + 8\sin\theta\cos\theta = 8\sin^{-1}\frac{x}{4} + 8 \cdot \frac{x}{4} \cdot \frac{\sqrt{16 - x^2}}{4} = 8\sin^{-1}\frac{x}{4} + \frac{1}{2}x\sqrt{16 - x^2}$$

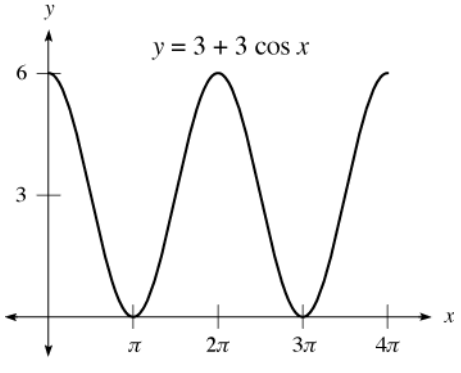
**68**. Since $x = 2\sin\theta$, $\sin\theta = \frac{x}{2}$ and thus $\theta = \sin^{-1}\frac{x}{2}$. Draw the triangle:



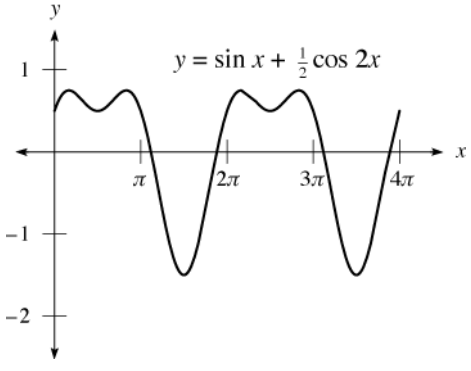

So $\tan\theta = \frac{x}{\sqrt{4 - x^2}}$. Thus the expression becomes:

$$2\theta - \tan 2\theta = 2\theta - \frac{2\tan\theta}{1 - \tan^2\theta} = 2\sin^{-1}\frac{x}{2} - \frac{2 \cdot \frac{x}{\sqrt{4 - x^2}}}{1 - \frac{x^2}{4 - x^2}} = 2\sin^{-1}\frac{x}{2} - \frac{2x\sqrt{4 - x^2}}{4 - 2x^2} = 2\sin^{-1}\frac{x}{2} - \frac{x\sqrt{4 - x^2}}{2 - x^2}$$

**70**. Graphing the curve:




**72**. Graphing the curve:



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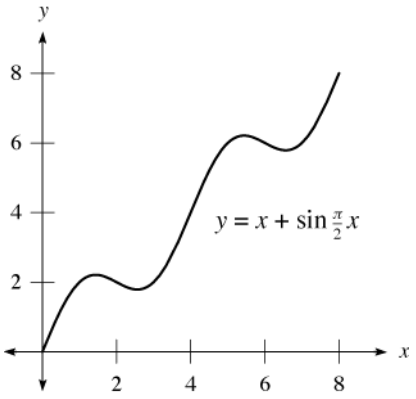
**74**. Graphing the curve:




**76**. Using the double-angle identity for cosine: $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12} = \cos\left(2 \cdot \frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$. The correct answer is a.

**78**. Working from the left side: $\dfrac{2\tan x}{1+\tan^2 x} = \dfrac{\dfrac{2\sin x}{\cos x}}{1+\dfrac{\sin^2 x}{\cos^2 x}} \cdot \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{2\sin x\cos x}{\cos^2 x+\sin^2 x} = 2\sin x\cos x = \sin 2x$

The correct answer is d.

**ODD SOLUTIONS**

**1.** $2\sin x\cos x$

**3.** $\dfrac{2\tan y}{1-\tan^2 y}$

**5.** False (See formula for $\cos 2A$)

**7.** If $\sin A = -\frac{3}{5}$ with $A$ in QIII, then

$$\cos A = -\sqrt{1-\sin^2 A}$$
$$= -\sqrt{1-\frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

Therefore,
$$\sin 2A = 2\sin A\cos A$$
$$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

**9.** $$\cos 2A = \cos^2 A - \sin^2 A$$
$$= \left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$
$$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{24/25}{7/25} = \frac{24}{7}$$



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**11.** If $\cos x = \frac{3}{4}$ with $x$ in QIV, then

$$\cos 2x = 2\cos^2 x - 1$$

$$= 2\left(\frac{3}{4}\right)^2 - 1$$

$$= 2\left(\frac{9}{16}\right) - 1$$

$$= \frac{9}{8} - 1 = \frac{1}{8}$$

**13.** $\sin 2x = -\sqrt{1-\cos^2(2x)}$

$$= -\sqrt{1-\left(\frac{1}{64}\right)} = -\sqrt{\frac{63}{64}}$$

$$= -\frac{3\sqrt{7}}{8}$$

$$\cot 2x = \frac{\cos 2x}{\sin 2x}$$

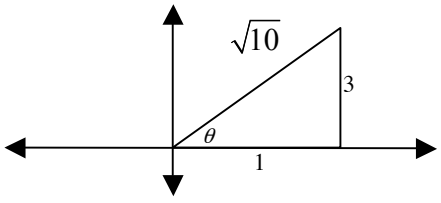
$$= \frac{1/8}{-3\sqrt{7}/8} = -\frac{1}{3\sqrt{7}} = -\frac{\sqrt{7}}{21}$$

**15.** If $\tan\theta = \frac{3}{1}$ with $\theta$ in QI, we can draw the triangle at the right and find the hypotenuse using the Pythagorean Theorem.

$$\text{hypotenuse} = \sqrt{1^2 + 3^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$




Then, $\sin\theta = \frac{3}{\sqrt{10}}$ and $\cos\theta = \frac{1}{\sqrt{10}}$.

Therefore, $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) = \frac{6}{10} = \frac{3}{5}$$

**17.** From problem 15, we know that $\sin\theta = \frac{3}{\sqrt{10}}$ and $\cos\theta = \frac{1}{\sqrt{10}}$.

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{1}{\sqrt{10}}\right)^2 - \left(\frac{3}{\sqrt{10}}\right)^2 = \frac{1-9}{10} = \frac{-8}{10} = -\frac{4}{5}$$

$$\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{1}{-4/5} = -\frac{5}{4}$$

**19.** If $\csc t = \sqrt{5}$ with $t$ in QII, then $\sin t = \frac{1}{\sqrt{5}}$.

Therefore, $\cos 2t = 1 - 2\sin^2 t$

$$= 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2$$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$

**21.** $\sec 2t = \frac{1}{\cos 2t}$

$$= \frac{1}{3/5} = \frac{5}{3}$$

**23.** $y = 2 - 4\sin^2 x$

$$= 2\left(1 - 2\sin^2 x\right)$$

$$= 2\cos 2x$$

The graph is a cosine curve with amplitude $= 2$ and period $= \frac{2\pi}{2} = \pi$.

**25.** $y = 6\cos^2 x - 3$

$$= 3\left(2\cos^2 x - 1\right)$$

$$= 3\cos 2x$$

The graph is a cosine curve with amplitude $= 3$ and period $= \frac{2\pi}{2} = \pi$.



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**27.** $y = 1 - 2\sin^2 2x$

$= \cos 2(2x)$

$= \cos 4x$

The graph is a cosine curve with amplitude $= 1$ and period $= \frac{2\pi}{4} = \frac{\pi}{2}$.

**29.** $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$2\sin 30^\circ \cos 30^\circ = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{2}$

Therefore, they are equal.

**31.** $\cos 120^\circ = -\cos 60^\circ$

$= -\frac{1}{2}$

$\cos^2 60^\circ - \sin^2 60^\circ = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} - \frac{3}{4}$

$= -\frac{1}{2}$

Therefore, they are equal.

**33.** $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

$= \frac{2\left(-\sqrt{3}\right)}{1 - \left(-\sqrt{3}\right)^2} = \frac{-2\sqrt{3}}{1-3}$

$= \frac{-2\sqrt{3}}{-2} = \sqrt{3}$

**35.** $2\sin 15^\circ \cos 15^\circ = \sin 2\left(15^\circ\right) = \sin 30^\circ = \frac{1}{2}$

**37.** $1 - 2\sin^2 75^\circ = \cos 2\left(75^\circ\right)$

$= \cos 150^\circ$

$= -\cos 30^\circ$

$= -\frac{\sqrt{3}}{2}$

**39.** $\sin\frac{\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}\left(2\sin\frac{\pi}{12}\cos\frac{\pi}{12}\right)$

$= \frac{1}{2}\sin 2\left(\frac{\pi}{12}\right)$

$= \frac{1}{2}\sin\frac{\pi}{6}$

$= \frac{1}{2}\left(\frac{1}{2}\right)$

$= \frac{1}{4}$

**41.** $\frac{\tan 22.5^\circ}{1 - \tan^2 22.5^\circ} = \frac{1}{2}\tan 2\left(22.5^\circ\right)$

$= \frac{1}{2}\tan 45^\circ$

$= \frac{1}{2}(1) = \frac{1}{2}$

**43.** $\left(\sin x - \cos x\right)^2 = \sin^2 x - 2\sin x\cos x + \cos^2 x$ Expand

$= \left(\sin^2 x + \cos^2 x\right) - 2\sin x\cos x$ Commutative property

$= 1 - 2\sin x\cos x$ Pythagorean identity

$= 1 - \sin 2x$ Double-angle identity



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**45.** $\dfrac{1+\cos 2\theta}{2} = \dfrac{1+2\cos^2\theta - 1}{2}$ Double-angle identity

$= \dfrac{2\cos^2\theta}{2}$ Combine

$= \cos^2\theta$ Reduce

**47.** $\dfrac{\sin 2\theta}{1-\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{1-\left(1-2\sin^2\theta\right)}$ Double-angle identities

$= \dfrac{2\sin\theta\cos\theta}{2\sin^2\theta}$ Subtract

$= \dfrac{\cos\theta}{\sin\theta}$ Reduce

$= \cot\theta$ Ratio identity

**49.** $\tan x + \cot x = \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$ Ratio identities

$= \dfrac{\sin x}{\cos x}\cdot\dfrac{\sin x}{\sin x} + \dfrac{\cos x}{\sin x}\cdot\dfrac{\cos x}{\cos x}$ LCD is $\sin x \cos x$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$ Multiply and add fractions

$= \dfrac{1}{\sin x \cos x}$ Pythagorean identity

$= \dfrac{1}{\frac{1}{2}\sin 2x}$ Double-angle identity

$= 2\csc 2x$ Reciprocal identity

**51.** $\sin 3\theta = \sin\left(2\theta + \theta\right)$ Addition

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$ Sum formula

$= \left(2\sin\theta\cos\theta\right)\cos\theta + \left(1-2\sin^2\theta\right)\sin\theta$ Double-angle identities

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$ Multiply

$= 2\sin\theta\left(1-\sin^2\theta\right) + \sin\theta - 2\sin^3\theta$ Pythagorean identity

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ Multiply

$= 3\sin\theta - 4\sin^3\theta$ Combine

**53.** $\cos^4 x - \sin^4 x = \left(\cos^2 x + \sin^2 x\right)\left(\cos^2 x - \sin^2 x\right)$ Factor

$= 1\left(\cos^2 x - \sin^2 x\right)$ Pythagorean identity

$= \cos 2x$ Double-angle identity

**55.** $\dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$ Double-angle identity

$= \dfrac{\cos^2\theta}{\sin\theta\cos\theta} - \dfrac{\sin^2\theta}{\sin\theta\cos\theta}$ Separate into 2 fractions

$= \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$ Reduce

$= \cot\theta - \tan\theta$ Ratio identities

**57.** $\sin 4A = 2\sin 2A\cos 2A$ Double-angle identity

$= 2\left(2\sin A\cos A\right)\left(\cos^2 A - \sin^2 A\right)$ Double-angle identities

$= 4\sin A\cos^3 A - 4\sin^3 A\cos A$ Distributive property



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**59.**
$$\frac{1-\tan x}{1+\tan x}=\frac{1-\dfrac{\sin x}{\cos x}}{1+\dfrac{\sin x}{\cos x}}$$ Ratio identity

$$=\frac{\cos x\left(1-\dfrac{\sin x}{\cos x}\right)}{\cos x\left(1+\dfrac{\sin x}{\cos x}\right)}$$ Multiply numerator and denominator by LCD

$$=\frac{\cos x-\sin x}{\cos x+\sin x}$$ Distributive property

$$=\frac{\cos x-\sin x}{\cos x+\sin x}\cdot\frac{\cos x-\sin x}{\cos x-\sin x}$$ Multiply by a fraction equal to 1

$$=\frac{\cos^2 x-2\sin x\cos x+\sin^2 x}{\cos^2 x-\sin^2 x}$$ Multiply fractions

$$=\frac{1-2\sin x\cos x}{\cos^2 x-\sin^2 x}$$ Pythagorean identity

$$=\frac{1-\sin 2x}{\cos 2x}$$ Double-angle identities

**61.**
$$\cot 2x=\frac{\cos 2x}{\sin 2x}$$ Ratio identity

$$=\frac{\cos^2 x-\sin^2 x}{\sin 2x}$$ Double-angle identity

$$=\frac{\dfrac{\cos^2 x}{\cos x}-\dfrac{\sin x}{\cos x}(\sin x)}{\sin 2x\left(\dfrac{1}{\cos x}\right)}$$ Multiply numerator and denominator by $\dfrac{1}{\cos x}$

$$=\frac{\cos x-\sin x\tan x}{\sin 2x\sec x}$$ Ratio identity and reciprocal identity

**63.** Let $x=\dfrac{\pi}{6}$, $\sec\left(2\left(\dfrac{\pi}{6}\right)\right)\neq\dfrac{\sec^2(\pi/6)\csc^2(\pi/6)}{\csc^2(\pi/6)+\sec^2(\pi/6)}$

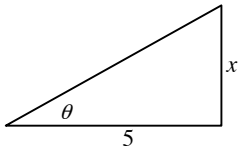
$$\sec\frac{\pi}{3}\neq\frac{\left(2\sqrt{3}/3\right)^2(2)^2}{(2)^2+\left(2\sqrt{3}/3\right)^2}$$

$$\frac{2\sqrt{3}}{3}\neq\frac{16/3}{16/3}\quad\text{or}\quad\frac{2\sqrt{3}}{3}\neq 1$$

**65.** First, we find $\theta$: $x=5\tan\theta$

$$\tan\theta=\frac{x}{5}$$

$$\theta=\tan^{-1}\frac{x}{5}$$




Next, we find $\sin 2\theta$ by drawing the triangle at the right and using the Pythagorean Theorem:

hypotenuse $=\sqrt{x^2+5^2}$

$=\sqrt{x^2+25}$

Then $\sin 2\theta=2\sin\theta\cos\theta$

$$=2\left(\frac{x}{\sqrt{x^2+25}}\right)\left(\frac{5}{\sqrt{x^2+25}}\right)$$

$$=\frac{10x}{x^2+25}$$



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Last, we evaluate: $\dfrac{\theta}{2}-\dfrac{\sin 2\theta}{4}=\dfrac{1}{2}\tan^{-1}\dfrac{x}{5}-\dfrac{1}{4}\left(\dfrac{10x}{x^2+25}\right)$

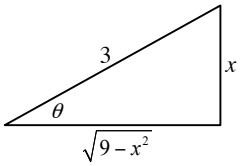
$$=\frac{1}{2}\tan^{-1}\frac{x}{5}-\frac{1}{2}\left(\frac{5x}{x^2+25}\right)$$

$$=\frac{1}{2}\left(\tan^{-1}\frac{x}{5}-\frac{5x}{x^2+25}\right)$$

**67.** First we find $\theta$: $x=3\sin\theta$

$$\sin\theta=\frac{x}{3}$$

$$\theta=\sin^{-1}\frac{x}{3}$$




Next, we find $\sin 2\theta$ by drawing the triangle at the right and using the Pythagorean Theorem:

adjacent side $=\sqrt{3^2-x^2}$

$=\sqrt{9-x^2}$

Then, $\sin 2\theta=2\sin\theta\cos\theta$

$$=2\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right)$$

$$=\frac{2}{9}x\sqrt{9-x^2}$$

Last, we evaluate: $\dfrac{\theta}{2}-\dfrac{\sin 2\theta}{4}=\dfrac{1}{2}\sin^{-1}\dfrac{x}{3}-\dfrac{1}{4}\left(\dfrac{2}{9}x\sqrt{9-x^2}\right)$

$$=\frac{1}{2}\sin^{-1}\frac{x}{3}-\frac{1}{18}x\sqrt{9-x^2}$$

$$=\frac{1}{2}\left(\sin^{-1}\frac{x}{3}-\frac{x\sqrt{9-x^2}}{9}\right)$$

**69.** Let $y_1=2$ (a horizontal line) and $y_2=-2\cos x$ (a cosine curve reflected about the $x$-axis with amplitude of 2). Graph $y_1, y_2$ and $y=y_1+y_2$ on the same coordinate system.

**71.** Let $y_1=\cos x$ and $y_2=\dfrac{1}{2}\sin 2x$ (a sine curve with amplitude of $\dfrac{1}{2}$ and a period of $\dfrac{2\pi}{2}$ or $\pi$). Graph $y_1, y_2$ and $y=y_1+y_2$ on the same coordinate system.

**73.** Let $y_1=\dfrac{1}{2}x$ (a line) and $y_2=\sin\pi x$ (a sine curve with amplitude of 1 and period $=\dfrac{2\pi}{\pi}=2$). Graph $y_1, y_2$ and $y=y_1+y_2$ on the same coordinate system



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## 5.4 Half-Angle Formulas

**EVEN SOLUTIONS**

**2.** The half-angle formula for $\cos\frac{A}{2}$ allows us to find the cosine of half the angle if we know the cosine of the angle.

**4.** The identity is: $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

**6.** The statement is false.

**8.** The statement is false.

**10.** If $90° < A < 180°$, then $45° < \frac{A}{2} < 90°$, thus $\frac{A}{2}$ terminates in quadrant I.

**12.** If $270° < A < 360°$, then $135° < \frac{A}{2} < 180°$, thus $\frac{A}{2}$ terminates in quadrant II.

**14.** If $180° < A < 270°$, then $90° < \frac{A}{2} < 135°$, thus $\frac{A}{2}$ terminates in quadrant II. So $\sin\frac{A}{2}$ is positive.

**16.** False. For example, $\cos 120° = -\frac{1}{2}$ while $\cos 60° = \frac{1}{2}$.

**18.** Using the half-angle formula for tangent: $\tan 165° = \frac{1-\cos 330°}{\sin 330°} = \frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -2+\sqrt{3}$

**20.** Using the half-angle formula for cosine (note that $\cos 75° > 0$):

$$\sin 75° = \sqrt{\frac{1-\cos 150°}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

**22.** Using the half-angle formula for sine (note that $\sin 105° > 0$):

$$\cos 105° = -\sqrt{\frac{1+\cos 210°}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$$

Note the negative value since 105° lies in the second quadrant.

**24.** Since $270° < A < 360°$, $135° < \frac{A}{2} < 180°$. Thus $\frac{A}{2}$ terminates in quadrant II, so $\cos\frac{A}{2} < 0$ and therefore:

$$\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}} = -\sqrt{\frac{1+\frac{1}{2}}{2}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$

**26.** Since $270° < A < 360°$, $135° < \frac{A}{2} < 180°$. Thus $\frac{A}{2}$ terminates in quadrant II, so $\sin\frac{A}{2} > 0$ and therefore:

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Then: $\csc\frac{A}{2} = \frac{1}{\sin\frac{A}{2}} = \frac{1}{1/2} = 2$



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**28**. Since $A$ terminates in quadrant III, $\cos A < 0$ and thus:

$$\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\left(\frac{5}{13}\right)^2} = -\sqrt{1-\frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

Since $180° < A < 270°$, $90° < \frac{A}{2} < 135°$. Thus $\frac{A}{2}$ terminates in quadrant II, so $\sin\frac{A}{2} > 0$ and therefore:

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\left(-\frac{12}{13}\right)}{2}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$$

**30**. Since, from Problem 28, $\sin\frac{A}{2} = \frac{5}{\sqrt{26}}$, $\csc\frac{A}{2} = \frac{1}{\sin\frac{A}{2}} = \frac{1}{5/\sqrt{26}} = \frac{\sqrt{26}}{5}$.

**32**. Since $B$ terminates in quadrant III, $\cos B < 0$ and thus:

$$\cos B = -\sqrt{1-\sin^2 B} = -\sqrt{1-\left(-\frac{1}{3}\right)^2} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$$

Since $180° < B < 270°$, $90° < \frac{B}{2} < 135°$. Thus $\frac{B}{2}$ terminates in quadrant II, so $\sin\frac{B}{2} > 0$ and therefore:

$$\sin\frac{B}{2} = \sqrt{\frac{1-\cos B}{2}} = \sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}} = \sqrt{\frac{3+2\sqrt{2}}{6}}$$

Thus $\csc\frac{B}{2} = \frac{1}{\sin\frac{B}{2}} = \frac{1}{\sqrt{\frac{3+2\sqrt{2}}{6}}} = \sqrt{\frac{6}{3+2\sqrt{2}}}$.

**34**. First find $\cos\frac{B}{2}$ (note that $\frac{B}{2}$ terminates in quadrant II, thus $\cos\frac{B}{2} < 0$):

$$\cos\frac{B}{2} = -\sqrt{\frac{1+\cos B}{2}} = -\sqrt{\frac{1-\frac{2\sqrt{2}}{3}}{2}} = -\sqrt{\frac{3-2\sqrt{2}}{6}}$$

Thus $\sec\frac{B}{2} = \frac{1}{\cos\frac{B}{2}} = \frac{1}{-\sqrt{\frac{3-2\sqrt{2}}{6}}} = -\sqrt{\frac{6}{3-2\sqrt{2}}}$.

**36**. Using the results from Problems 32 and 34:

$$\tan\frac{B}{2} = \frac{\sin\frac{B}{2}}{\cos\frac{B}{2}} = -\frac{\sqrt{\frac{3+2\sqrt{2}}{6}}}{\sqrt{\frac{3-2\sqrt{2}}{6}}} = -\sqrt{\frac{3+2\sqrt{2}}{3-2\sqrt{2}}} \cdot \frac{\sqrt{3-2\sqrt{2}}}{\sqrt{3-2\sqrt{2}}} = -\frac{1}{3-2\sqrt{2}} \cdot \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = -\frac{3+2\sqrt{2}}{1} = -3-2\sqrt{2}$$

**38**. Since $A$ terminates in quadrant II, $\cos A < 0$ and thus: $\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\left(\frac{4}{5}\right)^2} = -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$

Also note that $\frac{A}{2}$ terminates in quadrant I, thus $\sin\frac{A}{2} > 0$ and therefore:

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$



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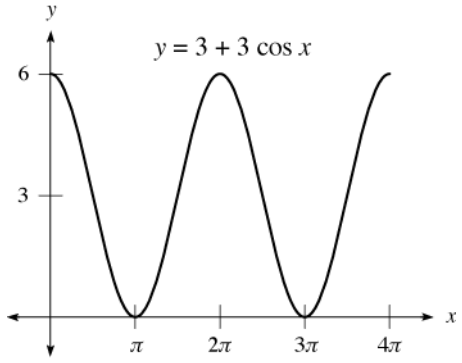
**40**. Since $B$ terminates in quadrant I, $\cos B > 0$ and thus: $\cos B = \sqrt{1-\sin^2 B} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Note that $\frac{B}{2}$ terminates in quadrant I, thus $\cos\frac{B}{2} > 0$ and therefore:

$$\cos\frac{B}{2} = \sqrt{\frac{1+\cos B}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

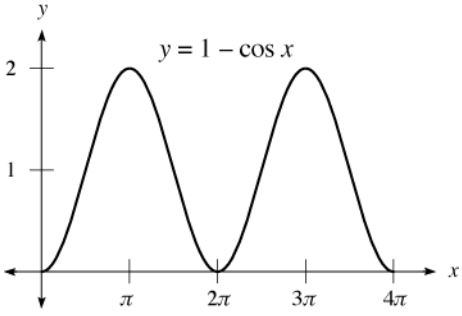
**42**. Using the half-angle formula for cosine: $y = 6\cos^2\frac{x}{2} = 6\left(\frac{1+\cos x}{2}\right) = 3(1+\cos x) = 3+3\cos x$

The amplitude is 3, the period is $2\pi$, and the vertical translation is 3 units. Now graphing the curve:




**44**. Using the half-angle formula for cosine: $y = 2\sin^2\frac{x}{2} = 2\left(\frac{1-\cos x}{2}\right) = 1-\cos x$

Now graphing the curve:




**46**. Working from the left side: $2\cos^2\frac{\theta}{2} = 2\left(\frac{1+\cos\theta}{2}\right) = 1+\cos\theta$

Working from the right side: $\frac{\sin^2\theta}{1-\cos\theta} = \frac{1-\cos^2\theta}{1-\cos\theta} = \frac{(1+\cos\theta)(1-\cos\theta)}{1-\cos\theta} = 1+\cos\theta$

Since both sides simplify to the same expression, they are equal.

**48**. Working from the left side: $\csc^2\frac{A}{2} = \frac{1}{\sin^2\frac{A}{2}} = \frac{1}{\frac{1-\cos A}{2}} = \frac{2}{1-\cos A}$

Working from the right side: $\frac{2\sec A}{\sec A - 1} = \frac{\frac{2}{\cos A}}{\frac{1}{\cos A} - 1} \cdot \frac{\cos A}{\cos A} = \frac{2}{1-\cos A}$

Since both sides simplify to the same expression, they are equal.



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**50**. Working from the right side: $\dfrac{\sec B}{\sec B\csc B+\csc B}=\dfrac{\dfrac{1}{\cos B}}{\dfrac{1}{\cos B}\cdot\dfrac{1}{\sin B}+\dfrac{1}{\sin B}}\cdot\dfrac{\sin B\cos B}{\sin B\cos B}=\dfrac{\sin B}{1+\cos B}=\tan\dfrac{B}{2}$

**52**. Working from the left side:

$$\begin{aligned}\tan\frac{x}{2}-\cot\frac{x}{2}&=\tan\frac{x}{2}-\frac{1}{\tan\frac{x}{2}}\\&=\frac{\sin x}{1+\cos x}-\frac{\sin x}{1-\cos x}\\&=\frac{\sin x(1-\cos x)-\sin x(1+\cos x)}{1-\cos^2 x}\\&=\frac{\sin x(1-\cos x-1-\cos x)}{\sin^2 x}\\&=\frac{-2\cos x}{\sin x}\\&=-2\cot x\end{aligned}$$

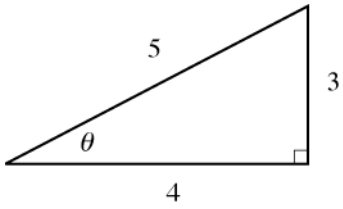
**54**. Working from the left side: $2\sin^2\dfrac{\theta}{2}=2\left(\dfrac{1-\cos\theta}{2}\right)=1-\cos\theta$

Working from the right side: $\dfrac{\sin^2\theta}{1+\cos\theta}\cdot\dfrac{1-\cos\theta}{1-\cos\theta}=\dfrac{\sin^2\theta(1-\cos\theta)}{1-\cos^2\theta}=\dfrac{\sin^2\theta(1-\cos\theta)}{\sin^2\theta}=1-\cos\theta$

Since both sides simplify to the same expression, they are equal.

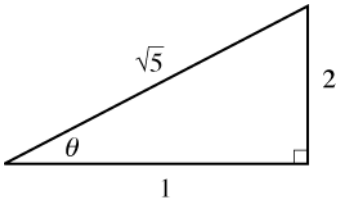
**56**. Working from the right side: $1-2\cos2\theta+\cos^2 2\theta=(1-\cos2\theta)^2=\left(2\sin^2\theta\right)^2=4\sin^4\theta$

**58**. Let $\theta=\arcsin\dfrac{3}{5}$, so $\sin\theta=\dfrac{3}{5}$. Draw the triangle:



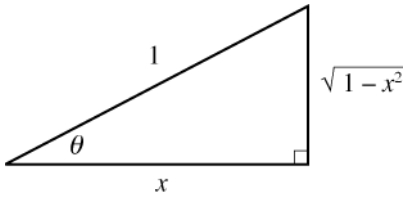

Therefore: $\sin\left(\arcsin\dfrac{3}{5}\right)=\sin\theta=\dfrac{3}{5}$

**60**. Let $\theta=\arctan 2$, so $\tan\theta=2$. Draw the triangle:



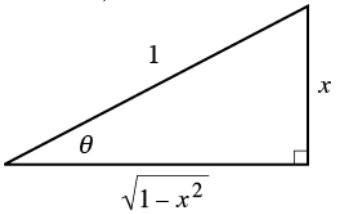

Therefore: $\cos(\arctan 2)=\cos\theta=\dfrac{1}{\sqrt{5}}=\dfrac{\sqrt{5}}{5}$

**62**. Let $\theta=\cos^{-1}x$, so $\cos\theta=x$. Draw the triangle:




Therefore: $\sin\left(\cos^{-1}x\right)=\sin\theta=\sqrt{1-x^2}$

**64**. Let $\theta=\sin^{-1}x$, so $\sin\theta=x$. Draw the triangle:




Therefore: $\tan\left(\sin^{-1}x\right)=\tan\theta=\dfrac{x}{\sqrt{1-x^2}}$

**66**. Since $270°<\theta<360°$, $135°<\dfrac{\theta}{2}<180°$. Thus $\dfrac{\theta}{2}$ terminates in quadrant II. The correct answer is b.

**68**. Using the half-angle identity for cosine: $y=4\cos^2\dfrac{x}{2}=4\left(\dfrac{1+\cos x}{2}\right)=2+2\cos x$

This graph is a cosine curve with amplitude 2 and vertical displacement 2. The correct answer is c.



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**ODD SOLUTIONS**

**1.** $\frac{A}{2}$

**3.** $\pm\sqrt{\frac{1-\cos x}{2}}$

**5.** $\frac{1-\cos y}{\sin y}$

**7.** False (See formula for $\sin\frac{A}{2}$)

**9.** If $0^\circ < A < 90^\circ$, then $\frac{0^\circ}{2} < \frac{A}{2} < \frac{90^\circ}{2}$ or $0^\circ < A < 45^\circ$. Therefore, $A$ terminates in quadrant I.

**11.** If $180^\circ < A < 270^\circ$, then $\frac{180^\circ}{2} < \frac{A}{2} < \frac{270^\circ}{2}$ or $90^\circ < A < 135^\circ$. Therefore, $A$ terminates in quadrant II.

**13.** If $270^\circ < A < 360^\circ$, then $\frac{270^\circ}{2} < \frac{A}{2} < \frac{360^\circ}{2}$ or $135^\circ < A < 180^\circ$. Therefore, $A$ terminates in quadrant II and $\cos\frac{A}{2}$ is negative.

**15.** False: If $A = 400^\circ$, then $\sin A$ is positive. However, $\frac{A}{2} = 200^\circ$ and $\sin\frac{A}{2}$ is negative.

**17.**
$$\cos 15^\circ = \cos\left(\frac{30^\circ}{2}\right)$$
$$= \sqrt{\frac{1+\cos 30^\circ}{2}}$$
$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

**19.**
$$\tan 75^\circ = \tan\left(\frac{150^\circ}{2}\right)$$
$$= \frac{1-\cos 150^\circ}{\sin 150^\circ}$$
$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}}$$
$$= 2+\sqrt{3}$$

**21.**
$$\sin 105^\circ = \sin\left(\frac{210^\circ}{2}\right)$$
$$= \sqrt{\frac{1-\cos 210^\circ}{2}}$$
$$= \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

**23.** If $A$ is in QIV, then $270^\circ < A < 360^\circ$ and $135^\circ < \frac{A}{2} < 180^\circ$. Therefore $\frac{A}{2}$ is in QII.

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

**25.**
$$\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}}$$
$$= -\sqrt{\frac{1+\frac{1}{2}}{2}} = -\sqrt{\frac{3/2}{2}} = -\frac{\sqrt{3}}{2}$$

$$\sec\frac{A}{2} = \frac{1}{\cos\frac{A}{2}} = \frac{1}{-\sqrt{3}/2} = -\frac{2\sqrt{3}}{3}$$



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**27.** If $A$ is in QII, then $90^\circ < A < 180^\circ$ and $45^\circ < \frac{A}{2} < 90^\circ$. Therefore, $\frac{A}{2}$ is in QI.

If $\sin A = \frac{5}{13}$, then $\cos A = -\sqrt{1-\sin^2 A}$

$$= -\sqrt{1-\frac{25}{169}}$$

$$= -\sqrt{\frac{144}{169}}$$

$$= -\frac{12}{13}$$

Therefore, $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$

$$= \sqrt{\frac{1-\frac{12}{13}}{2}}$$

$$= \sqrt{\frac{\frac{1}{13}}{2}}$$

$$= \sqrt{\frac{1}{26}} = \frac{\sqrt{26}}{26}$$

**29.** $\sec\frac{A}{2} = \frac{1}{\cos\frac{A}{2}}$

$$= \frac{1}{\frac{1}{\sqrt{26}}} = \sqrt{26}$$

**31.** If $B$ is in QIII, then $\frac{B}{2}$ is in QII. (See Problem 21)

If $\sin B = -\frac{1}{3}$, then $\cos B = -\sqrt{1-\sin^2 B}$

$$= -\sqrt{1-\frac{1}{9}}$$

$$= -\sqrt{\frac{8}{9}}$$

$$= -\frac{2\sqrt{2}}{3}$$

Therefore $\sin\frac{B}{2} = -\sqrt{\frac{1-\cos B}{2}}$

$$= \sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}}$$

$$= \sqrt{\frac{3+2\sqrt{2}}{6}}$$ Multiply numerator and denominator by 3

*Use the information from problem 31 to solve problems 33 and 35:*

**33.** $\cos\frac{B}{2} = -\sqrt{\frac{1+\cos B}{2}}$

$$= -\sqrt{\frac{1-\frac{2\sqrt{2}}{3}}{2}}$$

$$= -\sqrt{\frac{3-2\sqrt{2}}{6}}$$

**35.** $\cot\frac{B}{2} = \frac{1+\cos B}{\sin B}$

$$= \frac{1-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}}$$

$$= \frac{3-2\sqrt{2}}{-1} = -3+2\sqrt{2}$$



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**37.** $\cos A = -\sqrt{1-\sin^2 A}$

$= -\sqrt{1-\frac{16}{25}}$

$= -\sqrt{\frac{9}{25}} = -\frac{3}{5}$

$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$ (in QI)

$= \sqrt{\frac{1+(-3/5)}{2}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$

**39.** If $B$ is in QI, then $\frac{B}{2}$ must be in QI.

If $\sin B = \frac{3}{5}$, then $\cos B = \sqrt{1-\sin^2 B}$

$= \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Therefore, $\sin\frac{B}{2} = \sqrt{\frac{1-\cos B}{2}}$

$= \sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{1}{10}} = \frac{\sqrt{10}}{10}$

**41.** $y = 4\sin^2\frac{x}{2}$

$= 4\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2$

$= 4\left(\frac{1-\cos x}{2}\right) = 2-2\cos x$

This graph is a cosine curve with amplitude of 2. It has been reflected about the $x$-axis and has a vertical translation of 2.

**43.** $y = 2\cos^2\frac{x}{2}$

$= 2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$

$= 2\left(\frac{1+\cos x}{2}\right)$

$= 1+\cos x$ This graph is the standard cosine curve with a vertical translation of 1.

**45.** $\frac{\csc\theta-\cot\theta}{2\csc\theta} = \frac{\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}{\frac{2}{\sin\theta}}$ Reciprocal and ratio identities

$= \frac{\frac{1-\cos\theta}{\sin\theta}}{\frac{2}{\sin\theta}}$ Subtract

$= \frac{1-\cos\theta}{2}$ Divide

$= \sin^2\frac{\theta}{2}$ Half-angle formula



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**47.**

$$\sec^2\frac{A}{2}=\frac{1}{\cos^2\frac{A}{2}}$$ Reciprocal identity

$$=\frac{1}{\left(\pm\sqrt{\frac{1+\cos A}{2}}\right)^2}$$ Half-angle identity

$$=\frac{1}{\frac{1+\cos A}{2}}$$ Simplify

$$=\frac{2}{1+\cos A}$$ Divide

$$=\frac{2}{1+\frac{1}{\sec A}}$$ Reciprocal identity

$$=\frac{2\sec A}{\sec A+1}$$ Multiply numerator and denominator by sec$A$

**49.**

$$\tan\frac{B}{2}=\frac{1-\cos B}{\sin B}$$ Half-angle identity

$$=\frac{1}{\sin B}-\frac{\cos B}{\sin B}$$ Separate fractions

$$=\csc B-\cot B$$ Reciprocal and ratio identities

**51.**

$$\tan\frac{x}{2}+\cot\frac{x}{2}=\frac{1-\cos x}{\sin x}+\frac{1}{\frac{\sin x}{1+\cos x}}$$ Half-angle identities and reciprocal identity

$$=\frac{1-\cos x}{\sin x}+\frac{1+\cos x}{\sin x}$$ Divide second fraction

$$=\frac{1-\cos x+1+\cos x}{\sin x}$$ Add

$$=\frac{2}{\sin x}$$ Combine

$$=2\csc x$$ Reciprocal identity

**53.**

$$\frac{\tan\theta+\sin\theta}{2\tan\theta}=\frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{2\sin\theta}{\cos\theta}}$$ Ratio identity

$$=\frac{\sin\theta+\sin\theta\cos\theta}{2\sin\theta}$$ Multiply numerator and denominator by $\cos\theta$

$$=\frac{\sin\theta(1+\cos\theta)}{2\sin\theta}$$ Factor

$$=\frac{1+\cos\theta}{2}$$ Reduce

$$=\cos^2\frac{\theta}{2}$$ Half-angle identity



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**55.**
$$\frac{1}{4}+\frac{\cos 2\theta}{2}+\frac{\cos^2 2\theta}{4}=\frac{1}{4}+\frac{2\cos^2\theta-1}{2}+\frac{\left(2\cos^2\theta-1\right)^2}{4}$$ Double-angle identity

$$=\frac{1}{4}+\frac{2\left(2\cos^2\theta-1\right)}{4}+\frac{4\cos^4\theta-4\cos^2\theta+1}{4}$$ LCD is 4

$$=\frac{1+4\cos^2\theta-2+4\cos^4\theta-4\cos^2\theta+1}{4}$$ Combine

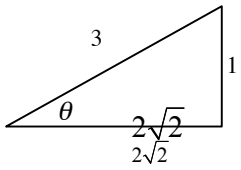
$$=\frac{4\cos^4\theta}{4}$$ Simplify

$$=\cos^4\theta$$ Reduce

**59.** Let $\theta=\arcsin\frac{1}{3}$. Then $\sin\theta=\frac{1}{3}$ and $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$.

We draw a triangle and label the opposite side and the hypotenuse. Then we find the adjacent side using the Pythagorean Theorem:

adjacent side $=\sqrt{3^2-1^2}=\sqrt{8}=2\sqrt{2}$



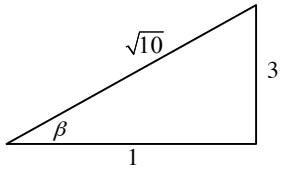

Using the figure, $\cos\theta=\frac{2\sqrt{2}}{3}$

**61.** Let $\beta=\arctan 3$. Then $\tan\beta=\frac{3}{1}$ and $-\frac{\pi}{2}<\beta<\frac{\pi}{2}$.

We draw a triangle and label the opposite and adjacent sides. Then we find the hypotenuse using the Pythagorean Theorem:

hypotenuse $=\sqrt{1^2+3^2}$

$=\sqrt{1+9}=\sqrt{10}$

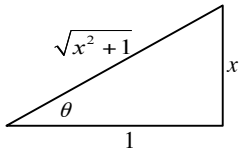



Using the figure, $\sin\beta=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}$

**63.** Let $\theta=\tan^{-1}x$. Then $\tan\theta=\frac{x}{1}$ and $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$.

We draw a triangle and label the opposite and adjacent sides. Then we find the hypotenuse using the Pythagorean Theorem:

hypotenuse $=\sqrt{x^2+1^2}=\sqrt{x^2+1}$



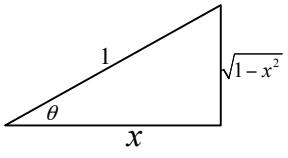

Using the figure, $\cos\theta=\frac{1}{\sqrt{x^2+1}}$

**65.** Let $\theta=\cos^{-1}x$. Then $\cos\theta=\frac{x}{1}$ and $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$.

We draw a triangle, label the sides, and then find the missing side:

opposite side $=\sqrt{1^2-x^2}$

$=\sqrt{1-x^2}$




Using the figure, $\tan\theta=\frac{\sqrt{1-x^2}}{x}$



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## 5.5 Additional Identities

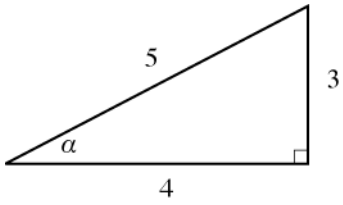
**EVEN SOLUTIONS**

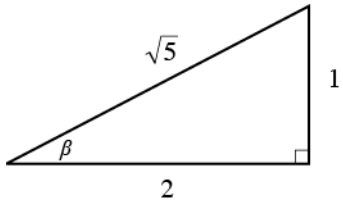
**2.** After assigning a new variable to each inverse trigonometric function, draw a right triangle and use right triangle trigonometry to label two of the three sides.

**4.** The identity is: $\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$

**6.** The identity is: $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

**8.** Let $\alpha = \arcsin\frac{3}{5}$, so $\sin\alpha = \frac{3}{5}$. Let $\beta = \arctan\frac{1}{2}$, so $\tan\beta = \frac{1}{2}$. Draw the triangles:

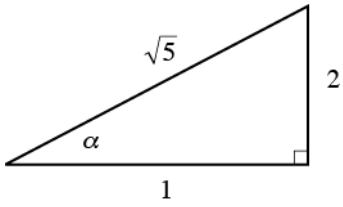


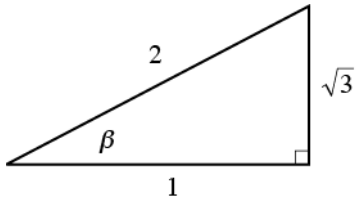




Therefore:

$$\sin\left(\arcsin\frac{3}{5} + \arctan\frac{1}{2}\right) = \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \left(\frac{3}{5}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{\sqrt{5}}\right) = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

**10.** Let $\alpha = \tan^{-1} 2$, so $\tan\alpha = 2$. Let $\beta = \cos^{-1}\frac{1}{2}$, so $\cos\beta = \frac{1}{2}$. Draw the triangles:

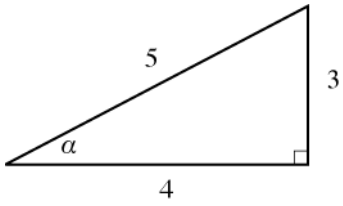



Therefore:

$$\sin\left(\tan^{-1}2 - \cos^{-1}\frac{1}{2}\right) = \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{2-\sqrt{3}}{2\sqrt{5}} = \frac{2\sqrt{5}-\sqrt{15}}{10}$$

**12.** Let $\alpha = \tan^{-1}\frac{3}{4}$, so $\tan\alpha = \frac{3}{4}$. Draw the triangle:




Therefore: $\cos\left(2\tan^{-1}\frac{3}{4}\right) = \cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$



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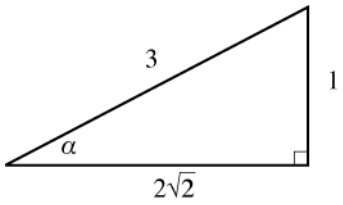
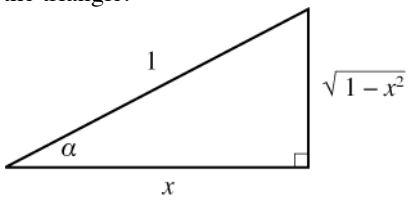
**14**. Let $\alpha = \sin^{-1}\frac{1}{3}$, so $\sin\alpha = \frac{1}{3}$. Draw the triangle:



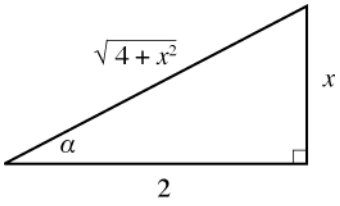

Therefore: $\sin\left(2\sin^{-1}\frac{1}{3}\right) = \sin(2\alpha) = 2\sin\alpha\cos\alpha = 2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right) = \frac{4\sqrt{2}}{9}$

**16**. Let $\alpha = \cos^{-1}x$, so $\cos\alpha = x$. Draw the triangle:




Therefore: $\tan\left(\cos^{-1}x\right) = \tan\alpha = \frac{\sqrt{1-x^2}}{x}$

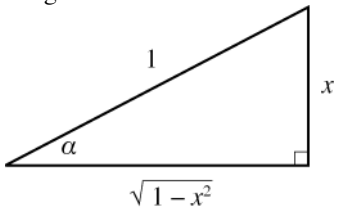
**18**. Let $\alpha = \tan^{-1}\frac{x}{2}$, so $\tan\alpha = \frac{x}{2}$. Draw the triangle:




Therefore: $\sin\left(\tan^{-1}\frac{x}{2}\right) = \sin\alpha = \frac{x}{\sqrt{x^2+4}}$

**20**. Using the triangle from Problem 16: $\sin\left(2\cos^{-1}x\right) = \sin(2\alpha) = 2\sin\alpha\cos\alpha = 2\cdot\sqrt{1-x^2}\cdot x = 2x\sqrt{1-x^2}$

**22**. Let $\alpha = \sin^{-1}x$, so $\sin\alpha = x$. Draw the triangle:




Therefore: $\cos\left(2\sin^{-1}x\right) = \cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = \left(\sqrt{1-x^2}\right)^2 - x^2 = 1 - x^2 - x^2 = 1 - 2x^2$



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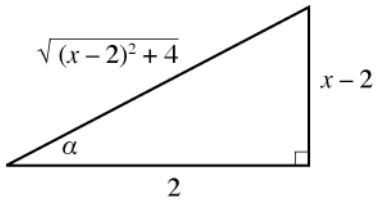
**24.** Let $\alpha = \tan^{-1}\dfrac{x-2}{2}$, so $\tan\alpha = \dfrac{x-2}{2}$. Draw the triangle:




Therefore: $\sec\left(\tan^{-1}\dfrac{x-2}{2}\right) = \sec\alpha = \dfrac{\sqrt{(x-2)^2+4}}{2} = \dfrac{\sqrt{x^2-4x+8}}{2}$

**26.** Substituting $A = 150°$ and $B = 30°$ into product formula (1):

$$\frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right] = \frac{1}{2}\left(\sin 180° + \sin 120°\right) = \frac{1}{2}\left(0+\frac{\sqrt{3}}{2}\right) = \frac{1}{2}\bullet\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

$$\sin A\cos B = \sin 150°\cos 30° = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$$

Both sides of the formula evaluate to the same number.

**28.** Using product formula (4): $10\sin 5x\sin 3x = 10\bullet\dfrac{1}{2}\left[\cos(5x-3x)-\cos(5x+3x)\right] = 5\left(\cos 2x-\cos 8x\right)$

**30.** Using product formula (2): $\cos 2x\sin 8x = \dfrac{1}{2}\left[\sin(2x+8x)-\sin(2x-8x)\right] = \dfrac{1}{2}\left[\sin 10x-\sin(-6x)\right] = \dfrac{1}{2}\left(\sin 10x+\sin 6x\right)$

**32.** Using product formula (1): $\sin 60°\cos 30° = \dfrac{1}{2}\left[\sin(60°+30°)+\sin(60°-30°)\right] = \dfrac{1}{2}\left[\sin 90°+\sin 30°\right] = \dfrac{1}{2}\left(1+\dfrac{1}{2}\right) = \dfrac{3}{4}$

**34.** Using product formula (3): $\cos 4\pi\cos 2\pi = \dfrac{1}{2}\left[\cos(4\pi+2\pi)+\cos(4\pi-2\pi)\right] = \dfrac{1}{2}\left[\cos 6\pi+\cos 2\pi\right] = \dfrac{1}{2}(1+1) = 1$

**36.** Substituting $\alpha = 180°$ and $\beta = 60°$ into sum formula (8):

$$\cos\alpha-\cos\beta = \cos 180°-\cos 60° = -1-\frac{1}{2} = -\frac{3}{2}$$

$$-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = -2\sin\frac{240°}{2}\sin\frac{120°}{2} = -2\sin 120°\sin 60° = -2\bullet\frac{\sqrt{3}}{2}\bullet\frac{\sqrt{3}}{2} = -\frac{3}{2}$$

**38.** Using sum formula (8): $\cos 5x-\cos x = -2\sin\left(\dfrac{5x+x}{2}\right)\sin\left(\dfrac{5x-x}{2}\right) = -2\sin\left(\dfrac{6x}{2}\right)\sin\left(\dfrac{4x}{2}\right) = -2\sin 3x\sin 2x$

**40.** Using sum formula (6):

$$\sin 75°-\sin 15° = 2\cos\left(\frac{75°+15°}{2}\right)\sin\left(\frac{75°-15°}{2}\right) = 2\cos\left(\frac{90°}{2}\right)\sin\left(\frac{60°}{2}\right) = 2\cos 45°\sin 30° = 2\bullet\frac{1}{\sqrt{2}}\bullet\frac{1}{2} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

**42.** Using sum formula (7):

$$\cos\frac{\pi}{8}+\cos\frac{3\pi}{8} = 2\cos\left(\frac{\frac{\pi}{8}+\frac{3\pi}{8}}{2}\right)\cos\left(\frac{\frac{\pi}{8}-\frac{3\pi}{8}}{2}\right) = 2\cos\left(\frac{\pi}{4}\right)\cos\left(-\frac{\pi}{8}\right) = 2\bullet\frac{\sqrt{2}}{2}\bullet\cos\left(-\frac{\pi}{8}\right) = \sqrt{2}\cos\frac{\pi}{8}$$

**44.** Working from the right side: $\dfrac{\cos 3x+\cos x}{\sin 3x-\sin x} = \dfrac{2\cos\left(\dfrac{3x+x}{2}\right)\cos\left(\dfrac{3x-x}{2}\right)}{2\cos\left(\dfrac{3x+x}{2}\right)\sin\left(\dfrac{3x-x}{2}\right)} = \dfrac{2\cos 2x\cos x}{2\cos 2x\sin x} = \dfrac{\cos x}{\sin x} = \cot x$

**46.** Working from the right side: $\dfrac{\sin 4x+\sin 6x}{\cos 4x-\cos 6x} = \dfrac{2\sin\left(\dfrac{4x+6x}{2}\right)\cos\left(\dfrac{4x-6x}{2}\right)}{-2\sin\left(\dfrac{4x+6x}{2}\right)\sin\left(\dfrac{4x-6x}{2}\right)} = \dfrac{2\sin 5x\cos(-x)}{-2\sin 5x\sin(-x)} = -\dfrac{\cos x}{-\sin x} = \cot x$



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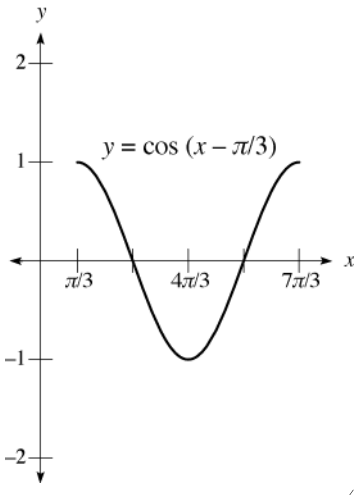
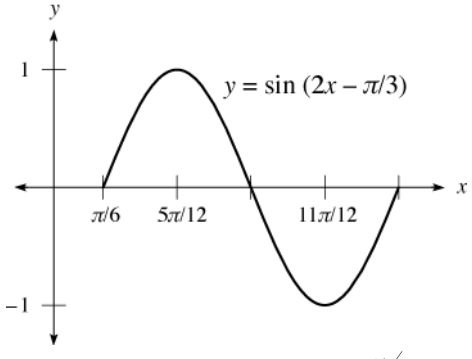
**48**. Working from the right side:

$$\frac{\cos 3x - \cos 5x}{\sin 3x - \sin 5x} = \frac{-2\sin\left(\frac{3x+5x}{2}\right)\sin\left(\frac{3x-5x}{2}\right)}{2\cos\left(\frac{3x+5x}{2}\right)\sin\left(\frac{3x-5x}{2}\right)} = \frac{-2\sin 4x\sin(-x)}{2\cos 4x\sin(-x)} = \frac{2\sin 4x\sin x}{-2\cos 4x\sin x} = -\tan 4x$$

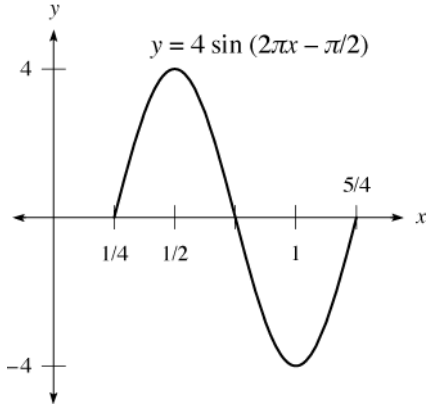
**50**. The amplitude is 1, the period is $2\pi$, and the horizontal shift is $\frac{\pi}{3}$:




**52**. The amplitude is 1, the period is $\frac{2\pi}{2} = \pi$, and the horizontal shift is $\frac{\pi/3}{2} = \frac{\pi}{6}$:




**54**. The amplitude is 4, the period is $\frac{2\pi}{2\pi} = 1$, and the horizontal shift is $\frac{\pi/2}{2\pi} = \frac{1}{4}$:



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**56**. Using the product formula (4): $6\sin 4x\sin 3x = 6\cdot\frac{1}{2}\left[\cos(4x-3x)-\cos(4x+3x)\right] = 3\cos x - 3\cos 7x$

The correct answer is b.

**58**. Working from the left side: $\dfrac{\sin 4x+\sin 2x}{\cos 4x-\cos 2x} = \dfrac{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)}{-2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)} = \dfrac{2\sin 3x\cos x}{-2\sin 3x\sin x} = -\dfrac{\cos x}{\sin x} = -\cot x$

The correct answer is a.

**ODD SOLUTIONS**

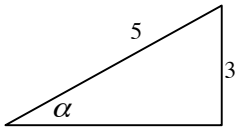
**1.** angle **3.** $\frac{1}{2}\left[\sin(x+y)-\sin(x-y)\right]$ **5.** $2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

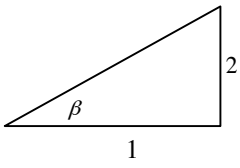
**7.** Let $\alpha = \arcsin\frac{3}{5}$ and $\beta = \arctan 2$.

Then $\sin\alpha = \frac{3}{5}$ and $-90^\circ \le \alpha \le 90^\circ$ and $\tan\beta = \frac{2}{1}$ and $-90^\circ < \beta < 90^\circ$.

Also, $\cos\left(\arcsin\frac{3}{5} - \arctan 2\right) = \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

We draw and label a triangle for $\alpha$ and another for $\beta$:




Using the Pythagorean Theorem we find the missing sides:

adjacent side $= \sqrt{5^2-3^2} = \sqrt{25-9} = \sqrt{16} = 4$

hypotenuse $= \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$

*This problem is continued on the next page.*

Then, $\sin\alpha = \frac{3}{5}$ $\quad\sin\beta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\cos\alpha = \frac{4}{5}$ $\quad\cos\beta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

Substituting these above, we get: $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{4}{5}\left(\frac{\sqrt{5}}{5}\right) + \frac{3}{5}\left(\frac{2\sqrt{5}}{5}\right) = \frac{4\sqrt{5}}{25} + \frac{6\sqrt{5}}{25} = \frac{10\sqrt{5}}{25} = \frac{2\sqrt{5}}{5}$

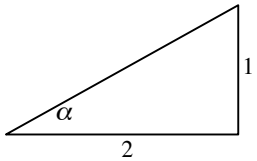
**9.** Let $\alpha = \tan^{-1}\frac{1}{2}$ and $\beta = \sin^{-1}\frac{1}{2}$.

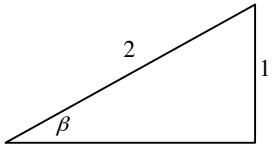
Then $\tan\alpha = \frac{1}{2}$ and $-90^\circ < \alpha < 90^\circ$ and $\sin\beta = \frac{1}{2}$ and $-90^\circ \le \beta \le 90^\circ$.

Also, $\cos\left(\tan^{-1}\frac{1}{2} + \sin^{-1}\frac{1}{2}\right) = \cos(\alpha+\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

We draw and label a triangle for $\alpha$ and another for $\beta$:



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Using the Pythagorean Theorem we find the missing sides:

hypotenuse $= \sqrt{1^2 + 2^2}$ adjacent side $= \sqrt{2^2 - 1^2}$

$= \sqrt{1+4} = \sqrt{5}$ $= \sqrt{4-1} = \sqrt{3}$

Then, $\sin\alpha = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$ $\sin\beta = \dfrac{1}{2}$

$\cos\alpha = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$ $\cos\beta = \dfrac{\sqrt{3}}{2}$

Substituting these above we get:

$$\cos\alpha\cos\beta - \sin\alpha\sin\beta = \left(\frac{2\sqrt{5}}{5}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{5}}{5}\right)\left(\frac{1}{2}\right)$$

$$= \frac{2\sqrt{15}}{10} - \frac{\sqrt{5}}{10} = \frac{2\sqrt{15} - \sqrt{5}}{10}$$

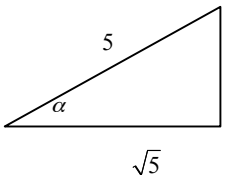
**11.** Let $\alpha = \cos^{-1}\dfrac{\sqrt{5}}{5}$. Then $\cos\alpha = \dfrac{\sqrt{5}}{5}$ and $0^\circ \le \alpha \le 180^\circ$.

Also, $\sin\left(2\cos^{-1}\dfrac{\sqrt{5}}{5}\right) = \sin 2\alpha$

$= 2\sin\alpha\cos\alpha$

We draw and label the sides of a triangle and using the Pythagorean Theorem, we find the opposite side:

opposite side $= \sqrt{(5)^2 - \left(\sqrt{5}\right)^2} = \sqrt{20} = 2\sqrt{5}$




From the figure, we have

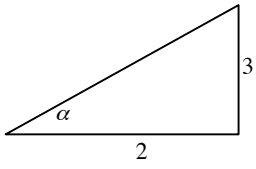
$\sin\alpha = \dfrac{2\sqrt{5}}{5}$ and $\cos\alpha = \dfrac{\sqrt{5}}{5}$

Substituting these above, we get $2\sin\alpha\cos\alpha = 2\left(\dfrac{2\sqrt{5}}{5}\right)\left(\dfrac{\sqrt{5}}{5}\right) = \dfrac{20}{25} = \dfrac{4}{5}$

**13.** Let $\alpha = \tan^{-1}\dfrac{3}{2}$. Then $\tan\alpha = \dfrac{3}{2}$ and $-90^\circ < \alpha < 90^\circ$.

Also, $\cos\left(2\tan^{-1}\dfrac{3}{2}\right) = \cos 2\alpha$

$= \cos^2\alpha - \sin^2\alpha$




We draw and label the sides of a triangle and find the missing side:

hypotenuse $= \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$

From the figure, we find $\cos\alpha = \dfrac{2}{\sqrt{13}}$ and $\sin\alpha = \dfrac{3}{\sqrt{13}}$.

Substituting these above, we get $\cos^2\alpha - \sin^2\alpha = \left(\dfrac{2}{\sqrt{13}}\right)^2 - \left(\dfrac{3}{\sqrt{13}}\right)^2 = \dfrac{4}{13} - \dfrac{9}{13} = -\dfrac{5}{13}$



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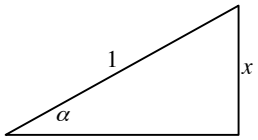
**15.** Let $\alpha = \sin^{-1} x$. Then $\sin\alpha = \frac{x}{1}$ and $-90^\circ \le \alpha \le 90^\circ$.

We draw and label the sides of a triangle and find the adjacent side:

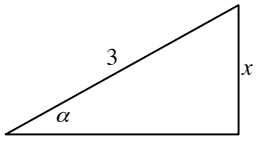



adjacent side $= \sqrt{1^2 - x^2} = \sqrt{1 - x^2}$

From the figure, we find $\tan\alpha = \frac{x}{\sqrt{1-x^2}}$

**17.** Let $\alpha = \sin^{-1}\frac{x}{3}$. Then $\sin\alpha = \frac{x}{3}$ and $-90^\circ \le \alpha \le 90^\circ$.

We draw and label the sides of a triangle and find the adjacent side:




adjacent side $= \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

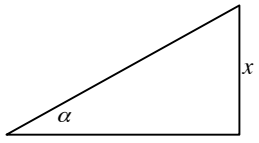
From the figure, we find $\cot\alpha = \frac{\sqrt{9-x^2}}{x}$.

**19.** Let $\alpha = \tan^{-1} x$. Then $\tan\alpha = \frac{x}{1}$ and $-90^\circ \le \alpha \le 90^\circ$.

Also, $\sin\left(2\tan^{-1} x\right) = \sin 2\alpha$

$= 2\sin\alpha\cos\alpha$

We draw and label the sides of a triangle and find the missing side:




hypotenuse $= \sqrt{x^2 + 1}$

$\sin\alpha = \frac{x}{\sqrt{x^2+1}}$ and $\cos\alpha = \frac{1}{\sqrt{x^2+1}}$

Substituting these above, we get $2\sin\alpha\cos\alpha = 2\frac{x}{\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}} = \frac{2}{x^2+1}$

**21.** Let $\alpha = \cos^{-1} x$. Then $\cos\alpha = x$ and $0^\circ \le \alpha \le 180^\circ$.

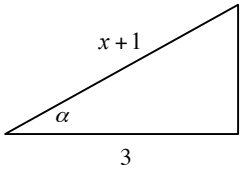
We want to find $\cos\left(2\cos^{-1} x\right) = \cos 2\alpha$

$= 2\cos^2\alpha - 1$

$= 2x^2 - 1$

**23.** Let $\alpha = \sec^{-1}\frac{x+1}{3}$. Then $\cos\alpha = \frac{3}{x+1}$ and $0^\circ < \alpha < 180^\circ$

We draw and label the sides of a triangle and find the missing side:




opposite side $= \sqrt{(x+1)^2 - (3)^2} = \sqrt{x^2 + 2x + 1 - 9} = \sqrt{x^2 + 2x - 8}$

From the figure, we find $\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{x^2+2x-8}}{x+1}$

**25.** $\sin 30^\circ \sin 120^\circ = \frac{1}{2}\left[\cos\left(-90^\circ\right) - \cos 150^\circ\right]$

$\frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{1}{2}\left[0 - \left(-\frac{\sqrt{3}}{2}\right)\right]$

$\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$

**27.** $10\sin 5x \cos 3x$

$= 10\left(\frac{1}{2}\right)\left[\sin(5x + 3x) + \sin(5x - 3x)\right]$

$= 5\left(\sin 8x + \sin 2x\right)$



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**29.** $\cos 8x \cos 2x =$

$$= \frac{1}{2}\left[\cos(8x+2x)+\cos(8x-2x)\right]$$
$$= \frac{1}{2}(\cos 10x + \cos 6x)$$

**31.** $\cos 90^\circ \sin 180^\circ$

$$= \frac{1}{2}\left[\sin(90^\circ+180^\circ)-\sin(90^\circ-180^\circ)\right]$$
$$= \frac{1}{2}\left(\sin 270^\circ - \sin(-90^\circ)\right)$$
$$= \frac{1}{2}\left[-1-(-1)\right] = 0$$

**33.** $\sin 3\pi \sin \pi = \frac{1}{2}\left[\cos(3\pi-\pi)-\cos(3\pi+\pi)\right]$

$$= \frac{1}{2}\left[\cos 2\pi - \cos 4\pi\right]$$
$$= \frac{1}{2}\left[1-1\right] = 0$$

**35.** $\sin\alpha - \sin\beta = \sin 30^\circ - \sin 90^\circ$

$$= \frac{1}{2} - 1$$
$$= -\frac{1}{2}$$

$$2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$= 2\cos\frac{30^\circ+90^\circ}{2}\sin\frac{30^\circ-90^\circ}{2}$$
$$= 2\cos 60^\circ \sin(-30^\circ)$$
$$= 2\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{2}$$

**37.** $\sin 7x + \sin 3x = 2\sin\frac{7x+3x}{2}\cos\frac{7x-3x}{2}$

$$= 2\sin 5x \cos 2x$$

**39.** $\cos 45^\circ + \cos 15^\circ$

$$= 2\cos\frac{45^\circ+15^\circ}{2}\cos\frac{45^\circ-15^\circ}{2}$$
$$= 2\cos 30^\circ \cos 15^\circ$$
$$= 2\left(\frac{\sqrt{3}}{2}\right)\cos 15^\circ = \sqrt{3}\cos 15^\circ$$

**41.** $\sin\frac{7\pi}{12} - \sin\frac{\pi}{12} = 2\cos\frac{\frac{7\pi}{12}+\frac{\pi}{12}}{2}\sin\frac{\frac{7\pi}{12}-\frac{\pi}{12}}{2}$

$$= 2\cos\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$$

**43.** $\dfrac{\sin 3x + \sin x}{\cos 3x - \cos x} = \dfrac{2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}}{-2\sin\frac{3x+x}{2}\sin\frac{3x-x}{2}}$ Sum to product formulas

$$= \frac{2\sin 2x \cos x}{-2\sin 2x \sin x}$$ Simplify

$$= -\frac{\cos x}{\sin x}$$ Reduce

$$= -\cot x$$ Ratio identity



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**45.** $\dfrac{\sin 3x - \sin x}{\cos x - \cos 3x} = \dfrac{2\cos 2x \sin x}{-2\sin 2x \sin(-x)}$ Sum to product formulas

$= -\dfrac{\cos 2x \sin x}{\sin 2x(-\sin x)}$ Reduce and sine is odd function

$= -\dfrac{\cos 2x}{-\sin 2x}$ Reduce

$= \cot 2x$ Ratio identity

**47.** $\dfrac{\sin 5x + \sin 3x}{\cos 3x + \cos 5x} = \dfrac{2\sin 4x \cos x}{2\cos 4x \cos x}$ Sum to product formulas

$= \dfrac{\sin 4x}{\cos 4x}$ Reduce

$= \tan 4x$ Ratio identity

**49.** The graph is a sine curve with a horizontal shift of $-\dfrac{\pi}{4}$.

**51.** Amplitude $= 1$ Period $= \dfrac{2\pi}{2} = \pi$ Horizontal shift $= \dfrac{\frac{\pi}{2}}{2} = \dfrac{\pi}{4}$

One Cycle: $0 \le 2x - \dfrac{\pi}{2} \le 2\pi$ Spacing $= \dfrac{1}{4}(\pi) = \dfrac{\pi}{4}$

$\dfrac{\pi}{2} \le 2x \le \dfrac{5\pi}{2}$

$\dfrac{\pi}{4} \le x \le \dfrac{5\pi}{4}$

The 5 points we use on the $x$-axis are: $\dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \pi, \dfrac{5\pi}{4}$.

The 2 points we use on the $y$-axis are: 1 and $-1$.

**53.** Amplitude $= \dfrac{1}{2}$ Period $= \dfrac{2\pi}{3}$ Horizontal shift $= \dfrac{\frac{\pi}{2}}{3} = \dfrac{\pi}{6}$

One Cycle: $0 \le 3x - \dfrac{\pi}{2} \le 2\pi$ Spacing $= \dfrac{1}{4}\left(\dfrac{2\pi}{3}\right) = \dfrac{\pi}{6}$

$\dfrac{\pi}{2} \le 3x \le \dfrac{5\pi}{2}$

$\dfrac{\pi}{6} \le x \le \dfrac{5\pi}{6}$

The 5 points we use on the $x$-axis are: $\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}$.

The 2 points we use on the $y$-axis are: $-\dfrac{1}{2}$ and $\dfrac{1}{2}$.



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# Chapter 5 Test

**1**. Working from the left side: $\dfrac{\cot\theta}{\csc\theta} = \dfrac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}} = \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{\sin\theta}{1} = \cos\theta$

**2**. Working from the left side: $(\sec x - 1)(\sec x + 1) = \sec^2 x - \sec x + \sec x - 1 = \sec^2 x - 1 = \tan^2 x$

**3**. Working from the left side: $\sec\theta - \cos\theta = \dfrac{1}{\cos\theta} - \cos\theta \cdot \dfrac{\cos\theta}{\cos\theta} = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta = \tan\theta\sin\theta$

**4**. Working from the left side: $\dfrac{\cos t}{1-\sin t} = \dfrac{\cos t}{1-\sin t} \cdot \dfrac{1+\sin t}{1+\sin t} = \dfrac{\cos t(1+\sin t)}{1-\sin^2 t} = \dfrac{\cos t(1+\sin t)}{\cos^2 t} = \dfrac{1+\sin t}{\cos t}$

**5**. Working from the left side: $\dfrac{1}{1-\sin t} + \dfrac{1}{1+\sin t} = \dfrac{1+\sin t}{1-\sin^2 t} + \dfrac{1-\sin t}{1-\sin^2 t} = \dfrac{2}{1-\sin^2 t} = \dfrac{2}{\cos^2 t} = 2\sec^2 t$

**6**. Using the addition formula for cosine: $\cos\left(\dfrac{\pi}{2} + \theta\right) = \cos\dfrac{\pi}{2}\cos\theta - \sin\dfrac{\pi}{2}\sin\theta = 0 \cdot \cos\theta - 1 \cdot \sin\theta = -\sin\theta$

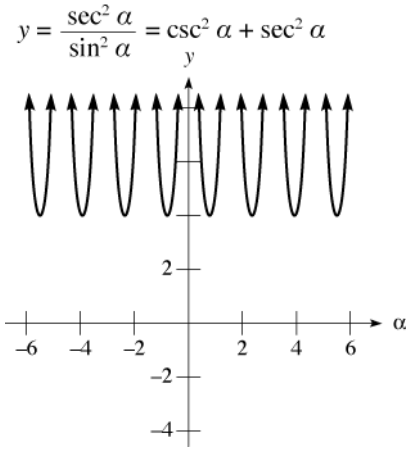
**7**. Working from the left side: $\cos^4 A - \sin^4 A = \left(\cos^2 A + \sin^2 A\right)\left(\cos^2 A - \sin^2 A\right) = 1 \cdot \cos 2A = \cos 2A$

**8**. Working from the right side: $\dfrac{\sin 2A}{1-\cos 2A} = \dfrac{2\sin A\cos A}{1-\left(1-2\sin^2 A\right)} = \dfrac{2\sin A\cos A}{2\sin^2 A} = \dfrac{\cos A}{\sin A} = \cot A$

**9**. Working from the left side: $\cot x - \tan x = \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x} = \dfrac{\cos^2 x - \sin^2 x}{\sin x\cos x} = \dfrac{\cos 2x}{\sin x\cos x}$

**10**. Working from the right side: $\dfrac{\tan x}{\sec x + 1} = \dfrac{\dfrac{\sin x}{\cos x}}{\dfrac{1}{\cos x} + 1} \cdot \dfrac{\cos x}{\cos x} = \dfrac{\sin x}{1+\cos x} = \tan\dfrac{x}{2}$

**11**. Sketching the graph:



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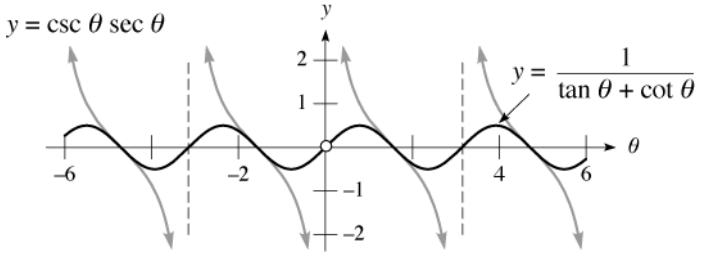
The equation appears to be an identity. Working from the right side:

$$\csc^2\alpha+\sec^2\alpha=\frac{1}{\sin^2\alpha}+\frac{1}{\cos^2\alpha}$$
$$=\frac{\cos^2\alpha}{\sin^2\alpha\cos^2\alpha}+\frac{\sin^2\alpha}{\sin^2\alpha\cos^2\alpha}$$
$$=\frac{\cos^2\alpha+\sin^2\alpha}{\sin^2\alpha\cos^2\alpha}$$
$$=\frac{1}{\sin^2\alpha\cos^2\alpha}$$
$$=\frac{1}{\sin^2\alpha}\cdot\frac{1}{\cos^2\alpha}$$
$$=\frac{\sec^2\alpha}{\sin^2\alpha}$$

**12**. Sketching the graph:




The equation does not appear to be an identity. Evaluating when $\theta=\frac{\pi}{4}$:

$$\csc\frac{\pi}{4}\sec\frac{\pi}{4}=\sqrt{2}\cdot\sqrt{2}=2 \qquad \frac{1}{\tan\frac{\pi}{4}+\cot\frac{\pi}{4}}=\frac{1}{1+1}=\frac{1}{2}$$

The two expressions are not equal.

**13**. Since $A$ terminates in quadrant IV, $\cos A>0$ and thus: $\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\left(-\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

Since $B$ terminates in quadrant II, $\sin B>0$ and thus:

$$\sin B=\sqrt{1-\cos^2 B}=\sqrt{1-\left(-\frac{8}{17}\right)^2}=\sqrt{1-\frac{64}{289}}=\sqrt{\frac{225}{289}}=\frac{15}{17}$$

Using the addition formula for sine: $\sin(A+B)=\sin A\cos B+\cos A\sin B=\left(-\frac{3}{5}\right)\left(-\frac{8}{17}\right)+\left(\frac{4}{5}\right)\left(\frac{15}{17}\right)=\frac{24}{85}+\frac{60}{85}=\frac{84}{85}$

**14**. Using the subtraction formula for cosine and the results from Problem 13:

$$\cos(A-B)=\cos A\cos B+\sin A\sin B=\left(\frac{4}{5}\right)\left(-\frac{8}{17}\right)+\left(-\frac{3}{5}\right)\left(\frac{15}{17}\right)=-\frac{32}{85}-\frac{45}{85}=-\frac{77}{85}$$

**15**. Using the double-angle formula for cosine and the results from Problem 13:

$$\cos 2B=\cos^2 B-\sin^2 B=\left(-\frac{8}{17}\right)^2-\left(\frac{15}{17}\right)^2=\frac{64}{289}-\frac{225}{289}=-\frac{161}{289}$$

**16**. Since $270°\le A\le 360°, 135°\le\frac{A}{2}\le 180°$. So $\sin\frac{A}{2}>0$, therefore (using the results from Problem 13):

$$\sin\frac{A}{2}=\sqrt{\frac{1-\cos A}{2}}=\sqrt{\frac{1-\frac{4}{5}}{2}}=\sqrt{\frac{1}{10}}=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10}$$



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**17**. Using the half-angle formula for sine (note that $\sin 75° > 0$):

$$\sin 75° = \sqrt{\frac{1-\cos 150°}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

We could also solve this using the addition formula for sine:

$$\sin 75° = \sin(45° + 30°) = \sin 45°\cos 30° + \cos 45°\sin 30° = \frac{\sqrt{2}}{2}\bullet\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\bullet\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

**18**. Using the difference formula for cosine:

$$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4} = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}+\sqrt{6}}{4}$$

**19**. Using the addition formula for cosine: $\cos 4x\cos 5x - \sin 4x\sin 5x = \cos(4x + 5x) = \cos 9x$

**20**. Using the addition formula for sine: $\sin 15°\cos 75° + \cos 15°\sin 75° = \sin(15° + 75°) = \sin 90° = 1$

**21**. Since $180° \le A \le 270°, \cos A < 0$ and thus: $\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\left(-\frac{1}{\sqrt{5}}\right)^2} = -\sqrt{1-\frac{1}{5}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}}$

Using the double-angle formula for cosine: $\cos 2A = \cos^2 A - \sin^2 A = \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(-\frac{1}{\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

Since $180° \le A \le 270°, 90° \le \frac{A}{2} \le 135°$ and thus $\cos\frac{A}{2} < 0$. Therefore:

$$\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}} = -\sqrt{\frac{1-\frac{2}{\sqrt{5}}}{2}} = -\sqrt{\frac{\sqrt{5}-2}{2\sqrt{5}}} = -\sqrt{\frac{5-2\sqrt{5}}{10}}$$

**22**. Since $\sec A = \sqrt{10}, \cos A = \frac{1}{\sqrt{10}}$. Since $0° \le A \le 90°, \sin A > 0$ and thus:

$$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{1}{\sqrt{10}}\right)^2} = \sqrt{1-\frac{1}{10}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

Using the double-angle formula for sine: $\sin 2A = 2\sin A\cos A = 2\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) = \frac{6}{10} = \frac{3}{5}$

Since $0° \le A \le 90°, 0° \le \frac{A}{2} \le 45°$, thus $\sin\frac{A}{2} > 0$. Therefore: $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{1}{\sqrt{10}}}{2}} = \sqrt{\frac{\sqrt{10}-1}{2\sqrt{10}}} = \sqrt{\frac{10-\sqrt{10}}{20}}$

**23**. Using the addition formula for tangent:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$3 = \frac{\tan A + \frac{1}{2}}{1 - \tan A\bullet\frac{1}{2}}$$
$$3 = \frac{2\tan A + 1}{2 - \tan A}$$
$$3(2 - \tan A) = 2\tan A + 1$$
$$6 - 3\tan A = 2\tan A + 1$$
$$5 = 5\tan A$$
$$\tan A = 1$$

**24**. Using the double-angle formula for cosine:

$$\cos 2x = 2\cos^2 x - 1$$
$$\frac{1}{2} = 2\cos^2 x - 1$$
$$2\cos^2 x = \frac{3}{2}$$
$$\cos^2 x = \frac{3}{4}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$



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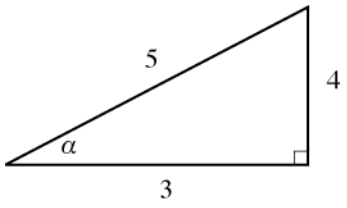
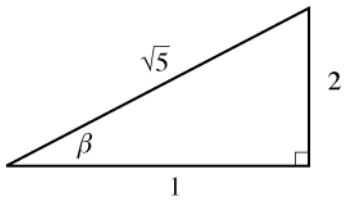
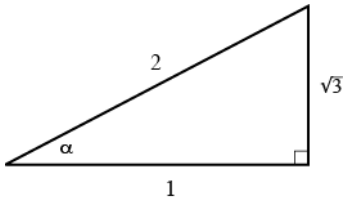
**25**. Let $\alpha = \arcsin\frac{4}{5}$, so $\sin\alpha = \frac{4}{5}$. Let $\beta = \arctan 2$, so $\tan\beta = 2$. Draw the triangles:

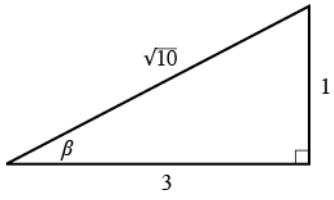



Therefore: $\cos\left(\arcsin\frac{4}{5} - \arctan 2\right) = \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \left(\frac{3}{5}\right)\left(\frac{1}{\sqrt{5}}\right) + \left(\frac{4}{5}\right)\left(\frac{2}{\sqrt{5}}\right) = \frac{11}{5\sqrt{5}} = \frac{11\sqrt{5}}{25}$

**26**. Let $\alpha = \arccos\frac{1}{2}$, so $\cos\alpha = \frac{1}{2}$. Let $\beta = \arctan\frac{1}{3}$, so $\tan\beta = \frac{1}{3}$. Draw the triangles:

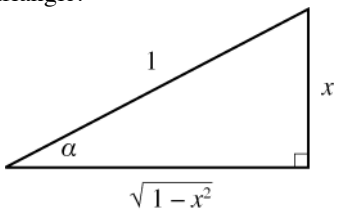



Therefore:

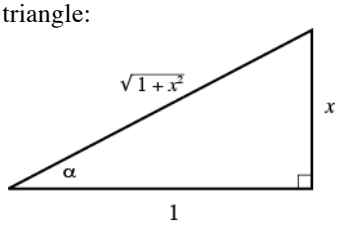
$$\sin\left(\arccos\frac{1}{2} + \arctan\frac{1}{3}\right) = \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{3}{\sqrt{10}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{10}}\right) = \frac{3\sqrt{3}+1}{2\sqrt{10}} = \frac{3\sqrt{30}+\sqrt{10}}{20}$$

**27**. Let $\alpha = \sin^{-1} x$, so $\sin\alpha = x$. Draw the triangle:




Therefore: $\cos\left(2\sin^{-1} x\right) = \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\sqrt{1-x^2}\right)^2 - x^2 = 1 - x^2 - x^2 = 1 - 2x^2$

**28**. Let $\alpha = \tan^{-1} x$, so $\tan\alpha = x$. Draw the triangle:




Therefore: $\sin\left(2\tan^{-1} x\right) = \sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{x}{\sqrt{1+x^2}}\right)\left(\frac{1}{\sqrt{1+x^2}}\right) = \frac{2x}{1+x^2}$

**29**. Using the product-to-sum formula: $\sin 6x\sin 4x = \frac{1}{2}\left[\cos(6x-4x) - \cos(6x+4x)\right] = \frac{1}{2}(\cos 2x - \cos 10x)$

**30**. Using the sum-to-product formula:

$$\cos 15° + \cos 105° = 2\cos\left(\frac{15°+105°}{2}\right)\cos\left(\frac{15°-105°}{2}\right) = 2\cos 60°\cos(-45°) = 2 \bullet \frac{1}{2} \bullet \left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$$



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# Chapter 6
# Equations

## 6.1 Solving Trigonometric Equations

**EVEN SOLUTIONS**

**2**. When solving a simple trigonometric equation, first isolate the trigonometric function, and then determine the values for the angle itself.

**4**. Graphically, each solution will appear as either an $x$-intercept or zero for a single graph, or a point of intersection of two graphs.

**6**. **a**. The solutions are $\theta = 180° + 360°k$.

**b**. The solution is $\theta = 180°$.

**8**. **a**. The solutions are $\theta = 135° + 360°k, 225° + 360°k$.

**b**. The solutions are $\theta = 135°, 225°$.

**10**. **a**. The solutions are $\theta = 135° + 180°k$.

**b**. The solutions are $\theta = 135°, 315°$.

**12**. **a**. Note that $\cos^{-1}(0.75) \approx 0.72$ and $2\pi - \cos^{-1}(0.75) \approx 5.56$.

The solutions are $x = 0.72 + 2k\pi, 5.56 + 2k\pi$.

**b**. The solutions are $x = 0.72, 5.56$.

**14**. **a**. Note that $\sin^{-1}(0.8125) \approx 0.95$, so $\pi + \sin^{-1}(0.8125) \approx 4.09$ and $2\pi - \sin^{-1}(0.8125) \approx 5.33$.

The solutions are $x = 4.09 + 2k\pi, 5.33 + 2k\pi$.

**b**. The solutions are $x = 4.09, 5.33$.

**16**. **a**. Note that $\tan^{-1}(0.25) \approx 0.24$, so $\pi - \tan^{-1}(0.25) \approx 2.90$.

The solutions are $x = 2.90 + k\pi$.

**b**. The solutions are $x = 2.90, 6.04$.

**18**. **a**. Solving the equation:

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60° + 360°k, 300° + 360°k$$

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta = 60°, 300°$.

**20**. **a**. Solving the equation:

$$2\cos\theta + \sqrt{3} = 0$$

$$2\cos\theta = -\sqrt{3}$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = 150° + 360°k, 210° + 360°k$$

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta = 150°, 210°$.



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**22**. **a**. Solving the equation:

$$2\tan\theta + 2 = 0$$
$$2\tan\theta = -2$$
$$\tan\theta = -1$$
$$\theta = 135° + 180°k$$

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta = 135°, 315°$.

**24**. **a**. Solving the equation:

$$\sqrt{3} + 5\sin t = 3\sin t$$
$$2\sin t = -\sqrt{3}$$
$$\sin t = -\frac{\sqrt{3}}{2}$$
$$t = \frac{4\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$$

**b**. In the interval $0 \le t < 2\pi$, the solutions are $t = \frac{4\pi}{3}, \frac{5\pi}{3}$.

**26**. **a**. Solving the equation:

$$5\cos t + 2\sqrt{3} = \cos t$$
$$4\cos t = -2\sqrt{3}$$
$$\cos t = -\frac{\sqrt{3}}{2}$$
$$t = \frac{5\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi$$

**b**. In the interval $0 \le t < 2\pi$, the solutions are $t = \frac{5\pi}{6}, \frac{7\pi}{6}$.

**28**. **a**. Solving the equation:

$$3\sin t + 4 = 4$$
$$3\sin t = 0$$
$$\sin t = 0$$
$$t = k\pi$$

**b**. In the interval $0 \le t < 2\pi$, the solutions are $t = 0, \pi$.

**30**. **a**. Solving the equation:

$$4\sin\theta + 3 = 0$$
$$4\sin\theta = -3$$
$$\sin\theta = -\frac{3}{4}$$
$$\theta \approx 228.6° + 360°k, 311.4° + 360°k$$

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta \approx 228.6°, 311.4°$.

**32**. **a**. Solving the equation:

$$4\cos\theta - 1 = 3\cos\theta + 4$$
$$\cos\theta = 5$$

Since $-1 \le \cos\theta \le 1, \cos\theta = 5$ is impossible. This equation has no solution $(\varnothing)$.

**b**. In the interval $0° \le \theta < 360°$, the equation has no solution $(\varnothing)$.



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**34**. **a**. Solving the equation:

$$\sin\theta - 4 = -2\sin\theta$$
$$3\sin\theta = 4$$
$$\sin\theta = \frac{4}{3}$$

Since $-1 \le \sin\theta \le 1, \sin\theta = \frac{4}{3}$ is impossible. This equation has no solution $(\varnothing)$.

**b**. In the interval $0° \le \theta < 360°$, the equation has no solution $(\varnothing)$.

**36**. **a**. Factoring: $y + 2xy = y(1 + 2x)$

**b**. Factoring: $\tan\theta + 2\cos\theta\tan\theta = \tan\theta(1 + 2\cos\theta)$

**38**. **a**. Factoring: $2x^2 + 3x + 1 = (2x + 1)(x + 1)$

**b**. Factoring: $2\sin^2\theta + 3\sin\theta + 1 = (2\sin\theta + 1)(\sin\theta + 1)$

**40**. **a**. Setting each factor equal to 0, $\tan x = 0$, or $\tan x - 1 = 0$ when $\tan x = 1$.

Thus the solutions are $x = k\pi, \frac{\pi}{4} + k\pi$.

**b**. In the interval $0 \le x < 2\pi$, the solutions are $x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$.

**42**. **a**. Factoring the equation:

$$\cos x - 2\sin x\cos x = 0$$
$$(\cos x)(1 - 2\sin x) = 0$$

Thus either $\cos x = 0$, or $1 - 2\sin x = 0$, which occurs when $\sin x = \frac{1}{2}$.

Thus the solutions are $x = \frac{\pi}{6} + 2k\pi, \frac{\pi}{2} + k\pi, \frac{5\pi}{6} + 2k\pi$.

**b**. In the interval $0 \le x < 2\pi$, the solutions are $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$.

**44**. **a**. Factoring the equation:

$$2\cos^2 x + \cos x - 1 = 0$$
$$(2\cos x - 1)(\cos x + 1) = 0$$

Thus either $2\cos x - 1 = 0$, so $\cos x = \frac{1}{2}$, or $\cos x + 1 = 0$, so $\cos x = -1$.

Thus the solutions are $x = \frac{\pi}{3} + 2k\pi, \pi + 2k\pi, \frac{5\pi}{3} + 2k\pi$.

**b**. In the interval $0 \le x < 2\pi$, the solutions are $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$.

**46**. **a**. Setting each factor equal to 0, $2\sin\theta - \sqrt{3} = 0$ when $\sin\theta = \frac{\sqrt{3}}{2}$, or $2\sin\theta - 1 = 0$ when $\sin\theta = \frac{1}{2}$.

Thus the solutions are $\theta = 30° + 360°k, 60° + 360°k, 120° + 360°k, 150° + 360°k$.

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta = 30°, 60°, 120°, 150°$.

**48**. **a**. Factoring the equation:

$$\tan\theta - 2\cos\theta\tan\theta = 0$$
$$(\tan\theta)(1 - 2\cos\theta) = 0$$

Thus either $\tan\theta = 0$ or $1 - 2\cos\theta = 0$, so $\cos\theta = \frac{1}{2}$.

Thus the solutions are $\theta = 0° + 180°k, 60° + 360°k, 300° + 360°k$.

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta = 0°, 60°, 180°, 300°$.



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**50**. **a**. Factoring the equation:

$$2\sin^2\theta - 7\sin\theta = -3$$

$$2\sin^2\theta - 7\sin\theta + 3 = 0$$

$$(2\sin\theta - 1)(\sin\theta - 3) = 0$$

Thus either $2\sin\theta - 1 = 0$, so $\sin\theta = \frac{1}{2}$, or $\sin\theta - 3 = 0$, so $\sin\theta = 3$, which is impossible.

Thus the solutions are $\theta = 30° + 360°k, 150° + 360°k$.

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta = 30°, 150°$.

**52**. **a**. Using the quadratic formula with $a = 2$, $b = 2$, and $c = -1$:

$$\cos\theta = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2} \approx -1.37, 0.37$$

Since $-1 \le \cos\theta \le 1$, $\cos\theta = -1.37$ is impossible. Thus $\theta = 68.5° + 360°k, 291.5° + 360°k$.

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta \approx 68.5°, 291.5°$.

**54**. **a**. Using the quadratic formula with $a = 1$, $b = -1$, and $c = -1$:

$$\sin\theta = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2} \approx -0.62, 1.62$$

Since $-1 \le \sin\theta \le 1$, $\sin\theta = 1.62$ is impossible. Thus $\theta = 218.2° + 360°k, 321.8° + 360°k$.

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta \approx 218.2°, 321.8°$.

**56**. **a**. First write the equation as $2\cos^2\theta - 4\cos\theta + 1 = 0$. Using the quadratic formula with $a = 2$, $b = -4$, and $c = 1$:

$$\cos\theta = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} = \frac{4 \pm \sqrt{16-8}}{4} = \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2} \approx 0.29, 1.71$$

Since $-1 \le \cos\theta \le 1$, $\cos\theta = 1.71$ is impossible. Thus $\theta = 73.0° + 360°k, 287.0° + 360°k$.

**b**. In the interval $0° \le \theta < 360°$, the solutions are $\theta \approx 73.0°, 287.0°$.

**58**. The principal solutions are $\theta = 60°$ and $\theta = 120°$. Therefore:

$$A + 50° = 60° + 360°k \qquad A + 50° = 120° + 360°k$$

$$A = 10° + 360°k \qquad A = 70° + 360°k$$

**60**. The principal solutions are $\theta = 60°$ and $\theta = 300°$. Therefore:

$$A + 30° = 60° + 360°k \qquad A + 30° = 300° + 360°k$$

$$A = 30° + 360°k \qquad A = 270° + 360°k$$

**62**. The principal solutions are $\theta = \frac{7\pi}{6}$ and $\theta = \frac{11\pi}{6}$. Therefore:

$$A - \frac{\pi}{9} = \frac{7\pi}{6} + k\pi \qquad A - \frac{\pi}{9} = \frac{11\pi}{6} + k\pi$$

$$A = \frac{23\pi}{18} + k\pi \qquad A = \frac{35\pi}{18} + k\pi$$

**64**. The principal solutions are $\theta = \frac{3\pi}{4}$ and $\theta = \frac{5\pi}{4}$. Therefore:

$$A + \frac{\pi}{12} = \frac{3\pi}{4} + k\pi \qquad A + \frac{\pi}{12} = \frac{5\pi}{4} + k\pi$$

$$A = \frac{2\pi}{3} + k\pi \qquad A = \frac{7\pi}{6} + k\pi$$



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**66**. Solving the equation:

$$2-5\cos(x+1)=4$$
$$-5\cos(x+1)=2$$
$$\cos(x+1)=-\frac{2}{5}$$
$$x+1=\pi-\cos^{-1}\left(\frac{2}{5}\right),\pi+\cos^{-1}\left(\frac{2}{5}\right)$$
$$x+1\approx 1.98,4.30$$
$$x\approx 0.98,3.30$$

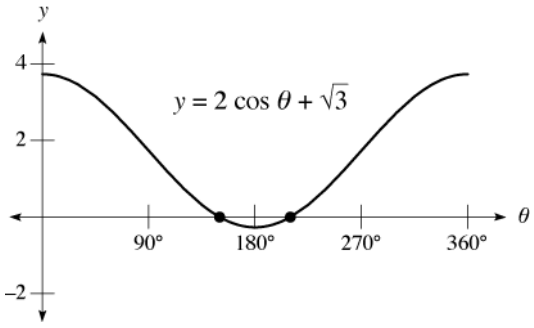
**68**. Solving the equation:

$$2\tan(x-1)+11=-9$$
$$2\tan(x-1)=-20$$
$$\tan(x-1)=-10$$
$$x-1=\pi-\tan^{-1}(10),2\pi-\tan^{-1}(10)$$
$$x-1\approx 1.67,4.81$$
$$x\approx 2.67,5.81$$

**70**. Solving the equation:

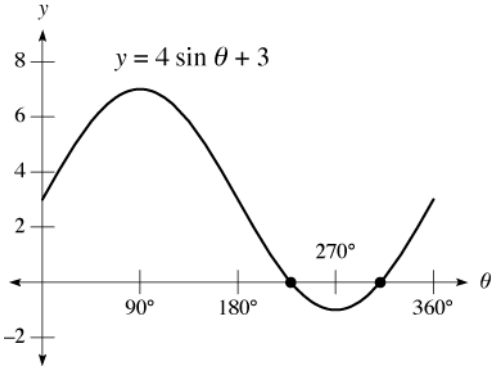
$$-5+16\sin^2 x=4$$
$$16\sin^2 x=9$$
$$\sin^2 x=\frac{9}{16}$$
$$\sin x=\pm\frac{3}{4}$$
$$x=\sin^{-1}\left(\frac{3}{4}\right),\pi-\sin^{-1}\left(\frac{3}{4}\right),\pi+\sin^{-1}\left(\frac{3}{4}\right),2\pi-\sin^{-1}\left(\frac{3}{4}\right)$$
$$x\approx 0.85,2.29,3.99,5.43$$

**72**. Graphing the equation:



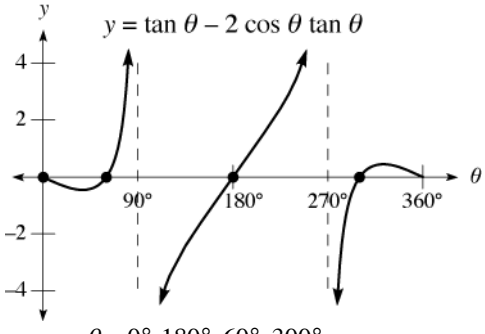

The zeros are $\theta=150°,210°$.

**74**. Graphing the equation:



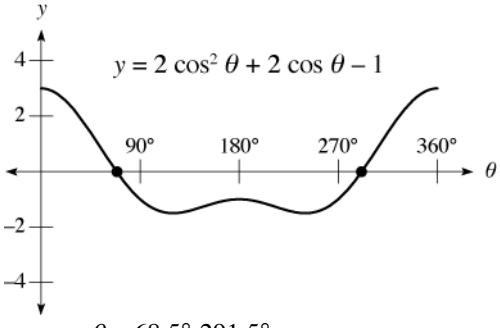

The zeros are $\theta\approx 228.6°,311.4°$.

**76**. Graphing the equation:




The zeros are $\theta=0°,180°,60°,300°$.

**78**. Graphing the equation:




The zeros are $\theta\approx 68.5°,291.5°$.



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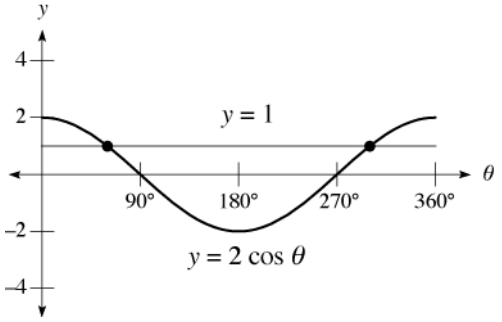
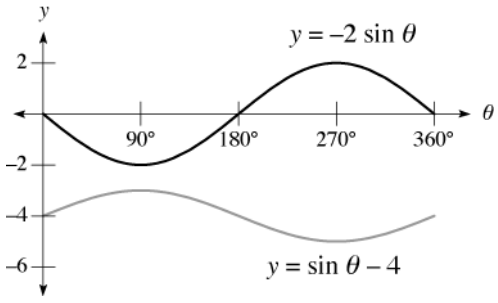
**80**. Graphing each side of the equation:



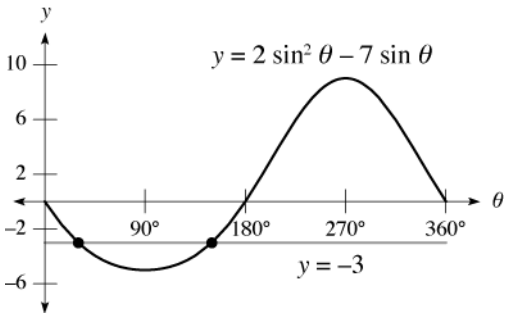

The intersection points are $\theta = 60°, 300°$.

**82**. Graphing each side of the equation:



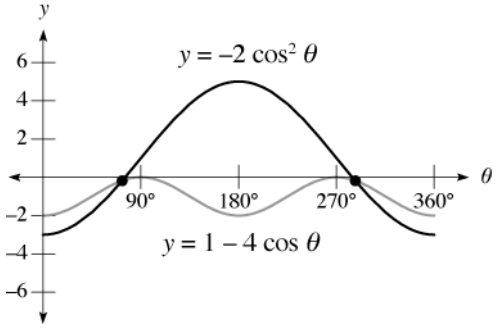

There are no intersection points.

**84**. Graphing each side of the equation:



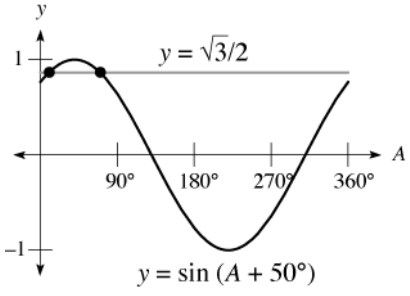

The intersection points are $\theta = 30°, 150°$.

**86**. Graphing each side of the equation:




The intersection points are $\theta \approx 73.0°, 287.0°$.

**88**. Graphing each side of the equation:




The intersection points are $A = 10°, 70°$.

**90**. Substituting $v = 600$ and $\theta = 45°$: $h = -16t^2 + 600t \sin 45° = -16t^2 + 600t \cdot \dfrac{\sqrt{2}}{2} = -16t^2 + 300\sqrt{2}\, t$

**92**. Let $t = \sqrt{3}$: $h = -16\left(\sqrt{3}\right)^2 + 300\sqrt{2} \cdot \sqrt{3} = -48 + 300\sqrt{6} \approx 686.8$ ft

**94**. Substituting $v = 1500$, $t = 3$, and $h = 750$:

$$750 = -16(3)^2 + 1500(3)\sin\theta$$
$$750 = -144 + 4500\sin\theta$$
$$894 = 4500\sin\theta$$
$$\sin\theta = \frac{894}{4500} \approx 0.1987$$
$$\theta \approx 11.5°$$

**96**. Using the double-angle formula for cosine:

$$\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - \left(1 - \cos^2 A\right) = \cos^2 A - 1 + \cos^2 A = 2\cos^2 A - 1$$

**98**. Using the addition formula for sine: $\sin(\theta + 30°) = \sin\theta \cos 30° + \cos\theta \sin 30° = \dfrac{\sqrt{3}}{2}\sin\theta + \dfrac{1}{2}\cos\theta$



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**100**. Using the addition formula for cosine:

$$\cos 105° = \cos(45° + 60°) = \cos 45° \cos 60° - \sin 45° \sin 60° = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$$

**102**. Working from the right side: $\dfrac{2}{\tan x + \cot x} = \dfrac{2}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}} \bullet \dfrac{\sin x \cos x}{\sin x \cos x} = \dfrac{2 \sin x \cos x}{\sin^2 x + \cos^2 x} = \dfrac{\sin 2x}{1} = \sin 2x$

**104**. The correct answer is b.

**106**. First write the equation as $\cos(A + 40°) = \dfrac{\sqrt{2}}{2}$. Then:

$A + 40° = 45° + 360°k$ $\qquad$ $A + 40° = 315° + 360°k$

$A = 5° + 360°k$ $\qquad$ $A = 275° + 360°k$

The sum of the first two positive solutions is 5° + 275° = 280°. The correct answer is d.

**ODD SOLUTIONS**

**1.** angle, true

**3.** reference, $0^\circ$, $360^\circ$, $360^\circ$

**5.** $\cos\theta = 0$

**a.** $\theta = 90^\circ + 180^\circ k$

**b.** $\theta = 90^\circ$ or $270^\circ$

**7.** $\sin\theta = -\dfrac{\sqrt{3}}{2}$

**a.** $\hat{\theta} = 60^\circ$ and $\theta$ is in QIII and IV

$\theta = 240^\circ + 360^\circ k$ or $\theta = 300^\circ + 360^\circ k$

**b.** $\theta = 240^\circ$ or $\theta = 300^\circ$

**9.** $\tan\theta = \sqrt{3}$

**a.** $\hat{\theta} = 60^\circ$ and $\theta$ is in QI and III

$\theta = 60^\circ + 180^\circ k$

**b.** $\theta = 60^\circ$ or $240^\circ$

**11.** $\sin x = 0.25$

**a.** $\hat{x} = 0.25$ and $\theta$ is in QI and II

$x = 0.25 + 2k\pi$ or $x = 2.89 + 2k\pi$

**b.** $x = 0.25$ or $x = 2.89$

**13.** $\cos x = -0.1250$

**a.** $\hat{x} = 1.45$ and $\theta$ is in QII and III

$x = 1.69 + 2k\pi$ or $x = 4.59 + 2k\pi$

**b.** $x = 1.69$ or $x = 4.59$

**15.** $\tan x = 2.5$

**a.** $\hat{x} = 1.19$ and $\theta$ is in QI and III

$x = 1.19 + k\pi$

**b.** $x = 1.19$ or $x = 4.33$

**17.** $2\sin\theta = 1$

$\sin\theta = \dfrac{1}{2}$

$\hat{\theta} = 30^\circ$ and $\theta$ is in QI or QII

**a.** $\theta = 30^\circ + 360^\circ k$ *or* $150^\circ + 360^\circ k$

**b.** $\theta = 30^\circ$ *or* $150^\circ$

**19.** $2\cos\theta - \sqrt{3} = 0$

$2\cos\theta = \sqrt{3}$

$\cos\theta = \dfrac{\sqrt{3}}{2}$

$\hat{\theta} = 30^\circ$ and $\theta$ is in QI or QIV

**a.** $\theta = 30^\circ + 360^\circ k$ or $330^\circ + 360^\circ k$

**b.** $\theta = 30^\circ$ or $330^\circ$



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**21.** $\sqrt{3}\cot\theta - 1 = 0$

$\sqrt{3}\cot\theta = 1$

$\cot\theta = \frac{1}{\sqrt{3}}$

$\hat{\theta} = 60^\circ$ and $\theta$ is in QI or QIII

**a.** $\theta = 60^\circ + 180^\circ k$

**b.** $\theta = 60^\circ \text{ or } 240^\circ$

**23.** $4\sin t - \sqrt{3} = 2\sin t$

$2\sin t = \sqrt{3}$

$\sin t = \frac{\sqrt{3}}{2}$

$\hat{t} = \frac{\pi}{3}$ and $t$ is in QI or QII

**a.** $t = \frac{\pi}{3} + 2k\pi \text{ or } \frac{2\pi}{3} + 2k\pi$

**b.** $t = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$

**25.** $2\cos t = 6\cos t - \sqrt{12}$

$-4\cos t = -2\sqrt{3}$

$\cos t = \frac{\sqrt{3}}{2}$

$\hat{t} = \frac{\pi}{6}$ and $t$ is in QI or QIV

**a.** $t = \frac{\pi}{6} + 2k\pi \text{ or } \frac{11\pi}{6} + 2k\pi$

**b.** $t = \frac{\pi}{6} \text{ or } \frac{11\pi}{6}$

**27.** $3\sin t + 5 = -2\sin t$

$5\sin t = -5$

$\sin t = -1$

$\hat{t} = \frac{3\pi}{2}$

**a.** $t = \frac{3\pi}{2} + 2k\pi$

**b.** $t = \frac{3\pi}{2}$

**29.** $4\sin\theta - 3 = 0$

$4\sin\theta = 3$

$\sin\theta = 0.75$

$\hat{\theta} = 48.6^\circ$ and $\theta$ is in QI or QII

**a.** $\theta = 48.6^\circ + 360^\circ k \text{ or } 131.4^\circ + 360^\circ k$

**b.** $\theta = 48.6^\circ \text{ or } 131.4^\circ$

**31.** $2\cos\theta - 5 = 3\cos\theta - 2$

$-\cos\theta = 3$

$\cos\theta = -3$

There is no solution because $\cos\theta$ must be between $-1$ *and* $1$.

**33.** $\sin\theta - 3 = 5\sin\theta$

$-3 = 4\sin\theta$

$\sin\theta = -0.75$

$\hat{\theta} = 48.6^\circ$ and $\theta$ is in QIII or QIV

**a.** $\theta = 228.6^\circ + 360^\circ k \text{ or } 311.4^\circ + 360^\circ k$

**b.** $\theta = 228.6^\circ \text{ or } 311.4^\circ$

**39.** $(\sin x - 1)(2\sin x - 1) = 0$

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$2\sin x = 1$ $\sin x = 1$

$\sin x = \frac{1}{2}$

$\hat{x} = \frac{\pi}{6}$ (QI or II

**a.** $x = \frac{\pi}{6} + 2k\pi \text{ or } \frac{5\pi}{6} + 2k\pi \text{ or } x = \frac{\pi}{2} + 2k\pi$

**b.** $x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \text{ or } x = \frac{\pi}{2}$

**41.** $\sin x + 2\sin x\cos x = 0$

$\sin x(1 + 2\cos x) = 0$

$\sin x = 0$ or $1 + 2\cos x = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$\hat{x} = \frac{\pi}{3}$ (QII or III)

**a.** $x = k\pi \text{ or } x = \frac{2\pi}{3} + 2k\pi \text{ or } \frac{4\pi}{3} + 2k\pi$ **b.** $x = 0, \pi \text{ or } x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$



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**43.** $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0$ or $\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$ $\sin x = 1$

**a.** $x = \frac{7\pi}{6} + 2k\pi$ or $\frac{11\pi}{6} + 2k\pi$ or $x = \frac{\pi}{2} + 2k\pi$

**b.** $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ or $x = \frac{\pi}{2}$

**45.** $(2\cos\theta + \sqrt{3})(2\cos\theta + 1) = 0$

$2\cos\theta + \sqrt{3} = 0$ or $2\cos\theta + 1 = 0$

$2\cos\theta = -\sqrt{3}$ $2\cos\theta = -1$

$\cos\theta = -\frac{\sqrt{3}}{2}$ $\cos\theta = -\frac{1}{2}$

$\hat{\theta} = 30^\circ$ (QII or III) $\hat{\theta} = 60^\circ$ (QII or III)

**a.** $\theta = 150^\circ + 360^\circ k$ or $210^\circ + 360^\circ k$ or $\theta = 120^\circ + 360^\circ k$ *or* $240^\circ + 360^\circ k$

**b.** $\theta = 150^\circ$ or $210^\circ$ or $\theta = 120^\circ$ *or* $240^\circ$

**47.** $\sqrt{3}\tan\theta - 2\sin\theta\tan\theta = 0$

$\tan\theta(\sqrt{3} - 2\sin\theta) = 0$

$\tan\theta = 0$ or $\sqrt{3} - 2\sin\theta = 0$

$-2\sin\theta = -\sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

**a.** $\theta = 0^\circ + 180^\circ k$ or $\theta = 60^\circ + 360^\circ k$ or $120^\circ + 360^\circ k$

**b.** $\theta = 0^\circ$ or $180^\circ$ or $\theta = 60^\circ$ or $120^\circ$

**49.** $2\cos^2\theta + 11\cos\theta + 5 = 0$

$(2\cos\theta + 1)(\cos\theta + 5) = 0$

$2\cos\theta + 1 = 0$ or $\cos\theta + 5 = 0$

$2\cos\theta = -1$ $\cos\theta = -5$

$\cos\theta = -\frac{1}{2}$ No solution

$\hat{\theta} = 60^\circ$ (QII or III)

**a.** $\theta = 120^\circ + 360^\circ k$ or $240^\circ + 360^\circ k$

**b.** $\theta = 120^\circ$ or $240^\circ$

**51.** $2\sin^2\theta - 2\sin\theta - 1 = 0$ where $a = 2, b = -2, c = -1$

$\sin\theta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{12}}{4}$

$\sin\theta = -0.3660$ or $\sin\theta = 1.3666$

$\hat{\theta} = 21.5^\circ$ and $\theta$ is in QIII or QIV No solution

**a.** $\theta = 201.5^\circ + 360^\circ k$ or $338.5^\circ + 360^\circ k$

**b.** $\theta = 201.5^\circ$ or $338.5^\circ$



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**53.** $\cos^2\theta + \cos\theta - 1 = 0$ where $a = 1, \quad b = 1, \quad c = -1$

$$\cos\theta = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

$\cos\theta = 0.6180$ or $\cos\theta = -1.6180$

$\hat{\theta} = 51.8^\circ$ (QI or II) No solution

**a.** $\theta = 51.8^\circ + 360^\circ k$ or $308.2^\circ + 360^\circ k$

**b.** $\theta = 51.8^\circ$ or $308.2^\circ$

**55.** $2\sin^2\theta + 1 = 4\sin\theta$

$2\sin^2\theta - 4\sin\theta + 1 = 0$ where $a = 2, \quad b = -4, \quad c = 1$

$$\sin\theta = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} = \frac{4 \pm \sqrt{8}}{4}$$

$\sin\theta = 0.2929$ or $\sin\theta = 1.7071$

$\hat{\theta} = 17.0^\circ$ (QI or II) No solution

**a.** $\theta = 17.0^\circ + 360^\circ k$ or $163.0^\circ + 360^\circ k$

**b.** $\theta = 17.0^\circ$ or $163.0^\circ$

**57.** $\cos(A - 50^\circ) = \dfrac{\sqrt{3}}{2}$

$A - 50^\circ = 30^\circ + 360^\circ k$ or $A - 50^\circ = 330^\circ + 360^\circ k$

$A = 80^\circ + 360^\circ k$ $A = 380^\circ + 360^\circ k$ or $A = 20^\circ + 360^\circ k$

**59.** $\sin(A + 30^\circ) = \dfrac{1}{2}$

$A + 30^\circ = 30^\circ + 360^\circ k$ or $A + 30^\circ = 150^\circ + 360^\circ k$

$A = 360^\circ k$ $A = 120^\circ + 360^\circ k$

**61.** $\cos\left(A - \dfrac{\pi}{9}\right) = -\dfrac{1}{2}$

$A - \dfrac{\pi}{9} = \dfrac{2\pi}{3} + 2k\pi$ or $A - \dfrac{\pi}{9} = \dfrac{4\pi}{3} + 2k\pi$

$A = \dfrac{7\pi}{9} + 2k\pi$ $A = \dfrac{13\pi}{9} + 2k\pi$

**63.** $\sin\left(A + \dfrac{\pi}{12}\right) = -\dfrac{\sqrt{2}}{2}$

$A + \dfrac{\pi}{12} = \dfrac{5\pi}{4} + 2k\pi$ or $A + \dfrac{\pi}{12} = \dfrac{7\pi}{4} + 2k\pi$

$A = \dfrac{7\pi}{6} + 2k\pi$ $A = \dfrac{5\pi}{3} + 2k\pi$

**65.** $4\sin(x - 2) + 3 = 6$ $x - 2 = 0.85$ or $x - 2 = 2.29$

$\sin(x - 2) = \dfrac{3}{4}$, let $u = x - 2$ $x = 2.85$ $x = 4.29$

$\hat{u} = 0.85$ and $u$ is in QI and II

$u = 0.85$ or $u = 2.29$

**67.** $7 - 5\tan(x + 3) = -13$ $x + 3 = 1.33$ or $x + 3 = 4.47$

$\tan(x + 3) = 4$, let $u = x + 3$ $x = -1.67$ * $x = 1.47$

$\hat{u} = 1.33$ and $u$ is in QI and III

$u = 1.33$ or $u = 4.47$

* $x = -1.67 + 2\pi = 4.61$ because we want values between 0 and $2\pi$.



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**69.** $7+9\cos^2 x=8$ $\hat{x}=1.23$ and $x$ is in all quadrants

$\cos^2 x=\frac{1}{9}$ $x=1.23, 1.91, 4.37, 5.05$

$\cos x=\pm\frac{1}{3}$

For problems 71 through 87, see textbook answer section for graphs.

**89.** $h=-16t^2+vt\sin\theta$ where $v$ = 1,500 ft/sec and $\theta=30^\circ$

$=-16t^2+1500t\sin 30^\circ$

$=-16t^2+1500t\left(\frac{1}{2}\right)$

$=-16t^2+750t$

**91.** $h=-16t^2+750t$ where $t=2$ sec

$=-16\left(2^2\right)+750(2)$

$=-64+1500$

$=1{,}436$ ft

**93.** $h=-16t^2+vt\sin\theta$ where $v$=1,500 ft/sec, $t$=2 sec, and $h$=750 ft

$750=-16\left(2^2\right)+1{,}500(2)\sin\theta$

$750=-64+3{,}000\sin\theta$

$814=3{,}000\sin\theta$

$\sin\theta=\frac{814}{3{,}000}$

$\theta=15.7^\circ$

**95.** $\cos 2A=\cos^2 A-\sin^2 A$

$=\left(1-\sin^2 A\right)-\sin^2 A$

$=1-2\sin^2 A$

**97.** $\sin\left(\theta+45^\circ\right)=\sin\theta\cos 45^\circ+\cos\theta\sin 45^\circ$

$=\sin\theta\left(\frac{\sqrt{2}}{2}\right)+\cos\theta\left(\frac{\sqrt{2}}{2}\right)$

$=\frac{\sqrt{2}}{2}\sin\theta+\frac{\sqrt{2}}{2}\cos\theta$

**99.** $\sin 75^\circ=\sin\left(45^\circ+30^\circ\right)$

$=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$

$=\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)+\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}=\frac{\sqrt{6}+\sqrt{2}}{4}$

**101.** $\frac{1-\tan^2 x}{1+\tan^2 x}=\frac{1-\frac{\sin^2 x}{\cos^2 x}}{1+\frac{\sin^2 x}{\cos^2 x}}$ Ratio identity

$=\frac{\cos^2 x-\sin^2 x}{\cos^2 x+\sin^2 x}$ Multiply numerator and denominator by $\cos^2 x$

$=\frac{\cos 2x}{1}$ Double-angle identity and Pythagorean identity

$=\cos 2x$ Simplify



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# 6.2 More on Trigonometric Equations

**EVEN SOLUTIONS**

**2**. For equations containing trigonometric functions with different arguments, use an appropriate identity to write all of the functions in terms of the same angle.

**4**. To solve a trigonometric equations that is quadratic in form, try factoring or else use the quadratic formula.

**6**. Since $\csc\theta = \frac{1}{\sin\theta}$, the equation becomes:

$$\sqrt{2} \cdot \frac{1}{\sin\theta} = 2$$
$$\sqrt{2} = 2\sin\theta$$
$$\sin\theta = \frac{\sqrt{2}}{2}$$
$$\theta = 45°, 135°$$

**8**. Since $\sec\theta = \frac{1}{\cos\theta}$, the equation becomes:

$$2\sqrt{3} \cdot \frac{1}{\cos\theta} + 7 = 3$$
$$\frac{2\sqrt{3}}{\cos\theta} = -4$$
$$2\sqrt{3} = -4\cos\theta$$
$$\cos\theta = -\frac{\sqrt{3}}{2}$$
$$\theta = 150°, 210°$$

**10**. Since $\sec\theta = \frac{1}{\cos\theta}$, the equation becomes:

$$4\cos\theta - 3 \cdot \frac{1}{\cos\theta} = 0$$
$$4\cos^2\theta - 3 = 0$$
$$4\cos^2\theta = 3$$
$$\cos^2\theta = \frac{3}{4}$$
$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$
$$\theta = 30°, 150°, 210°, 330°$$

**12**. Since $\csc\theta = \frac{1}{\sin\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$, the equation becomes:

$$\frac{1}{\sin\theta} + 2 \cdot \frac{\cos\theta}{\sin\theta} = 0$$
$$1 + 2\cos\theta = 0$$
$$2\cos\theta = -1$$
$$\cos\theta = -\frac{1}{2}$$
$$\theta = 120°, 240°$$

**14**. Using the double-angle formula for sine:

$$2\sin\theta + \sin 2\theta = 0$$
$$2\sin\theta + 2\sin\theta\cos\theta = 0$$
$$2\sin\theta(1 + \cos\theta) = 0$$

Thus either $\sin\theta = 0$ or $\cos\theta = -1$. Thus $\theta = 0°, 180°$.



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**16**. Since $\sec\theta = \dfrac{1}{\cos\theta}$, the equation becomes:

$$2\cos\theta + 1 = \frac{1}{\cos\theta}$$
$$2\cos^2\theta + \cos\theta = 1$$
$$2\cos^2\theta + \cos\theta - 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$
$$\cos\theta = \frac{1}{2}, -1$$
$$\theta = 60°, 180°, 300°$$

**18**. Using the double-angle formula for cosine:

$$\cos 2x - \cos x - 2 = 0$$
$$2\cos^2 x - 1 - \cos x - 2 = 0$$
$$2\cos^2 x - \cos x - 3 = 0$$
$$(2\cos x - 3)(\cos x + 1) = 0$$
$$\cos x = \frac{3}{2}, -1$$
$$x = \pi$$

Note that $\cos x = \dfrac{3}{2}$ has no solution.

**20**. Using the double-angle formula for cosine:

$$\sin x = -\cos 2x$$
$$\sin x = -(1 - 2\sin^2 x)$$
$$\sin x = -1 + 2\sin^2 x$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$\sin x = -\frac{1}{2}, 1$$
$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

**22**. Substituting $\sin^2 x = 1 - \cos^2 x$ into the equation:

$$2\sin^2 x - \cos x - 1 = 0$$
$$2(1 - \cos^2 x) - \cos x - 1 = 0$$
$$2 - 2\cos^2 x - \cos x - 1 = 0$$
$$2\cos^2 x + \cos x - 1 = 0$$
$$(2\cos x - 1)(\cos x + 1) = 0$$
$$\cos x = \frac{1}{2}, -1$$
$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

**24**. Substituting $\cos^2 x = 1 - \sin^2 x$ into the equation:

$$4\cos^2 x - 4\sin x - 5 = 0$$
$$4(1 - \sin^2 x) - 4\sin x - 5 = 0$$
$$4 - 4\sin^2 x - 4\sin x - 5 = 0$$
$$-4\sin^2 x - 4\sin x - 1 = 0$$
$$4\sin^2 x + 4\sin x + 1 = 0$$
$$(2\sin x + 1)^2 = 0$$
$$\sin x = -\frac{1}{2}$$
$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$



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**26**. Since $\tan x = \dfrac{\sin x}{\cos x}$ and $\sec x = \dfrac{1}{\cos x}$, the equation becomes:

$$2\cos x + \frac{\sin x}{\cos x} = \frac{1}{\cos x}$$
$$2\cos^2 x + \sin x = 1$$
$$2\left(1 - \sin^2 x\right) + \sin x = 1$$
$$2 - 2\sin^2 x + \sin x - 1 = 0$$
$$-2\sin^2 x + \sin x + 1 = 0$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$\sin x = -\frac{1}{2}, 1$$
$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$$

Note that $x = \dfrac{\pi}{2}$ is impossible, since $\tan\dfrac{\pi}{2}$ is undefined. So the solutions are $x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$.

**28**. Isolating $\sin x$ and then squaring:

$$\sin x - \cos x = \sqrt{2}$$
$$\sin x = \cos x + \sqrt{2}$$
$$\sin^2 x = \left(\cos x + \sqrt{2}\right)^2$$
$$\sin^2 x = \cos^2 x + 2\sqrt{2}\cos x + 2$$
$$1 - \cos^2 x = \cos^2 x + 2\sqrt{2}\cos x + 2$$
$$2\cos^2 x + 2\sqrt{2}\cos x + 1 = 0$$
$$\left(\sqrt{2}\cos x + 1\right)^2 = 0$$
$$\sqrt{2}\cos x = -1$$
$$\cos x = -\frac{1}{\sqrt{2}}$$
$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

Since this equation was solved by squaring, each solution must be checked:

$$\sin\frac{3\pi}{4} - \cos\frac{3\pi}{4} = \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \quad \text{(checks)}$$

$$\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0 \quad \text{(doesn't check)}$$

Only $x = \dfrac{3\pi}{4}$ checks in the original equation.



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**30**. Isolating $\sin\theta$ and then squaring:

$$\sin\theta - \sqrt{3}\cos\theta = \sqrt{3}$$
$$\sin\theta = \sqrt{3} + \sqrt{3}\cos\theta$$
$$\sin^2\theta = \left(\sqrt{3} + \sqrt{3}\cos\theta\right)^2$$
$$\sin^2\theta = 3 + 6\cos\theta + 3\cos^2\theta$$
$$1 - \cos^2\theta = 3 + 6\cos\theta + 3\cos^2\theta$$
$$4\cos^2\theta + 6\cos\theta + 2 = 0$$
$$2\cos^2\theta + 3\cos\theta + 1 = 0$$
$$(2\cos\theta + 1)(\cos\theta + 1) = 0$$
$$\cos\theta = -\frac{1}{2}, -1$$
$$\theta = 120°, 240°, 180°$$

Since this equation was solved by squaring, each solution must be checked:

$$\sin 120° - \sqrt{3}\cos 120° = \frac{\sqrt{3}}{2} - \sqrt{3}\left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3} \text{ (checks)}$$

$$\sin 240° - \sqrt{3}\cos 240° = -\frac{\sqrt{3}}{2} - \sqrt{3}\left(-\frac{1}{2}\right) = -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 0 \text{ (doesn't check)}$$

$$\sin 180° - \sqrt{3}\cos 180° = 0 - \sqrt{3}(-1) = 0 + \sqrt{3} = \sqrt{3} \text{ (checks)}$$

Thus $\theta = 120°, 180°$ are the solutions.

**32**. Isolating $\sin\theta$ and then squaring:

$$\sin\theta - \sqrt{3}\cos\theta = 1$$
$$\sin\theta = 1 + \sqrt{3}\cos\theta$$
$$\sin^2\theta = \left(1 + \sqrt{3}\cos\theta\right)^2$$
$$\sin^2\theta = 1 + 2\sqrt{3}\cos\theta + 3\cos^2\theta$$
$$1 - \cos^2\theta = 1 + 2\sqrt{3}\cos\theta + 3\cos^2\theta$$
$$4\cos^2\theta + 2\sqrt{3}\cos\theta = 0$$
$$2\cos\theta\left(2\cos\theta + \sqrt{3}\right) = 0$$
$$\cos\theta = 0, -\frac{\sqrt{3}}{2}$$
$$\theta = 90°, 270°, 150°, 210°$$

Since this equation was solved by squaring, each solution must be checked:

$$\sin 90° - \sqrt{3}\cos 90° = 1 - \sqrt{3}\cdot 0 = 1 \text{ (checks)}$$

$$\sin 270° - \sqrt{3}\cos 270° = -1 - \sqrt{3}\cdot 0 = -1 \text{ (doesn't check)}$$

$$\sin 150° - \sqrt{3}\cos 150° = \frac{1}{2} - \sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}\right) = \frac{1}{2} + \frac{3}{2} = 2 \text{ (doesn't check)}$$

$$\sin 210° - \sqrt{3}\cos 210° = -\frac{1}{2} - \sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}\right) = -\frac{1}{2} + \frac{3}{2} = 1 \text{ (checks)}$$

Thus $\theta = 90°, 210°$ are the solutions.



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**34**. Isolating $\sin\frac{\theta}{2}$ and then squaring:

$$\sin\frac{\theta}{2}+\cos\theta=1$$
$$\sin\frac{\theta}{2}=1-\cos\theta$$
$$\sin^2\frac{\theta}{2}=(1-\cos\theta)^2$$
$$\frac{1-\cos\theta}{2}=1-2\cos\theta+\cos^2\theta$$
$$1-\cos\theta=2-4\cos\theta+2\cos^2\theta$$
$$2\cos^2\theta-3\cos\theta+1=0$$
$$(2\cos\theta-1)(\cos\theta-1)=0$$
$$\cos\theta=\frac{1}{2},1$$
$$\theta=60°,300°,0°$$

Since this equation was solved by squaring, each solution must be checked:

$\sin\frac{60°}{2}+\cos 60°=\sin 30°+\cos 60°=\frac{1}{2}+\frac{1}{2}=1$ (checks)

$\sin\frac{300°}{2}+\cos 300°=\sin 150°+\cos 300°=\frac{1}{2}+\frac{1}{2}=1$ (checks)

$\sin\frac{0°}{2}+\cos 0°=\sin 0°+\cos 0°=0+1=1$ (checks)

Thus $\theta=0°,60°,300°$ are the solutions.

**36**. Isolating $\cos\frac{\theta}{2}$ and then squaring:

$$\cos\frac{\theta}{2}-\cos\theta=0$$
$$\cos\frac{\theta}{2}=\cos\theta$$
$$\cos^2\frac{\theta}{2}=\cos^2\theta$$
$$\frac{1+\cos\theta}{2}=\cos^2\theta$$
$$1+\cos\theta=2\cos^2\theta$$
$$2\cos^2\theta-\cos\theta-1=0$$
$$(2\cos\theta+1)(\cos\theta-1)=0$$
$$\cos\theta=-\frac{1}{2},1$$
$$\theta=120°,240°,0°$$

Checking each solution in the original equation:

$\cos\frac{120°}{2}-\cos 120°=\cos 60°-\cos 120°=\frac{1}{2}-\left(-\frac{1}{2}\right)=\frac{1}{2}+\frac{1}{2}=1$ (doesn't check)

$\cos\frac{240°}{2}-\cos 240°=\cos 120°-\cos 240°=-\frac{1}{2}-\left(-\frac{1}{2}\right)=-\frac{1}{2}+\frac{1}{2}=0$ (checks)

$\cos\frac{0°}{2}-\cos 0°=\cos 0°-\cos 0°=1-1=0$ (checks)

Thus $\theta=0°,240°$ are the solutions.



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**38**. Since $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ and $\csc\theta = \dfrac{1}{\sin\theta}$, the equation becomes:

$$13 \cdot \frac{\cos\theta}{\sin\theta} + \frac{11}{\sin\theta} = 6\sin\theta$$
$$13\cos\theta + 11 = 6\sin^2\theta$$
$$13\cos\theta + 11 = 6\left(1 - \cos^2\theta\right)$$
$$13\cos\theta + 11 = 6 - 6\cos^2\theta$$
$$6\cos^2\theta + 13\cos\theta + 5 = 0$$
$$(3\cos\theta + 5)(2\cos\theta + 1) = 0$$
$$\cos\theta = -\frac{5}{3}, -\frac{1}{2}$$
$$\theta = 120°, 240°$$

Note that $\cos\theta = -\dfrac{5}{3}$ has no solutions. These solutions are verified graphically.

**40**. Since $\csc\theta = \dfrac{1}{\sin\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, the equation becomes:

$$\frac{23}{\sin^2\theta} - 22 \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} - 15 = 0$$
$$23 - 22\cos\theta - 15\sin^2\theta = 0$$
$$23 - 22\cos\theta - 15\left(1 - \cos^2\theta\right) = 0$$
$$23 - 22\cos\theta - 15 + 15\cos^2\theta = 0$$
$$15\cos^2\theta - 22\cos\theta + 8 = 0$$
$$(5\cos\theta - 4)(3\cos\theta - 2) = 0$$
$$\cos\theta = \frac{2}{3}, \frac{4}{5}$$
$$\theta \approx 36.9°48.2°, 360° - 48.2°, 360° - 36.9°$$
$$\theta \approx 36.9°, 48.2°, 311.8°, 323.1°$$

These solutions are verified graphically.

**42**. Using the double-angle formula for cosine:

$$16\cos 2\theta - 18\sin^2\theta = 0$$
$$16\left(1 - 2\sin^2\theta\right) - 18\sin^2\theta = 0$$
$$16 - 32\sin^2\theta - 18\sin^2\theta = 0$$
$$16 - 50\sin^2\theta = 0$$
$$50\sin^2\theta = 16$$
$$\sin^2\theta = \frac{8}{25}$$
$$\sin\theta = \pm\frac{2\sqrt{2}}{5}$$
$$\theta \approx 34.4°, 180° - 34.4°, 180° + 34.4°, 360° - 34.4°$$
$$\theta \approx 34.4°, 145.6°, 214.4°, 325.6°$$

These solutions are verified graphically.

**44**. Using the results of Problem 8, $\theta = 150° + 360°k$ or $\theta = 210° + 360°k$, where $k$ is any integer.

**46**. Using the results of Problem 28, $x = \dfrac{3\pi}{4} + 2k\pi$, where $k$ is any integer.



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**48**. Using the results of Problem 36, $\theta = 360°k$ or $\theta = 240° + 360°k$, where $k$ is any integer.

**50**. Substituting $r = 2$ and $R = 4$, then set the expression equal to 0:

$$2^4 \csc^2\theta - 4^4 \csc\theta\cot\theta = 0$$

$$\frac{16}{\sin^2\theta} - 256 \cdot \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta} = 0$$

$$16 - 256\cos\theta = 0$$

$$256\cos\theta = 16$$

$$\cos\theta = \frac{1}{16}$$

$$\theta \approx 86.4°$$

**52**. Substituting $\cos^2\theta = 1 - \sin^2\theta$ into the equation:

$$2\left(1 - \sin^2\theta\right) + 2\sin\theta - 1 = 0$$

$$2 - 2\sin^2\theta + 2\sin\theta - 1 = 0$$

$$-2\sin^2\theta + 2\sin\theta + 1 = 0$$

$$2\sin^2\theta - 2\sin\theta - 1 = 0$$

Using the quadratic formula with $a = 2$, $b = -2$, and $c = -1$:

$$\sin\theta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{4+8}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

Either $\sin\theta = \frac{1+\sqrt{3}}{2} \approx 1.37$, which is impossible, or $\sin\theta = \frac{1-\sqrt{3}}{2} \approx -0.37$.

Thus $\theta = 180° + 21.5° = 201.5°$ or $\theta = 360° - 21.5° = 338.5°$.

**54**. Substituting $\sin^2\theta = 1 - \cos^2\theta$ into the equation:

$$1 - \cos^2\theta = \cos\theta$$

$$\cos^2\theta + \cos\theta - 1 = 0$$

Using the quadratic formula with $a = 1$, $b = 1$, and $c = -1$: $\cos\theta = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

Either $\cos\theta = \frac{-1-\sqrt{5}}{2} \approx -1.62$, which is impossible, or $\cos\theta = \frac{-1+\sqrt{5}}{2} \approx 0.62$.

Thus $\theta \approx 51.8°$ or $\theta \approx 360° - 51.8° = 308.2°$.

**56**. Substituting $\cos^2\theta = 1 - \sin^2\theta$ into the equation:

$$4\sin\theta = 3 - 2\left(1 - \sin^2\theta\right)$$

$$4\sin\theta = 3 - 2 + 2\sin^2\theta$$

$$2\sin^2\theta - 4\sin\theta + 1 = 0$$

Using the quadratic formula with $a = 2$, $b = -4$, and $c = 1$:

$$\sin\theta = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} = \frac{4 \pm \sqrt{16-8}}{4} = \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2}$$

Either $\sin\theta = \frac{2+\sqrt{2}}{2} \approx 1.71$, which is impossible, or $\sin\theta = \frac{2-\sqrt{2}}{2} \approx 0.29$.

Thus $\theta \approx 17.0°$ or $\theta \approx 180° - 17.0° = 163.0°$.



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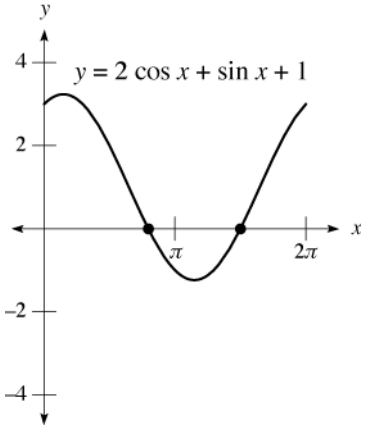
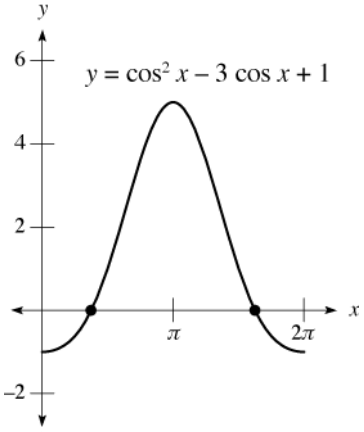
**58**. Sketching the graph:



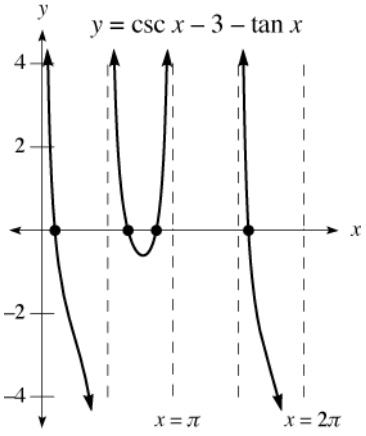

The solutions are $x \approx 2.4981, 4.7124$.

**60**. Sketching the graph:




The solutions are $x \approx 1.1789, 5.1043$.

**62**. Write the equation as $\csc x - \tan x - 3 = 0$. Sketching the graph:




The solutions are $x \approx 0.3063, 2.0629, 2.7435, 4.9556$.

**64**. Since $0° \le A \le 90°$, $\cos A > 0$ and thus: $\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \left(\frac{2}{3}\right)^2} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

Since $0° \le A \le 90°$, $0° \le \frac{A}{2} \le 45°$ and thus $\cos\frac{A}{2} > 0$. Therefore: $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \sqrt{\frac{1+\frac{\sqrt{5}}{3}}{2}} = \sqrt{\frac{3+\sqrt{5}}{6}}$

**66**. First find $\sin\frac{A}{2}$ (since $0° \le \frac{A}{2} \le 45°$, $\sin\frac{A}{2} > 0$): $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{\sqrt{5}}{3}}{2}} = \sqrt{\frac{3-\sqrt{5}}{6}}$

Therefore: $\cot\frac{A}{2} = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sqrt{\frac{3+\sqrt{5}}{6}}}{\sqrt{\frac{3-\sqrt{5}}{6}}} = \sqrt{\frac{3+\sqrt{5}}{3-\sqrt{5}}} \cdot \sqrt{\frac{3+\sqrt{5}}{3+\sqrt{5}}} = \frac{3+\sqrt{5}}{\sqrt{9-5}} = \frac{3+\sqrt{5}}{2}$



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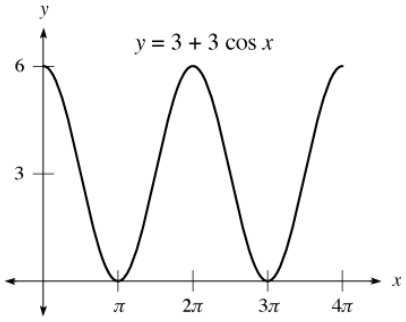
**68**. Using the half-angle formula for cosine: $y = 6\cos^2\frac{x}{2} = 6\left(\frac{1+\cos x}{2}\right) = 3(1+\cos x) = 3+3\cos x$

The amplitude is 3, the period is $2\pi$, and the vertical translation is 3 units. Now graphing the curve:




**70**. Since $\cos 75° > 0$, use the half-angle formula for cosine: $\cos 75° = \sqrt{\frac{1+\cos 150°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

**72**. Since $\csc\theta = \frac{1}{\sin\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$, the equation becomes:

$$\frac{1}{\sin\theta} - 2\cdot\frac{\cos\theta}{\sin\theta} = 0$$
$$1 - 2\cos\theta = 0$$
$$2\cos\theta = 1$$
$$\cos\theta = \frac{1}{2}$$

The correct answer is d.

**74**. Sketching the graph results in solutions (intersection) of $x \approx 3.8930, 6.0217$. The correct answer is d.

**ODD SOLUTIONS**

**1.** reciprocal, clear, multiplying

**3.** square, extraneous

**5.** $\sqrt{3}\sec\theta = 2$

$\sec\theta = \frac{2}{\sqrt{3}}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\hat{\theta} = 30°$ and $\theta$ is in QI or QIV

$\theta = 30°$ or $330°$

**7.** $\sqrt{2}\csc\theta + 5 = 3$

$\sqrt{2}\csc\theta = -2$

$\csc\theta = -\frac{2}{\sqrt{2}}$

$\sin\theta = -\frac{\sqrt{2}}{2}$

$\hat{\theta} = 45°$ and $\theta$ is in QIII or QIV

$\theta = 225°$ or $315°$

**9.** $4\sin\theta - 2\csc\theta = 0$

| | |
|---|---|
| $4\sin\theta - \frac{2}{\sin\theta} = 0$ | Reciprocal identity |
| $4\sin^2\theta - 2 = 0$ | Multiply both sides by $\sin\theta$ |
| $4\sin^2\theta = 2$ | Add 2 to both sides |
| $\sin^2\theta = \frac{1}{2}$ | Divide both sides by 4 |
| $\sin\theta = \pm\frac{\sqrt{2}}{2}$ | Take square root of both sides |

$\hat{\theta} = 45°$ and $\theta$ is in QI, QII, QIII, or QIV

$\theta = 45°, 135°, 225°, 315°$



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**11.** $\sec\theta - 2\tan\theta = 0$

$\dfrac{1}{\cos\theta} - 2\left(\dfrac{\sin\theta}{\cos\theta}\right) = 0$ Reciprocal and ratio identities

$1 - 2\sin\theta = 0$ Multiply both sides by $\cos\theta \quad (\cos\theta \neq 0)$

$-2\sin\theta = -1$ Solve equation and check

$\sin\theta = \dfrac{1}{2}$

$\theta = 30^\circ \text{ or } 150^\circ$

**13.** $\sin 2\theta - \cos\theta = 0$

$2\sin\theta\cos\theta - \cos\theta = 0$ Double-angle identity

$\cos\theta(2\sin\theta - 1) = 0$ Factor out $\cos\theta$

$\cos\theta = 0 \quad \text{or} \quad 2\sin\theta - 1 = 0$ Set each factor = 0

$2\sin\theta = 1$ Solve each equation

$\sin\theta = \dfrac{1}{2}$

$\theta = 90^\circ, 270^\circ \qquad \theta = 30^\circ \text{ or } 150^\circ$

**15.** $2\sin\theta + 1 = \csc\theta$

$2\sin\theta + 1 = \dfrac{1}{\sin\theta}$ Reciprocal identity

$2\sin^2\theta - \sin\theta = 1$ Multiply both sides by $\cos\theta \ (\cos\theta \neq 0)$

$2\sin^2\theta - \sin\theta - 1 = 0$ Add 1 to both sides

$(\sin\theta - 1)(2\sin\theta + 1) = 0$ Factor

$\sin\theta - 1 = 0 \quad \text{or} \quad 2\sin\theta + 1 = 0$ Set each factor = 0

$\sin\theta = 1 \qquad 2\sin\theta = -1$ Solve each equation and check

$\theta = 90^\circ \qquad \sin\theta = -\dfrac{1}{2}$

$\hat{\theta} = 30^\circ$ and $\theta$ is in QIII or QIV

$\theta = 210^\circ \text{ or } 330^\circ$

**17.** $\cos 2x - 3\sin x - 2 = 0$

$1 - 2\sin^2 x - 3\sin x - 2 = 0$ Double-angle identity

$2\sin^2 x + 3\sin x + 1 = 0$ Multiply both sides by $-1$ and simplify

$(2\sin x + 1)(\sin x + 1) = 0$ Factor

$2\sin x + 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$ Set each factor = 0

$2\sin x = -1 \qquad \sin x = -1$

$\sin x = -\dfrac{1}{2} \qquad x = \dfrac{3\pi}{2}$ Solve each equation

$x = \dfrac{7\pi}{6} \text{ or } \dfrac{11\pi}{6}$



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**19.** $\cos x - \cos 2x = 0$

$\cos x - \left(2\cos^2 x - 1\right) = 0$ Double-angle identity

$\cos x - 2\cos^2 x + 1 = 0$ Simplify

$2\cos^2 x - \cos x - 1 = 0$ Multiply both sides by $-1$

$\left(2\cos x + 1\right)\left(\cos x - 1\right) = 0$ Factor

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$ Set each factor = 0

$2\cos x = -1$ $\cos x = 1$ Solve each equation

$\cos x = -\frac{1}{2}$

$\hat{x} = \frac{\pi}{3}$ $x = 0$

$x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

**21.** $2\cos^2 x + \sin x - 1 = 0$

$2\left(1 - \sin^2 x\right) + \sin x - 1 = 0$ Pythagorean identity

$2 - 2\sin^2 x + \sin x - 1 = 0$ Simplify

$2\sin^2 x - \sin x - 1 = 0$ Multiply both sides by $-1$ and simplify

$\left(2\sin x + 1\right)\left(\sin x - 1\right) = 0$ Factor

$2\sin x + 1 = 0$ or $\sin x - 1 = 0$ Set each factor = 0

$2\sin x = -1$ $\sin x = 1$ Solve each equation

$\sin x = -\frac{1}{2}$ $x = \frac{\pi}{2}$

$x = \frac{7\pi}{6}$ *or* $\frac{11\pi}{6}$

**23.** $4\sin^2 x + 4\cos x - 5 = 0$

$4\left(1 - \cos^2 x\right) + 4\cos x - 5 = 0$ Pythagorean identity

$4 - 4\cos^2 x + 4\cos x - 5 = 0$ Simplify

$-4\cos^2 x + 4\cos x - 1 = 0$ Simplify

$4\cos^2 x - 4\cos\theta + 1 = 0$ Multiply both sides by $-1$

$\left(2\cos x - 1\right)\left(2\cos x - 1\right) = 0$ Factor

$2\cos x - 1 = 0$ or $2\cos x - 1 = 0$ Set each factor = 0

$2\cos x = 1$ Solve the equation

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$ *or* $\frac{5\pi}{3}$



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**25.** $2\sin x + \cot x - \csc x = 0$

$2\sin x + \dfrac{\cos x}{\sin x} - \dfrac{1}{\sin x} = 0$ Ratio and reciprocal identities

$2\sin^2 x + \cos x - 1 = 0$ Multiply both sides by $\sin x \left(\sin x \neq 0\right)$

$2\left(1 - \cos^2 x\right) + \cos x - 1 = 0$ Pythagorean identity

$2 - 2\cos^2 x + \cos x - 1 = 0$ Simplify

$2\cos^2 x - \cos x - 1 = 0$ Multiply both sides by $-1$ and simplify

$\left(2\cos x + 1\right)\left(\cos x - 1\right) = 0$ Factor

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$ Set each factor = 0

$2\cos x = -1$ $\quad$ $\cos x = 1$ Solve each equation and check

$\cos x = -\dfrac{1}{2}$ $\quad$ $x = 0$ This is not possible because $\sin x \neq 0$

$x = \dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$

**27.** $\sin x + \cos x = \sqrt{2}$

$\sin^2 x + 2\sin x \cos x + \cos^2 x = 2$ Square both sides

$2\sin x \cos x + 1 = 2$ Pythagorean identity

$\sin 2x = 1$ Double-angle identity

$2x = \dfrac{\pi}{2}$ or $2x = \dfrac{5\pi}{2}$ Solve equation and check

$x = \dfrac{\pi}{4}$ $\quad$ $x = \dfrac{5\pi}{4}$ Possible solutions

Check each possible solution:

$\sin\dfrac{\pi}{4} + \cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}$

$\sqrt{2} = \dfrac{2\sqrt{2}}{2}$

$\sqrt{2} = \sqrt{2}$ It checks

$\sin\dfrac{5\pi}{4} + \cos\dfrac{5\pi}{4} = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}$

$\sqrt{2} \neq 0$ Does not check

The answer is $\dfrac{\pi}{4}$ only.

**29.** $\sqrt{3}\sin\theta + \cos\theta = \sqrt{3}$

$\cos\theta = \sqrt{3} - \sqrt{3}\sin\theta$ Subtract $\sqrt{3}\sin\theta$ from both sides

$\cos\theta = \sqrt{3}\left(1 - \sin\theta\right)$ Factor out $\sqrt{3}$

$\cos^2\theta = 3\left(1 - 2\sin\theta + \sin^2\theta\right)$ Square both sides

$1 - \sin^2\theta = 3 - 6\sin\theta + 3\sin^2\theta$ Pythagorean identity

$4\sin^2\theta - 6\sin\theta + 2 = 0$ Simplify

$2\sin^2\theta - 3\sin\theta + 1 = 0$ Divide both sides by 2

$\left(2\sin\theta - 1\right)\left(\sin\theta - 1\right) = 0$ Factor

$2\sin\theta - 1 = 0$ or $\sin\theta - 1 = 0$ Set each factor = 0

$2\sin\theta = 1$ $\quad$ $\sin\theta = 1$ Solve each equation and check

$\sin\theta = \dfrac{1}{2}$ $\quad$ $\theta = 90^\circ$ Possible solutions

$\theta = 30^\circ$ *or* $150^\circ$



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Check each possible solution:

$\sqrt{3}\sin 30^\circ + \cos 30^\circ = \sqrt{3}\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}$

$\sqrt{3} = \sqrt{3}$ It checks

$\sqrt{3}\sin 90^\circ + \cos 90^\circ = \sqrt{3}(1) + 0$

$\sqrt{3} = \sqrt{3}$ It checks

$\sqrt{3}\sin 150^\circ = \cos 150^\circ = \sqrt{3}\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)$

$\sqrt{3} \neq 0$ Does not check

Answers: $30^\circ$ *or* $90^\circ$

**31.**

$\sqrt{3}\sin\theta - \cos\theta = 1$

$\sqrt{3}\sin\theta = \cos\theta + 1$ Isolate $\sqrt{3}\sin\theta$

$3\sin^2\theta = \cos^2\theta + 2\cos\theta + 1$ Square both sides

$3(1 - \cos^2\theta) = \cos^2\theta + 2\cos\theta + 1$ Pythagorean identity

$3 - 3\cos^2\theta = \cos^2\theta + 2\cos\theta + 1$ Simplify

$4\cos^2\theta + 2\cos\theta - 2 = 0$ Put in standard form

$2(2\cos\theta - 1)(\cos\theta + 1) = 0$ Factor

$2\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$ Set each factor = 0

$2\cos\theta = 1$ $\cos\theta = -1$ Solve each equation and check

$\cos\theta = \frac{1}{2}$ $\theta = 180^\circ$

$\theta = 60^\circ$ *or* $300^\circ$ Possible solutions

The solution is $60^\circ$ *or* $180^\circ$. ($300^\circ$ does not check.)

**33.**

$\sin\frac{\theta}{2} - \cos\theta = 0$

$\sin\frac{\theta}{2} = \cos\theta$ Add $\cos\theta$ to both sides

$\sin^2\frac{\theta}{2} = \cos^2\theta$ Square both sides

$\frac{1 - \cos\theta}{2} = \cos^2\theta$ $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$

$1 - \cos\theta = 2\cos^2\theta$ Multiply both sides by 2

$2\cos^2\theta + \cos\theta - 1 = 0$ Rewrite in standard form

$(2\cos\theta - 1)(\cos\theta + 1) = 0$ Factor

$2\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$ Set each factor = 0

$2\cos\theta = 1$ $\cos\theta = -1$ Solve each equation and check

$\cos\theta = \frac{1}{2}$ $\theta = 180^\circ$ Possible solutions

$\theta = 60^\circ$ *or* $300^\circ$

The solution is $60^\circ$ *or* $300^\circ$. ($180^\circ$ does not check.)



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**35.** $\cos\frac{\theta}{2}-\cos\theta=1$

$\pm\sqrt{\frac{1+\cos\theta}{2}}-\cos\theta=1$ Half-angle identity

$\pm\sqrt{\frac{1+\cos\theta}{2}}=\cos\theta+1$ Add $\cos\theta$ to both sides

$\frac{1+\cos\theta}{2}=\cos^2\theta+2\cos\theta+1$ Square both sides

$1+\cos\theta=2\cos^2\theta+4\cos\theta+2$ Multiply both sides by 2

$0=2\cos^2\theta+3\cos\theta+1$ Put in standard form

$0=(2\cos\theta+1)(\cos\theta+1)$ Factor

$2\cos\theta+1=0$ or $\cos\theta+1=0$ Set each factor = 0

$2\cos\theta=-1$ $\cos\theta=-1$ Solve each equation and check

$\cos\theta=-\frac{1}{2}$ $\theta=180^\circ$ Possible solutions

$\theta=120^\circ$ *or* $240^\circ$

The solution is $120^\circ$ *or* $180^\circ$. ($240^\circ$ does not check.)

**37.** $6\cos\theta+7\tan\theta=\sec\theta$

$6\cos\theta+\frac{7\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}$ Ratio and reciprocal identities

$6\cos^2\theta+7\sin\theta=1$ Multiply both sides by $\cos\theta\ (\cos\theta\neq 0)$

$6(1-\sin^2\theta)+7\sin\theta=1$ Pythagorean identity

$6-6\sin^2\theta+7\sin^2\theta=1$ Simplify

$-6\sin^2\theta+7\sin\theta+5=0$ Subtract 1 from both sides

$6\sin^2\theta-7\sin\theta-5=0$ Multiply both sides by $-1$

$(3\sin\theta-5)(2\sin\theta+1)=0$ Factor

$3\sin\theta-5=0$ or $2\sin\theta+1=0$ Set each factor = 0

$3\sin\theta=5$ $2\sin\theta=-1$ Solve each equation and check

$\sin\theta=\frac{5}{3}$ $\sin\theta=-\frac{1}{2}$

No solution $\theta=210^\circ$ *or* $330^\circ$ Both check

**39.** $18\sec^2\theta-17\tan\theta\sec\theta-12=0$

$18\left(\frac{1}{\cos^2\theta}\right)-17\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\right)-12=0$ Reciprocal and ratio identities

$\frac{18}{\cos^2\theta}-\frac{17\sin\theta}{\cos^2\theta}-12=0$ Simplify

$18-17\sin\theta-12\cos^2\theta=0$ Multiply both sides by $\sin^2\theta$

$18-17\sin\theta-12(1-\sin^2\theta)=0$ Pythagorean identity

$18-17\sin\theta-12+12\sin^2\theta=0$ Simplify

$12\sin^2\theta-17\sin\theta+6=0$ Put in standard form

$(4\sin\theta-3)(3\sin\theta-2)=0$ Factor

$4\sin\theta-3=0$ or $3\sin\theta-2=0$ Set each factor = 0

$4\sin\theta=3$ $3\sin\theta=2$ Solve each equations and check

$\sin\theta=\frac{3}{4}$ $\sin\theta=\frac{2}{3}$

$\theta=48.6^\circ$ or $131.4^\circ$ $\theta=41.8^\circ$ or $138.2^\circ$ All check



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**41.** $7\sin^2\theta - 9\cos 2\theta = 0$

$7\sin^2\theta - 9(1 - 2\sin^2\theta) = 0$ Double-angle identity

$7\sin^2\theta - 9 + 18\sin^2\theta = 0$ Simplify left side

$25\sin^2\theta = 9$ Add 9 to both sides

$\sin^2\theta = \frac{9}{25}$ Divide both sides by 25

$\sin\theta = \pm\frac{3}{5}$ Take square root of both sides

$\hat{\theta} = 36.9^\circ$ and $\theta = QI, QII, QIII, or\ QIV$

$\theta = 36.9^\circ, 143.1^\circ, 216.9^\circ\ or\ 323.1^\circ$

**43.** In problem 7, we get $225^\circ$ or $315^\circ$. All solutions would be $225^\circ + 360^\circ k$ or $315^\circ + 360^\circ k$.

**45.** In problem 27 we get $x = \frac{\pi}{4}$. All solutions would be $\frac{\pi}{4} + 2k\pi$.

**47.** In problem 35, we get $120^\circ$ or $180^\circ$. All solutions would be $120^\circ + 360^\circ k$ or $180^\circ + 360^\circ k$.

**49.** $r^4\csc^2\theta - R^4\csc\theta\cot\theta = r^4 \cdot \frac{1}{\sin^2\theta} - R^4 \cdot \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}$

$= \frac{r^4}{\sin^2\theta} - \frac{R^4\cos\theta}{\sin^2\theta}$

$= \frac{r^4 - R^4\cos\theta}{\sin^2\theta}$

This expression is zero only when the numerator is zero. Therefore,

$r^4 - R^4\cos\theta = 0$

$-R^4\cos\theta = -r^4$

$\cos\theta = \frac{r^4}{R^4}$

Therefore, when $\cos\theta = \frac{r^4}{R^4}$, then $r^4\csc^2\theta - R^4\csc\theta\cot\theta = 0$

**51.** $2\sin^2\theta - 2\cos\theta - 1 = 0$

$2(1 - \cos^2\theta) - 2\cos\theta - 1 = 0$ Pythagorean identity

$2 - 2\cos^2\theta - 2\cos\theta - 1 = 0$ Simplify

$-2\cos^2\theta - 2\cos\theta + 1 = 0$ Put in standard form

$2\cos^2\theta + 2\cos\theta - 1 = 0$ Multiply both sides by $-1$

We apply the quadratic formula with $a = 2$, $b = 2$, and $c = -1$:

$\cos\theta = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$

$= \frac{-2 \pm \sqrt{12}}{4}$

$\cos\theta = \frac{-2 + 3.464}{4}$ or $\cos\theta = \frac{-2 - 3.464}{4}$

$= 0.366$ $= -1.366$

$\hat{\theta} = 68.5^\circ$ and $\theta$ in in QI, QIV No solution

$\theta = 68.5^\circ\ or\ 291.5^\circ$



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**53.** $\cos^2\theta + \sin\theta = 0$

$1 - \sin^2\theta + \sin\theta = 0$ Pythagorean identity

$-\sin^2\theta + \sin\theta + 1 = 0$ Put in standard form

$\sin^2\theta - \sin\theta - 1 = 0$ Multiply both sides by $-1$

We apply the quadratic formula with $a = 1$, $b = -1$, and $c = -1$:

$$\sin\theta = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

$$\sin\theta = \frac{1 + 2.236}{2} \quad \text{or} \quad \sin\theta = \frac{1 - 2.236}{2}$$

$$= 1.618 \qquad = -0.618$$

No solution $\hat{\theta} = 38.2^\circ$ and $\theta$ in in QIII, QIV

$\theta = 218.2^\circ$ *or* $321.8^\circ$

**55.** $2\sin^2\theta = 3 - 4\cos\theta$

$2(1 - \cos^2\theta) = 3 - 4\cos\theta$ Pythagorean identity

$2 - 2\cos^2\theta = 3 - 4\cos\theta$ Simplify

$-2\cos^2\theta + 4\cos\theta - 1 = 0$ Put in standard form

$2\cos^2\theta - 4\cos\theta + 1 = 0$ Multiply both sides by $-1$

We apply the quadratic formula with $a = 2$, $b = -4$, and $c = 1$:

$$\cos\theta = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{8}}{4}$$

$$\cos\theta = \frac{4 + 2.828}{4} \quad \text{or} \quad \cos\theta = \frac{4 - 2.828}{4}$$

$$= 1.707 \qquad = 0.293$$

No solution $\hat{\theta} = 73.0^\circ$ and $\theta$ is in QI and QIV

$\theta = 73.0^\circ$ *or* $287.0^\circ$

**57.** Graph $y_1 = \cos(x) + 3\sin(x) - 2$ on your graphing calculator. Set the window for $x$ between 0 and 6.28 and $y$ between $-1$ and 1. Use the zero or root finder to locate the zeros. The solution is 0.3630 or 2.1351.

**59.** Graph $y_1 = (\sin(x))^2 - 3\sin(x) - 1$ on your graphing calculator. The window settings are the same as problem 57. Use the zero or root finder to locate the zeros. The solution is 3.4492 or 5.9756.

**61.** Graph $y_1 = \frac{1}{\cos(x)} + 2 - \frac{1}{\tan(x)}$ on your graphing calculator. The window settings are the same as problem 57. Use the zero or root finder to locate the zeros. The solution is 0.3166 or 1.9917.

**63.** $\cos A = \sqrt{1 - \sin^2 A}$ with $A$ in QI

$$= \sqrt{1 - \left(\frac{2}{3}\right)^2} = \sqrt{1 - \frac{4}{9}}$$

$$= \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

$$\sin\frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}}$$

$$= \sqrt{\frac{1 - \sqrt{5}/3}{2}} = \sqrt{\frac{3 - \sqrt{5}}{6}}$$



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**65.** $\tan\frac{A}{2}=\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}$

We know that $\cos A=\frac{\sqrt{5}}{3}$ and $\sin\frac{A}{2}=\sqrt{\frac{3-\sqrt{5}}{6}}$ from problem 59.

$\cos\frac{A}{2}=\sqrt{\frac{1+\cos A}{2}}$

$=\sqrt{\frac{1+\sqrt{5}/3}{2}}=\sqrt{\frac{3+\sqrt{5}}{6}}$

Therefore, $\tan\frac{A}{2}=\frac{\sqrt{\frac{3-\sqrt{5}}{6}}}{\sqrt{\frac{3+\sqrt{5}}{6}}}$

$=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}=\frac{3-\sqrt{5}}{2}$ (*Rationalize* the denominator)

**67.** $y=4\sin^2\frac{x}{2}$

$y=4\left[\sqrt{\frac{1-\cos x}{2}}\right]^2$

$y=4\left(\frac{1-\cos x}{2}\right)$

$y=2-2\cos x$

This graph is a cosine curve with amplitude of 2 that has been reflected about the *x*-axis and shifted up 2.

**69.** $\sin 22.5^\circ=\sin\frac{1}{2}\left(45^\circ\right)$

$=\sqrt{\frac{1-\cos 45^\circ}{2}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$

$=\sqrt{\frac{2-\sqrt{2}}{4}}=\frac{\sqrt{2-\sqrt{2}}}{2}$

## 6.3 Trigonometric Equations Involving Multiple Angles

**EVEN SOLUTIONS**

**2.** For a trigonometric equation involving $\cos n\theta$ in degrees, add $\frac{360^\circ}{n}k$ to find the values of $\theta$ to find all coterminal solutions. If solving in radians, add $\frac{2\pi}{n}k$ to find the values of $\theta$ to find all coterminal solutions.

**4.** False. It is not necessary to use the double-angle formula to rewrite the left side.

**6.** For $\cos 2\theta=-\frac{\sqrt{3}}{2}$, all values will take the form:

$2\theta=150^\circ+360^\circ k$ $\qquad$ $2\theta=210^\circ+360^\circ k$

$\theta=75^\circ+180^\circ k$ $\qquad$ $\theta=105^\circ+180^\circ k$

Substituting $k=0$ and $k=1$ results in the solutions $\theta=75^\circ,105^\circ,255^\circ,285^\circ$. These solutions are verified graphically.

**8.** For $\cot 2\theta=\sqrt{3}$, all values will take the form:

$2\theta=30^\circ+180^\circ k$

$\theta=15^\circ+90^\circ k$

Substituting $k=0$, 1, 2, and 3 results in the solutions $\theta=15^\circ,105^\circ,195^\circ,285^\circ$. These solutions are verified graphically.



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**10**. For $\sin 5\theta = -1$, all values will take the form:

$$5\theta = 270° + 360°k$$

$$\theta = 54° + 72°k$$

Substituting $k = 0, 1, 2, 3$ and 4 results in the solutions $\theta = 54°, 126°, 198°, 270°, 342°$.

These solutions are verified graphically.

**12**. For $\sin 2x = -\frac{1}{2}$, all values will take the form:

$$2x = \frac{7\pi}{6} + 2k\pi \qquad 2x = \frac{11\pi}{6} + 2k\pi$$

$$x = \frac{7\pi}{12} + k\pi \qquad x = \frac{11\pi}{12} + k\pi$$

Substituting $k = 0$ and $k = 1$ results in the solutions $x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$. These solutions are verified graphically.

**14**. For $\csc 3x = 2$, all values will take the form:

$$3x = \frac{\pi}{6} + 2k\pi \qquad 3x = \frac{5\pi}{6} + 2k\pi$$

$$x = \frac{\pi}{18} + \frac{2k\pi}{3} \qquad x = \frac{5\pi}{18} + \frac{2k\pi}{3}$$

Substituting $k = 0, 1$, and 2 results in the solutions $x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$.

These solutions are verified graphically.

**16**. For $\tan 2x = 0$, all values will take the form:

$$2x = 0 + k\pi$$

$$x = 0 + \frac{k\pi}{2}$$

Substituting $k = 0, 1, 2$, and 3 results in the solutions $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$. These solutions are verified graphically.

**18**. For $\cos 2\theta = -\frac{\sqrt{2}}{2}$, all values will take the form:

$$2\theta = 135° + 360°k \qquad 2\theta = 225° + 360°k$$

$$\theta = 67.5° + 180°k \qquad \theta = 112.5° + 180°k$$

**20**. For $\sin 3\theta = -1$, all values will take the form:

$$3\theta = 270° + 360°k$$

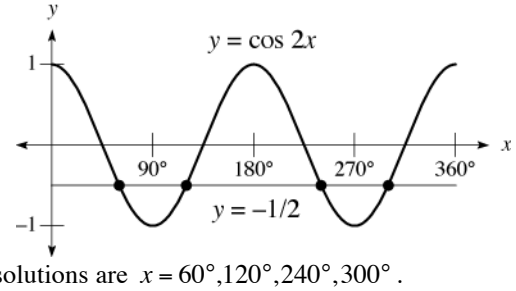
$$\theta = 90° + 120°k$$

**22**. For $\cos 8\theta = \frac{1}{2}$, all values will take the form:

$$8\theta = 60° + 360°k \qquad 8\theta = 300° + 360°k$$

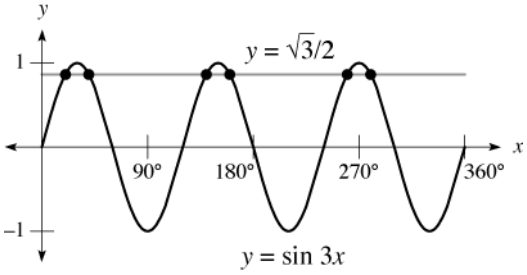
$$\theta = 7.5° + 45°k \qquad \theta = 37.5° + 45°k$$

**24**. Graphing each side of the equation:




The solutions are $x = 60°, 120°, 240°, 300°$.

**26**. Graphing each side of the equation:




The solutions are $x = 20°, 40°, 140°, 160°, 260°, 280°$.



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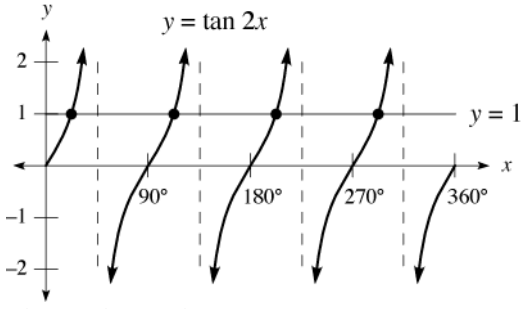
**28**. Graphing each side of the equation:




The solutions are $x = 22.5°, 112.5°, 202.5°, 292.5°$ .

**30**. Solving the equation:

$$\sin(3x+5) = 0.4$$

$$3x+5 = \sin^{-1}(0.4) + 2k\pi, \left(\pi - \sin^{-1}(0.4)\right) + 2k\pi$$

$$3x+5 \approx 0.41 + 2k\pi, 2.73 + 2k\pi$$

$$3x \approx -4.59 + 2k\pi, -2.27 + 2k\pi$$

$$x \approx -1.53 + \frac{2k\pi}{3}, -0.76 + \frac{2k\pi}{3}$$

**32**. Solving the equation:

$$\cos(\pi x - 3) = -0.41$$

$$\pi x - 3 = \left(\pi - \cos^{-1}(0.41)\right) + 2k\pi, \left(\pi + \cos^{-1}(0.41)\right) + 2k\pi$$

$$\pi x - 3 \approx 1.99 + 2k\pi, 4.29 + 2k\pi$$

$$\pi x \approx 4.99 + 2k\pi, 7.29 + 2k\pi$$

$$x \approx 1.59 + 2k, 2.32 + 2k$$

**34**. Solving the equation:

$$\tan(0.2x - \pi) = -9$$

$$0.2x - \pi = \left(\pi - \tan^{-1}(9)\right) + k\pi$$

$$0.2x - \pi \approx 1.68 + k\pi$$

$$0.2x \approx 4.82 + k\pi$$

$$x \approx 24.12 + 5k\pi$$

$$x \approx 8.41 + 5k\pi$$

The final answer was arrived at by subtracting $24.12 - 5\pi \approx 8.41$ .

**36**. Since $\sin 2x \cos x + \cos 2x \sin x = \sin(2x + x) = \sin 3x$ , solving $\sin 3x = 1$ results in all values of the form:

$$3x = \frac{\pi}{2} + 2k\pi$$

$$x = \frac{\pi}{6} + \frac{2k\pi}{3}$$

Substituting $k = 0$, 1, and 2 results in the solutions $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ .

**38**. Since $\cos 2x \cos x - \sin 2x \sin x = \cos(2x + x) = \cos 3x$ , solving $\cos 3x = \frac{\sqrt{2}}{2}$ results in all values of the form:

$$3x = \frac{\pi}{4} + 2k\pi \qquad 3x = \frac{7\pi}{4} + 2k\pi$$

$$x = \frac{\pi}{12} + \frac{2k\pi}{3} \qquad x = \frac{7\pi}{12} + \frac{2k\pi}{3}$$

Substituting $k = 0$, 1, and 2 results in the solutions $x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$ .



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**40**. Since $\sin 2x\cos 3x + \cos 2x\sin 3x = \sin(2x+3x) = \sin 5x$ , solving $\sin 5x = -1$ results in all values of the form:

$$5x = \frac{3\pi}{2} + 2k\pi$$

$$x = \frac{3\pi}{10} + \frac{2k\pi}{5}$$

**42**. Since $\cos^2 4x = 1$ , $\cos 4x = \pm 1$ . All values will take the form:

$$4x = 0 + 2k\pi \qquad\qquad 4x = \pi + 2k\pi$$

$$x = \frac{k\pi}{2} \qquad\qquad x = \frac{\pi}{4} + \frac{k\pi}{2}$$

Note that these solutions can be combined as $x = \frac{k\pi}{4}$ , where $k$ is any integer.

**44**. Since $\sin^3 5x = -1$ , $\sin 5x = -1$ . All values will take the form:

$$5x = \frac{3\pi}{2} + 2k\pi$$

$$x = \frac{3\pi}{10} + \frac{2k\pi}{5}$$

**46**. First factor the equation:

$$2\sin^2 3\theta + 3\sin 3\theta + 1 = 0$$

$$(2\sin 3\theta + 1)(\sin 3\theta + 1) = 0$$

$$\sin 3\theta = -\frac{1}{2}, -1$$

For $\sin 3\theta = -\frac{1}{2}$ , all values will take the form:

$$3\theta = 210° + 360°k \qquad\qquad 3\theta = 330° + 360°k$$

$$\theta = 70° + 120°k \qquad\qquad \theta = 110° + 120°k$$

For $\sin 3\theta = -1$ , all values will take the form:

$$3\theta = 270° + 360°k$$

$$\theta = 90° + 120°k$$

**48**. First factor the equation:

$$2\cos^2 2\theta - \cos 2\theta - 1 = 0$$

$$(2\cos 2\theta + 1)(\cos 2\theta - 1) = 0$$

$$\cos 2\theta = -\frac{1}{2}, 1$$

For $\cos 2\theta = -\frac{1}{2}$ , all values will take the form:

$$2\theta = 120° + 360°k \qquad\qquad 2\theta = 240° + 360°k$$

$$\theta = 60° + 180°k \qquad\qquad \theta = 120° + 180°k$$

For $\cos 2\theta = 1$ , all values will take the form:

$$2\theta = 0° + 360°k$$

$$\theta = 180°k$$

Note that these solutions can be combined as $\theta = 60°k$ , where $k$ is any integer.

**50**. Since $\cot^2 3\theta = 1$ , $\cot 3\theta = \pm 1$ . For $\cot 3\theta = 1$ , all values will take the form:

$$3\theta = 45° + 180°k$$

$$\theta = 15° + 60°k$$

For $\cot 3\theta = -1$ , all values will take the form:

$$3\theta = 135° + 180°k$$

$$\theta = 45° + 60°k$$

Note that these solutions can be combined as $\theta = 15° + 30°k$ , where $k$ is any integer.



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**52**. Solving the equation:

$$7+12\sin(5t+3)=2$$
$$12\sin(5t+3)=-5$$
$$\sin(5t+3)=-\frac{5}{12}$$
$$5t+3=\left(\pi+\sin^{-1}\left(\frac{5}{12}\right)\right)+2k\pi,\left(2\pi-\sin^{-1}\left(\frac{5}{12}\right)\right)+2k\pi$$
$$5t+3\approx 3.57+2k\pi,5.85+2k\pi$$
$$5t\approx 0.57+2k\pi,2.85+2k\pi$$
$$t\approx 0.11+\frac{2k\pi}{5},0.57+\frac{2k\pi}{5}$$

**54**. Solving the equation:

$$3\tan(0.5t-\pi)-1.5=0$$
$$3\tan(0.5t-\pi)=1.5$$
$$\tan(0.5t-\pi)=0.5$$
$$0.5t-\pi=\tan^{-1}(0.5)+k\pi$$
$$0.5t-\pi\approx 0.46+k\pi$$
$$0.5t\approx 3.61+k\pi$$
$$t\approx 7.21+2k\pi$$
$$t\approx 0.93+2k\pi$$

The final answer was arrived at by subtracting $7.21-2\pi\approx 0.93$.

**56**. Squaring each side of the equation:

$$\sin\theta-\cos\theta=1$$
$$(\sin\theta-\cos\theta)^2=1$$
$$\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta=1$$
$$1-\sin 2\theta=1$$
$$\sin 2\theta=0$$

All solutions will take the form:

$$2\theta=0°+360°k \qquad 2\theta=180°+360°k$$
$$\theta=180°k \qquad \theta=90°+180°k$$

Substituting $k=0$ and $k=1$ results in the possible solutions $\theta=0°,90°,180°,270°$. Since this equation was solved by squaring, these possible solutions must be checked:

$\sin 0°-\cos 0°=0-1=-1$ (doesn't check)

$\sin 90°-\cos 90°=1-0=1$ (checks)

$\sin 180°-\cos 180°=0-(-1)=1$ (checks)

$\sin 270°-\cos 270°=-1-0=-1$ (doesn't check)

Only $\theta=90°,180°$ are solutions.

**58**. Squaring each side of the equation:

$$\cos\theta-\sin\theta=-1$$
$$(\cos\theta-\sin\theta)^2=1$$
$$\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta=1$$
$$1-\sin 2\theta=1$$
$$\sin 2\theta=0$$



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The possible solutions $\theta = 0°, 90°, 180°, 270°$. Checking these solutions:

$\cos 0° - \sin 0° = 1 - 0 = 1$ (doesn't check)

$\cos 90° - \sin 90° = 0 - 1 = -1$ (checks)

$\cos 180° - \sin 180° = -1 - 0 = -1$ (checks)

$\cos 270° - \sin 270° = 0 - (-1) = 1$ (doesn't check)

Only $\theta = 90°, 180°$ are solutions.

**60**. Using $a = 1$, $b = -6$, and $c = 4$ in the quadratic formula:

$$\cos 3\theta = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} = \frac{6 \pm \sqrt{36 - 16}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$$

So either $\cos 3\theta = 3 + \sqrt{5} \approx 5.24$, which is impossible, or $\cos 3\theta = 3 - \sqrt{5} \approx 0.764$. All solutions will take the form:

$3\theta = 40.2° + 360°k$ $\qquad$ $3\theta = 319.8° + 360°k$

$\theta = 13.4° + 120°k$ $\qquad$ $\theta = 106.6° + 120°k$

Substituting $k = 0$, 1, and 2 results in the solutions $\theta = 13.4°, 106.6°, 133.4°, 226.6°, 253.4°, 346.6°$.

**62**. Using $a = 2$, $b = -6$, and $c = 3$ in the quadratic formula:

$$\sin 2\theta = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)} = \frac{6 \pm \sqrt{36 - 24}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

So either $\sin 2\theta = \frac{3 + \sqrt{3}}{2} \approx 2.37$, which is impossible, or $\sin 2\theta = \frac{3 - \sqrt{3}}{2} \approx 0.63$. All solutions will take the form:

$2\theta = 39.3° + 360°k$ $\qquad$ $2\theta = 140.7° + 360°k$

$\theta \approx 19.7° + 180°k$ $\qquad$ $\theta \approx 70.3° + 180°k$

Substituting $k = 0$ and $k = 1$ results in the solutions $\theta = 19.7°, 70.3°, 199.7°, 250.3°$.

**64**. Using $\sin^2 4\theta = 1 - \cos^2 4\theta$:

$$2(1 - \cos^2 4\theta) - 2\cos 4\theta = 1$$

$$2 - 2\cos^2 4\theta - 2\cos 4\theta = 1$$

$$-2\cos^2 4\theta - 2\cos 4\theta + 1 = 0$$

$$2\cos^2 4\theta + 2\cos 4\theta - 1 = 0$$

Using $a = 2$, $b = 2$, and $c = -1$ in the quadratic formula:

$$\cos 4\theta = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{4 + 8}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

So either $\cos 4\theta = \frac{-1 - \sqrt{3}}{2} \approx -1.37$, which is impossible, or $\cos 4\theta = \frac{-1 + \sqrt{3}}{2} \approx 0.37$. All solutions will take the form:

$4\theta = 68.5° + 360°k$ $\qquad$ $4\theta = 291.5° + 360°k$

$\theta = 17.1° + 90°k$ $\qquad$ $\theta = 72.9° + 90°k$

Substituting $k = 0$, 1, 2, and 3 results in the solutions $\theta = 17.1°, 72.9°, 107.1°, 162.9°, 197.1°, 252.9°, 287.1°, 342.9°$.

**66**. Substituting $h = 100$ ft:

$$100 = 110.5 - 98.5\cos\frac{2\pi}{15}t$$

$$-10.5 = -98.5\cos\frac{2\pi}{15}t$$

$$\cos\frac{2\pi}{15}t \approx 0.1066$$

$$\frac{2\pi}{15}t = \cos^{-1}(0.1066)$$



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Using radian mode (since $t$ is a real number) yields:

$$\frac{2\pi}{15}t = 1.46 \qquad\qquad \frac{2\pi}{15}t = 2\pi - 1.46 = 4.82$$

$$t \approx 3.5 \qquad\qquad t \approx 11.5$$

The time is either 3.5 min or 11.5 min. This solution is verified graphically.

**68**. Substituting $c = \sqrt{3}\,r$ :

$$2r\sin\frac{\theta}{2} = \sqrt{3}\,r$$

$$\sin\frac{\theta}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{\theta}{2} = 60°, 120°$$

$$\theta = 120°, 240°$$

**70**. Substituting $d$ = 100 ft:

$$100\tan\frac{1}{2}\pi t = 100$$

$$\tan\frac{1}{2}\pi t = 1$$

Using radian mode (since $t$ is a real number) yields:

$$\frac{1}{2}\pi t = \frac{\pi}{4}$$

$$\pi t = \frac{\pi}{2}$$

$$t = \frac{1}{2}$$

The other will see the light after $\frac{1}{2}$ sec if the person opposite the lighthouse first sees the light. Since the light makes a complete revolution every 2 seconds, the other will see the light after $2 - \frac{1}{2} = \frac{3}{2}$ sec if the second person sees the light first.

**72**. Setting the function equal to 0:

$$0.02\sin(880\pi t) = 0$$

$$\sin(880\pi t) = 0$$

$$880\pi t = 0 + k\pi$$

$$t = \frac{k}{880} \text{ seconds}$$

**74**. The magnitude is greatest when the function is equal to 20:

$$20\cos(3\pi t) = 20$$

$$\cos(3\pi t) = 1$$

$$3\pi t = 0 + k\pi$$

$$t = \frac{k}{3} \text{ seconds}$$

**76**. Working from the left side: $\dfrac{\sin^2 x}{(1-\cos x)^2} = \dfrac{1-\cos^2 x}{(1-\cos x)^2} = \dfrac{(1+\cos x)(1-\cos x)}{(1-\cos x)^2} = \dfrac{1+\cos x}{1-\cos x}$

**78**. Working from the left side: $\dfrac{1}{1-\sin t} + \dfrac{1}{1+\sin t} = \dfrac{1+\sin t + 1-\sin t}{(1-\sin t)(1+\sin t)} = \dfrac{2}{1-\sin^2 t} = \dfrac{2}{\cos^2 t} = 2\sec^2 t$



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**80**. Since $0° \le B \le 90°, \cos B > 0$ and thus: $\cos B = \sqrt{1-\sin^2 B} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Using the double-angle formula for cosine: $\cos 2B = \cos^2 B - \sin^2 B = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

**82**. Since $90° \le A \le 180°, \cos A < 0$ and thus: $\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\left(\frac{1}{3}\right)^2} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$

From Problem 80 $\cos B = \frac{4}{5}$, so using the addition formula for sine:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \left(\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(-\frac{2\sqrt{2}}{3}\right)\left(\frac{3}{5}\right) = \frac{4-6\sqrt{2}}{15}$$

**84**. Since $\sin 4x \cos x + \cos 4x \sin x = \sin(4x + x) = \sin 5x$, solving $\sin 5x = -1$ results in all values of the form:

$$5x = \frac{3\pi}{2} + 2\pi k$$

$$x = \frac{3\pi}{10} + \frac{2\pi}{5}k$$

The correct answer is b.

**86**. Substituting $h$ = 150 ft:

$$150 = 101 - 95\cos \pi t$$

$$49 = -95\cos \pi t$$

$$\cos \pi t = -\frac{49}{95}$$

$$\pi t = \cos^{-1}\left(-\frac{49}{95}\right)$$

Using radian mode (since $t$ is a real number) yields:

$\pi t \approx 2.113$ $\qquad$ $\pi t = 2\pi - 2.113 \approx 4.170$

$t \approx 0.7$ $\qquad$ $t \approx 1.3$

The time is either 0.7 min or 1.3 min. The correct answer is c.

**ODD SOLUTIONS**

**1.** $n\theta, n$

**3.** sum

**5.** $\sin 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = 60° + 360°k$ or $2\theta = 120° + 360°k$

$\theta = 30° + 180°k$ $\qquad$ $\theta = 60° + 180°k$

If we let $k$ = 0 and 1, we get

$\theta = 30°$ $\qquad$ $\theta = 60°$

$\theta = 210°$ $\qquad$ $\theta = 240°$

**7.** $\tan 2\theta = -1$

$2\theta = 135° + 360°k$ or $2\theta = 315° + 360°k$

$\theta = 67.5° + 180°k$ $\qquad$ $\theta = 157.5° + 180°k$

If we let $k$ = 0 and 1, we get

$\theta = 67.5°$ $\qquad$ $\theta = 157.5°$

$\theta = 247.5°$ $\qquad$ $\theta = 337.5°$



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**9.** $\cos 3\theta = -1$

$3\theta = 180^\circ + 360^\circ k$

$\theta = 60^\circ + 120^\circ k$

If we let $k = 0$, 1, and 2, we get

$\theta = 60^\circ$ $\quad$ $\theta = 300^\circ$

$\theta = 180^\circ$

**11.** $\cos 2x = \dfrac{\sqrt{2}}{2}$

$2x = \dfrac{\pi}{4} + 2k\pi$ or $2x = \dfrac{7\pi}{4} + 2k\pi$

$x = \dfrac{\pi}{8} + k\pi$ or $x = \dfrac{7\pi}{8} + k\pi$

If we let $k = 0$ and 1, we get

$x = \dfrac{\pi}{8}$ $\quad$ $x = \dfrac{7\pi}{8}$

$x = \dfrac{9\pi}{8}$ $\quad$ $x = \dfrac{15\pi}{8}$

**13.** $\sec 3x = -1$

$\cos 3x = -1$ (Reciprocal identity)

$3x = \pi + 2k\pi$

$x = \dfrac{\pi + 2k\pi}{3}$

If we let $k = 0$, 1, and 2, we get

$\theta = \dfrac{\pi}{3}$ $\quad$ $\theta = \dfrac{\pi}{3} + \dfrac{4\pi}{3} = \dfrac{5\pi}{3}$

$\theta = \dfrac{3\pi}{3} = \pi$

**15.** $\tan 2x = \sqrt{3}$

$2x = \dfrac{\pi}{3} + k\pi$

$x = \dfrac{\pi + 3k\pi}{6}$

If we let $k = 0$, 1, 2 and 3, we get

$x = \dfrac{\pi}{6}$ $\quad$ $x = \dfrac{7\pi}{6}$

$x = \dfrac{4\pi}{6} = \dfrac{2\pi}{3}$ $\quad$ $x = \dfrac{10\pi}{6} = \dfrac{5\pi}{3}$

**17.** $\sin 2\theta = \dfrac{1}{2}$

$2\theta = 30^\circ + 360^\circ k$ or $2\theta = 150^\circ + 360^\circ k$

$\theta = 15^\circ + 180^\circ k$ $\quad$ $\theta = 75^\circ + 180^\circ k$

**19.** $\cos 3\theta = 0$

$3\theta = 90^\circ + 360^\circ k$ or $3\theta = 270^\circ + 360^\circ k$

$\theta = 30^\circ + 120^\circ k$ $\quad$ $\theta = 90^\circ + 120^\circ k$

(We could also write this as $\theta = 30^\circ + 60^\circ k$ .)

**21.** $\sin 10\theta = \dfrac{\sqrt{3}}{2}$

$10\theta = 60^\circ + 360^\circ k$ or $10\theta = 120^\circ + 360^\circ k$

$\theta = 6^\circ + 36^\circ k$ $\quad$ $\theta = 12^\circ + 36^\circ k$

For problems 23 through 27, see textbook answer section for graphs.

**29.** $\cos(2x - 1) = 0.8$

$2x - 1 = 0.64 + 2k\pi$ or $2x - 1 = 5.64 + 2k\pi$

$2x = 1.64 + 2k\pi$ $\quad$ $2x = 6.64 + 2k\pi$

$x = 0.82 + k\pi$ $\quad$ $x = 3.32 + k\pi$

**31.** $\sin(\pi x + 2) = -0.88$

$\pi x + 2 = 4.22 + 2k\pi$ or $\pi x + 2 = 5.20 + 2k\pi$

$\pi x = 2.22 + 2k\pi$ $\quad$ $\pi x = 3.20 + 2k\pi$

$x = 0.71 + 2k$ $\quad$ $x = 1.02 + 2k$

**33.** $\tan(3x + \pi) = 4$

$3x + \pi = 1.33 + k\pi$

$3x = -1.82 + k\pi$

$x = -0.61 + \dfrac{k\pi}{3}$



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**35.** $2\sin 2x\cos x+\cos 2x\sin x=\dfrac{1}{2}$

$\sin(2x+x)=\dfrac{1}{2}$ (Sum formula)

$3x=\dfrac{\pi}{6}+2k\pi$ or $3x=\dfrac{5\pi}{6}+2k\pi$

$x=\dfrac{\pi}{18}+\dfrac{2k\pi}{3}$ $\quad x=\dfrac{5\pi}{18}+\dfrac{2k\pi}{3}$

$x=\dfrac{\pi+12k\pi}{18}$ $\quad x=\dfrac{5\pi+12k\pi}{3}$

If we let $k = 0, 1$, and $2$, we get

$x=\dfrac{\pi}{18}$ $\quad x=\dfrac{5\pi}{18}$

$x=\dfrac{13\pi}{18}$ $\quad x=\dfrac{17\pi}{18}$

$x=\dfrac{25\pi}{18}$ $\quad x=\dfrac{29\pi}{18}$

**37.** $\cos 2x\cos x-\sin 2x\sin x=-\dfrac{\sqrt{3}}{2}$

$\cos(2x+x)=-\dfrac{\sqrt{3}}{2}$ (Sum formula)

$\cos 3x=-\dfrac{\sqrt{3}}{2}$

$3x=\dfrac{5\pi}{6}+2k\pi$ or $3x=\dfrac{7\pi}{6}+2k\pi$

$x=\dfrac{5\pi}{18}+\dfrac{2k\pi}{3}$ $\quad x=\dfrac{7\pi}{18}+\dfrac{2k\pi}{3}$

$x=\dfrac{5\pi+12k\pi}{18}$ $\quad x=\dfrac{7\pi+12k\pi}{18}$

If we let $k = 0, 1$, and $2$, we get

$x=\dfrac{5\pi}{18}$ $\quad x=\dfrac{7\pi}{18}$

$x=\dfrac{17\pi}{18}$ $\quad x=\dfrac{19\pi}{18}$

$x=\dfrac{29\pi}{18}$ $\quad x=\dfrac{31\pi}{18}$

**39.** $\sin 3x\cos 2x+\cos 3x\sin 2x=1$

$\sin(3x+2x)=1$

$5x=\dfrac{\pi}{2}+2k\pi$

$x=\dfrac{\pi}{10}+\dfrac{2k\pi}{5}$

**41.** $\sin^2 4x=1$

$\sin 4x=\pm 1$

$4x=\dfrac{\pi}{2}+2k\pi$ or $4x=\dfrac{3\pi}{2}+2k\pi$

$x=\dfrac{\pi}{8}+\dfrac{k\pi}{2}$ $\quad x=\dfrac{3\pi}{8}+\dfrac{k\pi}{2}$

We could also write this as $4x=\dfrac{\pi}{2}+k\pi$ or $x=\dfrac{\pi}{8}+\dfrac{k\pi}{4}$

**43.** $\cos^3 5x=-1$

$\cos x=-1$

$5x=\pi+2k\pi$

$x=\dfrac{\pi}{5}+\dfrac{2k\pi}{5}$

**45.** $2\sin^2 3\theta+\sin 3\theta-1=0$

$(\sin 3\theta-1)(\sin 3\theta+1)=0$

$2\sin 3\theta-1=0$ or $\sin 3\theta+1=0$

$2\sin 3\theta=1$ $\quad \sin 3\theta=-1$

$\sin 3\theta=\dfrac{1}{2}$ $\quad 3\theta=270^\circ+360^\circ k$

$3\theta=30^\circ+360^\circ k$ or $3\theta=150^\circ+360^\circ k$ $\quad \theta=90^\circ+120^\circ k$

$\theta=10^\circ+120^\circ k$ $\quad \theta=50^\circ+120^\circ k$



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**47.** $2\cos^2 2\theta + 3\cos 2\theta + 1 = 0$

$(2\cos 2\theta + 1)(\cos 2\theta + 1) = 0$

$2\cos 2\theta + 1 = 0$ or $\cos 2\theta + 1 = 0$

$2\cos 2\theta = -1$ $\quad 2\theta = 180^\circ + 360^\circ k$

$\cos 2\theta = -\frac{1}{2}$ $\quad \theta = 90^\circ + 180^\circ k$

$2\theta = 120^\circ + 360^\circ k$ or $2\theta = 240^\circ + 360^\circ k$

$\theta = 60^\circ + 180^\circ k$ $\quad \theta = 120^\circ + 180^\circ k$

**49.** $\tan^2 3\theta = 3$

$\tan 3\theta = \pm\sqrt{3}$

$3\theta = 60^\circ + 180^\circ k$ or $3\theta = 120^\circ + 180^\circ k$

$\theta = 20^\circ + 60^\circ k$ $\quad \theta = 40^\circ + 60^\circ k$

**51.** $15 - 9\cos(6t - 5) = 11$

$\cos(6t - 5) = \frac{4}{9}$

$6t - 5 = 1.11 + 2k\pi$ or $6t - 5 = 5.17 + 2k\pi$

$6t = 6.11 + 2k\pi$ $\quad 6t = 10.17 + 2k\pi$

$t = 1.02 + \frac{k\pi}{3}$ $\quad t = 1.70 + \frac{k\pi}{3}$

**53.** $2\tan(4\pi + 1) - 3 = -11$

$\tan(4\pi t + 1) = -4$

$4\pi t + 1 = 1.82 + k\pi$

$4\pi t = 0.82 + k\pi$

$t = 0.06 + 0.25k$

**55.** $\cos\theta - \sin\theta = 1$

$\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = 1$ Square both sides

$-2\sin\theta\cos\theta + 1 = 1$ Pythagorean identity

$\sin 2\theta = 0$ Double-angle identity

$2\theta = 0^\circ + 360^\circ k$ or $2\theta = 180^\circ + 360^\circ k$

$\theta = 180^\circ k$ $\quad \theta = 90^\circ + 180^\circ k$

If we let $k = 0$ and 1, we get

$\theta = 0^\circ$ $\quad \theta = 90^\circ$

$\theta = 180^\circ$ $\quad \theta = 270^\circ$

Since we squared both sides, we must check. Only $0^\circ$ and $270^\circ$ check.

**57.** $\sin\theta + \cos\theta = -1$

$\sin^2 + 2\sin\theta\cos\theta + \cos^2\theta = 1$ Square both sides

$1 + 2\sin\theta\cos\theta = 1$ Pythagorean identity

$\sin 2\theta = 0$ Double-angle identity

$2\theta = 0^\circ + 360^\circ k$ or $2\theta = 180^\circ + 360^\circ k$

$\theta = 180^\circ k$ $\quad \theta = 90^\circ + 180^\circ k$

If we let $k = 0$ and 1, we get

$\theta = 0^\circ$ $\quad \theta = 90^\circ$

$\theta = 180^\circ$ $\quad \theta = 270^\circ$

Since we squared both sides, we must check. Only $180^\circ$ and $270^\circ$ check.



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**59.** $\sin^2 2\theta - 4\sin 2\theta - 1 = 0$

We use the quadratic formula with $a = 1, b = -4, c = -1$,

$$\sin 2\theta = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{20}}{2}$$

$$\sin 2\theta = \frac{4 + \sqrt{20}}{2} \quad \text{or} \quad \sin 2\theta = \frac{4 - \sqrt{20}}{2}$$

$$= 4.236 \qquad = -0.236$$

No solution

$2\theta = 193.7^\circ + 360^\circ k$ or $2\theta = 346.3^\circ + 360^\circ k$

$\theta = 96.8^\circ + 180^\circ k$ $\theta = 173.2^\circ + 180^\circ k$

If we let $k = 0$ and 1, we get:

$\theta = 96.8^\circ$ $\theta = 173.2^\circ$

$\theta = 276.8^\circ$ $\theta = 353.2^\circ$

**61.** $4\cos^2 3\theta - 8\cos 3\theta + 1 = 0$ $a = 4, b = -8, c = 1$

$$\cos 3\theta = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{8 \pm \sqrt{48}}{8}$$

$$\cos 3\theta = \frac{8 \pm 6.9282}{8}$$

$\cos 3\theta = 1.8660$ or $\cos 3\theta = 0.1340$

No solution

$3\theta = 82.3^\circ + 360^\circ k$ $3\theta = 277.7^\circ + 360^\circ k$

$\theta = 27.4^\circ + 120^\circ k$ $\theta = 92.6^\circ + 120^\circ k$

If we let $k = 0$, 1 and 2, we get

$\theta = 27.4^\circ$ $\theta = 92.6^\circ$

$\theta = 147.4^\circ$ $\theta = 212.6^\circ$

$\theta = 267.4^\circ$ $\theta = 332.6^\circ$

**63.** $2\cos^2 4\theta + 2\sin 4\theta = 1$

$2(1 - \sin^2 4\theta) + 2\sin 4\theta - 1 = 0$ Pythagorean identity

$2 - 2\sin^2 4\theta + 2\sin 4\theta - 1 = 0$

$-2\sin^2 4\theta + 2\sin 4\theta + 1 = 0$

$2\sin^2 4\theta - 2\sin 4\theta - 1 = 0$

We apply the quadratic formula with $a = 2, b = -2$, and $c = -1$:

$$\sin 4\theta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{12}}{4}$$

$$\sin 4\theta = \frac{2 + \sqrt{12}}{4} \quad \text{or} \quad \sin 4\theta = \frac{2 - \sqrt{12}}{4}$$

$$= 1.366 \qquad = -0.366$$

No solution

$4\theta = 201.5^\circ + 360^\circ k$ or $4\theta = 338.5^\circ + 360^\circ k$

$\theta = 50.4^\circ + 90^\circ k$ $\theta = 84.6^\circ + 90^\circ k$

If we let $k = 0$, 1, 2, and 3, we get

$\theta = 50.4^\circ, 140.4^\circ, 230.4^\circ, 320.4^\circ$ or $\theta = 84.6^\circ, 174.6^\circ, 264.6^\circ, 354.6^\circ$



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**65.** We want to find $t$ when $h = 100$.

$$100 = 139 - 125\cos\frac{\pi}{10}t$$

$$125\cos\frac{\pi}{10}t = 39$$

$$\cos\frac{\pi}{10}t = 0.312$$

$$\frac{\pi}{10}t = 1.253 + 2\pi k \quad \text{or} \quad \frac{\pi}{10}t = (2\pi - 1.253) + 2\pi k$$

$$t = 4.0 + 20k \qquad \frac{\pi}{10}t = 5.030 + 2\pi k$$

$$t = 16.0 + 20k \text{ (where } k = 0 \text{ and 1for one cycle)}$$

It will be at 100 ft after 4.0 min and 16.0 min.

**67.** $l = 2r\sin\frac{180^\circ}{n}$ where $l = r$

$$r = 2r\sin\frac{180^\circ}{n}$$

$$\sin\frac{180^\circ}{n} = \frac{r}{2r}$$

$$\sin\frac{180^\circ}{n} = \frac{1}{2}$$

$$\frac{180^\circ}{n} = 30^\circ \quad \text{or} \quad \frac{180^\circ}{n} = 150^\circ$$

$$180^\circ = 30^\circ n \qquad 180^\circ = 150^\circ n$$

$$n = 6 \qquad n = 1.2$$

The polygon has 6 sides. Not possible

**69.** $d = 10\tan\pi t$ where $d = 10$

$$10 = 10\tan\pi t$$

$$\tan\pi t = 1$$

$$\pi t = \frac{\pi}{4} + k\pi$$

$$t = \frac{1}{4} + k$$

$t$ is $\frac{1}{4}$ second and every second after that.

**71.** $163\sin(120\pi t) = 163$ The amplitude is 163. Therefore the maximum occurs at $y = 163$.

$$\sin(120\pi t) = 1$$

$$120\pi t = \frac{\pi}{2} + 2k\pi$$

$$t = \frac{1}{240} + \frac{k}{60} \text{ seconds}$$

**73.** $-3.5\cos(2\pi t) = 0$

$$\cos(2\pi t) = 0$$

$$2\pi t = \frac{\pi}{2} + k\pi$$

$$t = 0.25 + 0.5k \text{ seconds}$$

**75.**

| | |
|---|---|
| $\frac{\sin x}{1+\cos x} = \frac{\sin x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}$ | Multiply by a fraction equal to 1 |
| $= \frac{\sin x(1-\cos x)}{1-\cos^2 x}$ | Multiply |
| $= \frac{\sin x(1-\cos x)}{\sin^2 x}$ | Pythagorean identity |
| $= \frac{1-\cos x}{\sin x}$ | Reduce |



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**77.**

$$\frac{1}{1+\cos t}+\frac{1}{1-\cos t}=\frac{1}{1+\cos t}\cdot\frac{1-\cos t}{1-\cos t}+\frac{1}{1-\cos t}\cdot\frac{1+\cos t}{1+\cos t}$$ Multiply by a fraction equal to 1

$$=\frac{1-\cos t}{1-\cos^2 t}+\frac{1+\cos t}{1-\cos^2 t}$$ Multiply fractions

$$=\frac{1-\cos t+1+\cos t}{1-\cos^2 t}$$ Add fractions

$$=\frac{2}{\sin^2 t}$$ Pythagorean identity

$$=2\csc^2 t$$ Reciprocal identity

**79.** If $\sin A=\frac{1}{3}$ and $A$ is in QII, then

$$\cos A=-\sqrt{1-\sin^2 A}$$

$$=\sqrt{1-\frac{1}{9}}=-\sqrt{\frac{8}{9}}=-\frac{2\sqrt{2}}{3}$$

$$\sin 2A=2\sin A\cos A$$

$$=2\left(\frac{1}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right)$$

$$=-\frac{4\sqrt{2}}{9}$$

**81.** If $90^\circ\le A\le 180^\circ$, then $45^\circ\le\frac{A}{2}\le 90^\circ$

Also, $\sin A=\frac{1}{3}$ and $\cos A=-\frac{2\sqrt{2}}{3}$ (from problem 65)

Therefore, $\cos\frac{A}{2}=\sqrt{\frac{1+\cos A}{2}}$

$$=\sqrt{\frac{1+\left(\frac{-2\sqrt{2}}{3}\right)}{2}}=\sqrt{\frac{3-2\sqrt{2}}{6}}$$



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# 6.4 Parametric Equations and Further Graphing

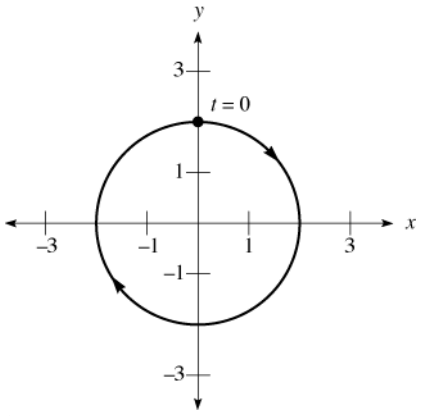
**EVEN SOLUTIONS**

**2**. When we sketch the graph for a pair of parametric equations, the resulting curve is called a plane curve.

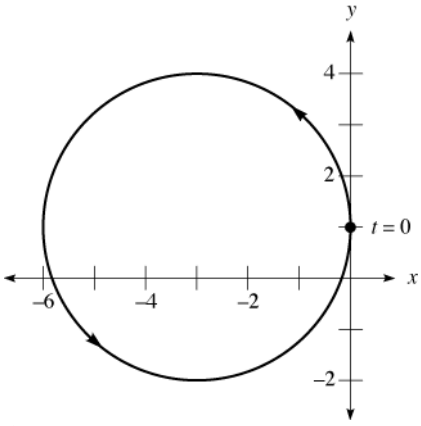
**4**. Eliminating the parameter means algebraically eliminating the variable $t$ from a pair of parametric equations to obtain a single equation in the variables $x$ and $y$ only.

**6**. Given values $|\mathbf{V}_0|$ and $\theta$, the plane curve for the parametric equations $x = (|\mathbf{V}_0|\cos\theta)t$, $y = (|\mathbf{V}_0|\sin\theta)t - 16t^2$ is a parabola passing through the point (0,0).

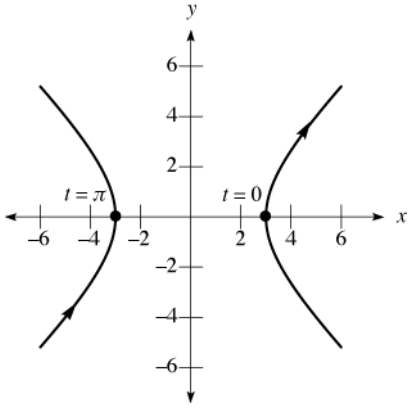
**8**. Sketching the graph:



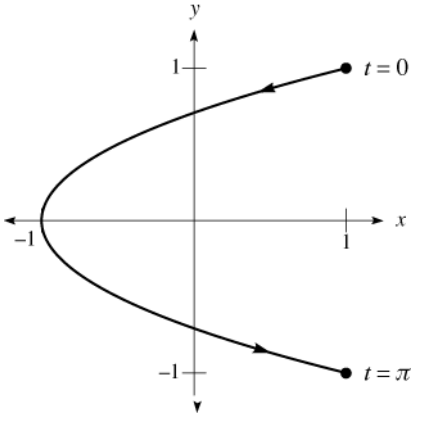

**10**. Sketching the graph:




**12**. Sketching the graph:




**14**. Sketching the graph:




**16**. Since $\sin t = -x$ and $\cos t = y$:

$$\sin^2 t + \cos^2 t = 1$$
$$(-x)^2 + y^2 = 1$$
$$x^2 + y^2 = 1$$

**18**. Since $\cos t = \frac{x}{2}$ and $\sin t = \frac{y}{2}$:

$$\sin^2 t + \cos^2 t = 1$$
$$\left(\frac{y}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = 1$$
$$\frac{y^2}{4} + \frac{x^2}{4} = 1$$
$$x^2 + y^2 = 4$$



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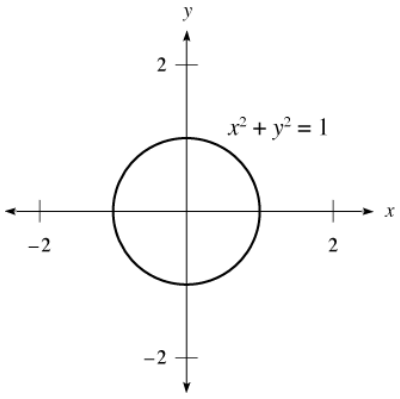
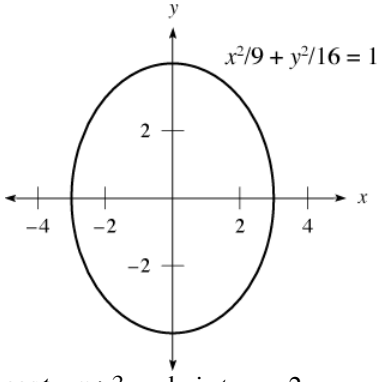
The graph is a circle with center = (0,0) and radius = 1:




**20**. Since $\sin t = \frac{x}{3}$ and $\cos t = \frac{y}{4}$:

$$\sin^2 t + \cos^2 t = 1$$
$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$
$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

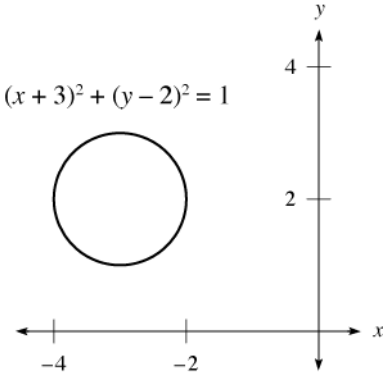
The graph is an ellipse:



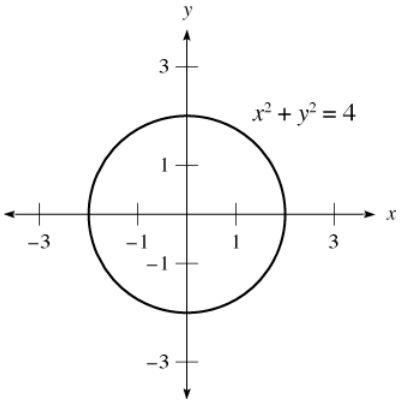

**24**. Since $\cos t = x + 3$ and $\sin t = y - 2$:

$$\sin^2 t + \cos^2 t = 1$$
$$(y-2)^2 + (x+3)^2 = 1$$
$$(x+3)^2 + (y-2)^2 = 1$$

The graph is a circle with center = (–3,2) and radius = 1:



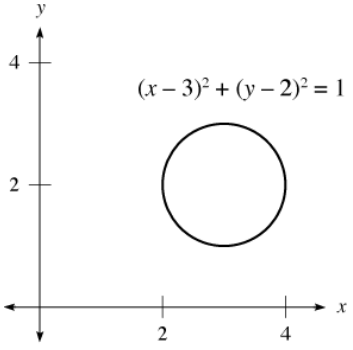

The graph is a circle with center = (0,0) and radius = 2:




**22**. Since $\sin t = x - 3$ and $\cos t = y - 2$:

$$\sin^2 t + \cos^2 t = 1$$
$$(x-3)^2 + (y-2)^2 = 1$$

The graph is a circle with center = (3,2) and radius = 1:



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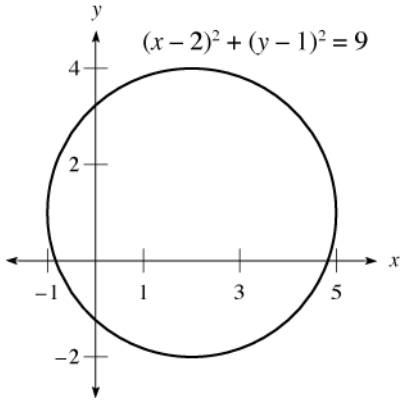
**26**. Since $x-2=3\sin t, \sin t=\dfrac{x-2}{3}$. Since $y-1=3\cos t, \cos t=\dfrac{y-1}{3}$. Therefore:

$$\sin^2 t+\cos^2 t=1$$
$$\left(\frac{x-2}{3}\right)^2+\left(\frac{y-1}{3}\right)^2=1$$
$$\frac{(x-2)^2}{9}+\frac{(y-1)^2}{9}=1$$
$$(x-2)^2+(y-1)^2=9$$

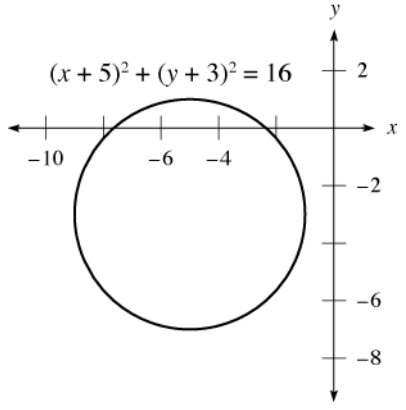
The graph is a circle with center = (2,1) and radius = 3:




**28**. Since $x+5=4\sin t, \sin t=\dfrac{x+5}{4}$. Since $y+3=4\cos t, \cos t=\dfrac{y+3}{4}$. Therefore:

$$\sin^2 t+\cos^2 t=1$$
$$\left(\frac{x+5}{4}\right)^2+\left(\frac{y+3}{4}\right)^2=1$$
$$\frac{(x+5)^2}{16}+\frac{(y+3)^2}{16}=1$$
$$(x+5)^2+(y+3)^2=16$$

The graph is a circle with center = (–5,–3) and radius = 4:



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**30**. Since $\tan t = x$ and $\sec t = y$:

$$1 + \tan^2 t = \sec^2 t$$
$$1 + x^2 = y^2$$
$$y^2 - x^2 = 1$$

The graph would be a hyperbola.

**32**. Since $\cot t = \frac{x}{4}$ and $\csc t = \frac{y}{4}$:

$$1 + \cot^2 t = \csc^2 t$$
$$1 + \left(\frac{x}{4}\right)^2 = \left(\frac{y}{4}\right)^2$$
$$1 + \frac{x^2}{16} = \frac{y^2}{16}$$
$$\frac{y^2}{16} - \frac{x^2}{16} = 1$$

The graph would be a hyperbola.

**34**. Since $x - 3 = 5\csc t, \csc t = \frac{x-3}{5}$. Since $y - 2 = 5\cot t, \cot t = \frac{y-2}{5}$. Therefore:

$$1 + \cot^2 t = \csc^2 t$$
$$1 + \left(\frac{y-2}{5}\right)^2 = \left(\frac{x-3}{5}\right)^2$$
$$1 + \frac{(y-2)^2}{25} = \frac{(x-3)^2}{25}$$
$$\frac{(x-3)^2}{25} - \frac{(y-2)^2}{25} = 1$$

The graph would be a hyperbola.

**36**. Since $x = \cos 2t = 2\cos^2 t - 1$:

$$\cos 2t = 2\cos^2 t - 1$$
$$x = 2y^2 - 1$$

The graph would be a portion of a parabola.

**38**. Since $\cos t = -\cos t$, the equation is $y = -x$. This graph would be a line segment.

**40**. Since $\frac{x}{2} = \sin t$ and $\frac{y}{3} = \sin t$:

$$\frac{x}{2} = \frac{y}{3}$$
$$3x = 2y$$

The graph would be a line segment.

**42**. Let $t$ represent the central angle in radians. Since the radius of the wheel is 67.5 feet, we have:

$$x = 67.5\cos t$$
$$y = 67.5\sin t$$

To shift the wheel so that the rider starts at the bottom, subtract $\frac{\pi}{2}$ from $t$ to obtain the equations:

$$x = 67.5\cos\left(t - \frac{\pi}{2}\right)$$
$$y = 67.5\sin\left(t - \frac{\pi}{2}\right)$$

We now need to find a connection between $t$ and the parameter $T$. Since it takes 30 min to make 1 revolution, then:

$$\frac{t}{T} = \frac{2\pi}{30}$$
$$t = \frac{\pi}{15}T$$



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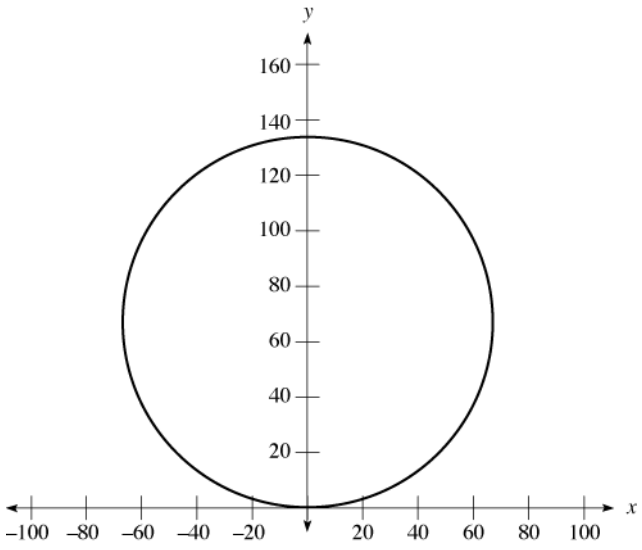
Therefore the parametric equations are:

$$x = 67.5\cos\left(\frac{\pi}{15}T - \frac{\pi}{2}\right)$$

$$y = 67.5\sin\left(\frac{\pi}{15}T - \frac{\pi}{2}\right)$$

Graphing these equations:




**44**. Let $t$ represent the central angle in radians. Since the radius of the wheel is 82.5 feet, we have:

$$x = 82.5\cos t$$

$$y = 82.5\sin t$$

To shift the wheel 100 feet to the right and 91.5 feet up (82.5 + 9 = 91.5), the parametric equations are:

$$x = 100 + 82.5\cos t$$

$$y = 91.5 + 82.5\sin t$$

To shift the wheel so that the rider starts at the bottom, subtract $\frac{\pi}{2}$ from $t$ to obtain the equations:

$$x = 100 + 82.5\cos\left(t - \frac{\pi}{2}\right)$$

$$y = 91.5 + 82.5\sin\left(t - \frac{\pi}{2}\right)$$

We now need to find a connection between $t$ and the parameter $T$. Since it takes 1.5 min = 90 sec to make 1 revolution, then:

$$\frac{t}{T} = \frac{2\pi}{90}$$

$$t = \frac{\pi}{45}T$$

Therefore the parametric equations are:

$$x = 100 + 82.5\cos\left(\frac{\pi}{45}T - \frac{\pi}{2}\right)$$

$$y = 91.5 + 82.5\sin\left(\frac{\pi}{45}T - \frac{\pi}{2}\right)$$



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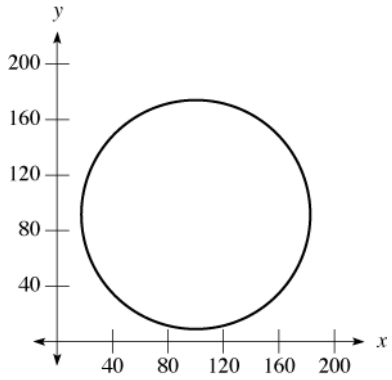
Graphing these equations:



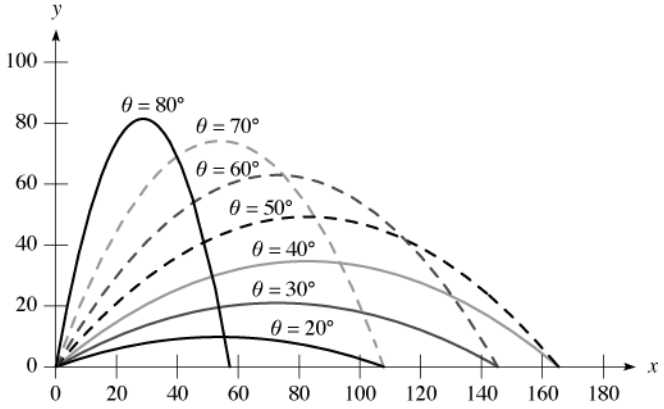

**46**. First convert the initial speed 50 mi/hr to ft/sec: $\frac{50 \text{ mi}}{1 \text{ hr}} \bullet \frac{5280 \text{ ft}}{1 \text{ mi}} \bullet \frac{1 \text{ hr}}{3600 \text{ sec}} \approx 73.3 \text{ ft/sec}$

The parametric equations for the path are then:

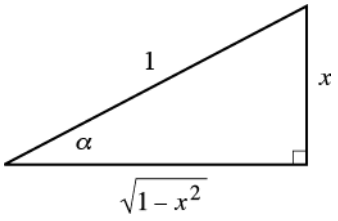
$$x = (73.3\cos\theta)t$$

$$y = (73.3\sin\theta)t - 16t^2$$

Now substituting $\theta = 20°, 30°, 40°, 50°, 60°, 70°, 80°$ results in the graphs:

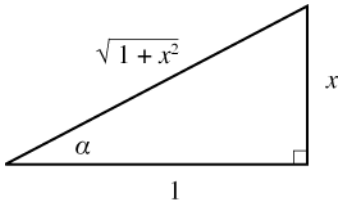



**48**. Let $\alpha = \sin^{-1} x$, so $\sin\alpha = x$. Drawing the triangle:




Therefore: $\cos\left(\sin^{-1} x\right) = \cos\alpha = \sqrt{1-x^2}$

**50**. Let $\alpha = \tan^{-1} x$, so $\tan\alpha = x$. Drawing the triangle:




Therefore: $\cos\left(2\tan^{-1} x\right) = \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 - \left(\frac{x}{\sqrt{1+x^2}}\right)^2 = \frac{1-x^2}{1+x^2}$



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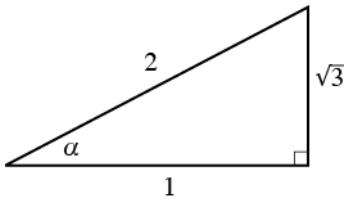
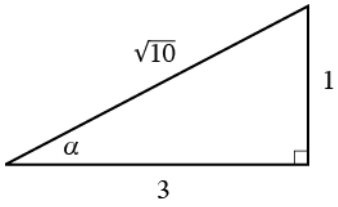
**52.** Using the product-to-sum formula: $8\sin 3x\cos 2x = \frac{8}{2}\left(\sin\left(3x+2x\right)+\sin\left(3x-2x\right)\right) = 4\left(\sin 5x+\sin x\right)$

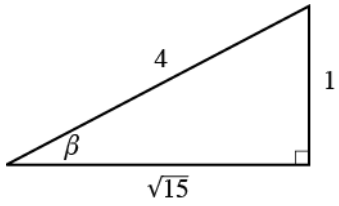
**54.** Let $\alpha = \cos^{-1}\frac{1}{2}$, so $\cos\alpha = \frac{1}{2}$. Drawing the triangle:




Therefore: $\sin\left(\cos^{-1}\frac{1}{2}\right) = \sin\alpha = \frac{\sqrt{3}}{2}$

**56.** Let $\alpha = \tan^{-1}\frac{1}{3}$, so $\tan\alpha = \frac{1}{3}$. Let $\beta = \sin^{-1}\frac{1}{4}$, so $\sin\beta = \frac{1}{4}$. Drawing the triangles:




Therefore:

$$\sin\left(\tan^{-1}\frac{1}{3}+\sin^{-1}\frac{1}{4}\right) = \sin\left(\alpha+\beta\right)$$
$$= \sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$= \left(\frac{1}{\sqrt{10}}\right)\left(\frac{\sqrt{15}}{4}\right)+\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{4}\right)$$
$$= \frac{\sqrt{15}+3}{4\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}$$
$$= \frac{\sqrt{150}+3\sqrt{10}}{40}$$
$$= \frac{5\sqrt{6}+3\sqrt{10}}{40}$$

**58.** When $t = 0$, $x = 6$ and $y = -1$. When $t = \frac{\pi}{2}$, $x = 2$ and $y = 2$. The correct answer is a.

**60.** Since $\sin t = \frac{x}{3}$ and $\cos t = y-2$:

$$\sin^2 t+\cos^2 t = 1$$
$$\left(\frac{x}{3}\right)^2+\left(y-2\right)^2 = 1$$
$$\frac{x^2}{9}+\left(y-2\right)^2 = 1$$

The correct answer is a.



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**ODD SOLUTIONS**

**1.** parametric, parameter **3.** orientation, direction, increases **5.** $\sin t, \cos t$, Pythagorean

For problems 7 through 13, see textbook answer section for graphs.

**15.** $\sin t = x$ and $\cos t = y$

$\sin^2 t + \cos^2 t = 1$ Pythagorean identity

$x^2 + y^2 = 1$ Substitute known values

The graph is a circle with its center at the origin and $r = 1$.

**17.** $3\cos t = x$ and $3\sin t = y$

$\cos t = \frac{x}{3}$ $\sin t = \frac{y}{3}$

$\cos^2 t + \sin^2 t = 1$ Pythagorean identity

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ Substitute known values

$\frac{x^2}{9} + \frac{y^2}{9} = 1$ Simplify

$x^2 + y^2 = 9$

The graph is a circle with its center at the origin and $r = 3$.

**19.** $2\sin t = x$ and $4\cos t = y$

$\sin t = \frac{x}{2}$ $\cos t = \frac{y}{4}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$ or $\frac{x^2}{4} + \frac{y^2}{16} = 1$

The graph is an ellipse with center at the origin. The intercepts on the major axis are $(0, 4)$ and $(0, -4)$. The intercepts on the minor axis are $(2, 0)$ and $(-2, 0)$.

**21.** $2 + \sin t = x$ and $3 + \cos t = y$

$\sin t = x - 2$ $\cos t = y - 3$

$\sin^2 t + \cos^2 t = 1$

$(x-2)^2 + (y-3)^2 = 1$

The graph is a circle with center at $(2, 3)$ and $r = 1$.

**23.** $\sin t - 2 = x$ and $\cos t - 3 = y$

$\sin t = x + 2$ $\cos t = y + 3$

$\sin^2 t + \cos^2 t = 1$

$(x+2)^2 + (y+3)^2 = 1$

The graph is a circle with center at $(-2, -3)$ and $r = 1$.



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**25.** $3+2\sin t = x$ and $1+2\cos t = y$

$2\sin t = x-3$ $\qquad$ $2\cos t = y-1$

$\sin t = \dfrac{x-3}{2}$ $\qquad$ $\cos t = \dfrac{y-1}{2}$

$\sin^2 t + \cos^2 t = 1$

$\left(\dfrac{x-3}{2}\right)^2 + \left(\dfrac{y-1}{2}\right)^2 = 1$

$\dfrac{(x-3)^2}{4} + \dfrac{(y-1)^2}{4} = 1$

$(x-3)^2 + (y-1)^2 = 4$

The graph is a circle with center at (3, 1) and $r = 2$.

**27.** $3\cos t - 3 = x$ and $3\sin t + 1 = y$

$3\cos t = x+3$ $\qquad$ $3\sin t = y-1$

$\cos t = \dfrac{x+3}{3}$ $\qquad$ $\sin t = \dfrac{y-1}{3}$

$\cos^2 t + \sin^2 t = 1$

$\left(\dfrac{x+3}{3}\right)^2 + \left(\dfrac{y-1}{3}\right)^2 = 1$

$\dfrac{(x+3)^2}{9} + \dfrac{(y-1)^2}{9} = 1$

$(x+3)^2 + (y-1)^2 = 9$

The graph is a circle with center at $(-3, 1)$ and $r = 3$.

**29.** $\sec t = x$ and $\tan t = y$

$\sec^2 t = \tan^2 t + 1$ Pythagorean identity

$x^2 = y^2 + 1$ Substitute known values

$x^2 - y^2 = 1$ Simplify

**31.** $3\csc t = x$ and $3\cot t = y$

$\csc t = \dfrac{x}{3}$ $\qquad$ $\cot t = \dfrac{y}{3}$

$\csc^2 t = \cot^2 t + 1$ Pythagorean identity

$\left(\dfrac{x}{3}\right)^2 = \left(\dfrac{y}{3}\right)^2 + 1$ Substitute known values

$\dfrac{x^2}{9} = \dfrac{y^2}{9} + 1$ Simplify

$\dfrac{x^2}{9} - \dfrac{y^2}{9} = 1$



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**33.** $2+3\tan t = x$ and $4+3\sec t = y$

$3\tan t = x-2$ $\quad$ $3\sec t = y-4$

$\tan t = \dfrac{x-2}{3}$ $\quad$ $\sec t = \dfrac{y-4}{3}$

$\sec^2 t = \tan^2 t + 1$ $\quad$ Pythagorean identity

$\left(\dfrac{y-4}{3}\right)^2 = \left(\dfrac{x-2}{3}\right)^2 + 1$ $\quad$ Substitute known values

$\dfrac{(y-4)^2}{9} = \dfrac{(x-2)^2}{9} + 1$ $\quad$ Simplify

$\dfrac{(y-4)^2}{9} - \dfrac{(x-2)^2}{9} = 1$

**35.** $\cos 2t = x$ and $\sin t = y$

$\cos 2t = 1 - 2\sin^2 t$ $\quad$ Double-angle identity

$x = 1 - 2y^2$ $\quad$ Substitute known values

**37.** $\sin t = x$ and $\sin t = y$

$\sin t = \sin t$ $\quad$ Reflective property of equality

$y = x$ $\quad$ Substitute known values

**39.** $3\cos t = x$ and $2\cos t = y$

$\cos t = \dfrac{x}{3}$ $\quad$ $\cos t = \dfrac{y}{2}$

$\cos t = \cos t$ $\quad$ Reflective property or equality

$\dfrac{x}{3} = \dfrac{y}{2}$ $\quad$ Substitute known values

$2x = 3y$ $\quad$ Multiply both sides by 6

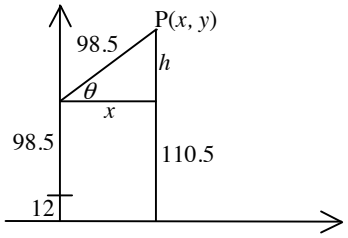
**41.** From the diagram at the right,

$\cos\theta = \dfrac{x}{98.5}$ $\quad$ $\sin\theta = \dfrac{h}{98.5}$

$x = 98.5\cos\theta$ $\quad$ $h = 98.5\sin\theta$

$y = 110.5 + h$

$= 110.5 + 98.5\sin\theta$




Next, we write $\theta$ in terms of $t$ using angular velocity:

$\omega = \dfrac{1\ rev}{15\min} \cdot \dfrac{2\pi\ rad}{1\ rev} = \dfrac{2\pi}{15}\ \dfrac{rad}{\min}$ $\quad$ $\omega = \dfrac{\theta}{t}$ (from Section 3.5)

$\dfrac{2\pi}{15} = \dfrac{\theta}{t}$ $\quad$ or $\quad$ $\theta = \dfrac{2\pi}{15}t$

We want the starting point to be at (0, 0). So we must subtract $\dfrac{\pi}{2}$ from $\dfrac{2\pi}{15}t$.

Therefore: $x = 98.5\cos\left(\dfrac{2\pi}{15}t - \dfrac{\pi}{2}\right)$ and $y = 110.5 + 98.5\sin\left(\dfrac{2\pi}{15}t - \dfrac{\pi}{2}\right)$

Check on your graphing calculator.

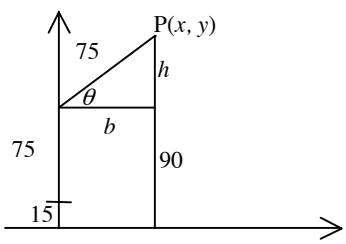
**43.** This problem is very similar to problem 41.

$\cos\theta = \dfrac{b}{75}$ $\quad$ $\sin\theta = \dfrac{h}{75}$

$b = 75\cos\theta$ $\quad$ $h = 75\sin\theta$

$x = 125 + b$ $\quad$ $y = 90 + h$

$= 125 + 75\cos\theta$ $\quad$ $= 90 + 75\sin\theta$



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Next, we write $\theta$ in terms of $t$ using angular velocity:

$$\omega = \frac{1\ rev}{32\min}\cdot\frac{2\pi\ rad}{1\ rev} = \frac{\pi}{16}\frac{rad}{\min} \qquad \omega = \frac{\theta}{t} \text{ (from Section 3.5)}$$

$$\frac{\pi}{16} = \frac{\theta}{t} \quad \text{or} \quad \theta = \frac{\pi}{16}t$$

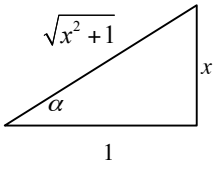
We want the starting point to be at (0, 0). So we must subtract $\frac{\pi}{2}$ from $\frac{\pi}{16}t$.

Therefore: $x = 125 + 75\cos\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$ and $y = 90 + 75\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$

Check on your graphing calculator.

**49.** Let $\alpha = \tan^{-1}x$. Then $\tan\alpha = \frac{x}{1}$ and $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$.

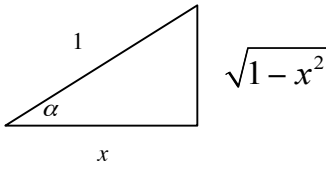
We draw a triangle and we find the hypotenuse using the Pythagorean Theorem.




Therefore, $\sin\alpha = \frac{x}{\sqrt{x^2+1}}$.

**51.** Let $\alpha = \cos^{-1}x$. Then $\cos\alpha = \frac{x}{1}$ and $0 < \alpha < \pi$.

We draw a triangle and we find the opposite side using the Pythagorean Theorem.




Therefore, $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$= 2\left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{x}{1}\right) = 2x\sqrt{1-x^2}$$

**53.** We must use the formula $\sin a\sin b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$.

Using algebra, we solve the equations: $a+b=10$ and $a-b=7$ and get $a=7$ and $b=3$.

Substituting these into the formula above, we get $\sin 7x\sin 3x = \frac{1}{2}\left[\sin(7x+3x) + \sin(7x-3x)\right]$.

Multiplying both sides by 2 gives us: $\sin(10x) + \sin(4x) = 2\sin 7x\sin 3x$

**55.** Let $\alpha = \sin^{-1}\frac{1}{3}$. Then $\sin\alpha = \frac{1}{3}$. We draw a triangle and we find the adjacent side using the Pythagorean Theorem.

Next we find the $\tan\alpha = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$.

**57.** Let $\alpha = \tan^{-1}\frac{2}{3}$ and let $\beta = \cos^{-1}\frac{1}{3}$. We draw 2 triangles and find the missing sides. Next we find the $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$. Substituting the values we found in the triangles, we get:

$$\cos(\alpha+\beta) = \left(\frac{3}{\sqrt{13}}\right)\left(\frac{1}{3}\right) - \left(\frac{2}{\sqrt{13}}\right)\left(\frac{2\sqrt{2}}{3}\right) = \frac{3-4\sqrt{2}}{3\sqrt{13}}.$$



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# Chapter 6 Test

**1**. Solving the equation:

$$2\sin\theta - 1 = 0$$
$$2\sin\theta = 1$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = 30°, 150°$$

**2**. Solving the equation:

$$\tan\theta + \sqrt{3} = 0$$
$$\tan\theta = -\sqrt{3}$$
$$\theta = 120°, 300°$$

**3**. Factoring the equation:

$$\cos\theta - 2\sin\theta\cos\theta = 0$$
$$\cos\theta(1 - 2\sin\theta) = 0$$

Thus either $\cos\theta = 0$ or $1 - 2\sin\theta = 0$, which occurs when $\sin\theta = \frac{1}{2}$. Thus the solutions are $\theta = 30°, 90°, 150°, 270°$.

**4**. Factoring the equation:

$$\tan\theta + 2\cos\theta\tan\theta = 0$$
$$\tan\theta(1 + 2\cos\theta) = 0$$

Thus either $\tan\theta = 0$ or $1 + 2\cos\theta = 0$, which occurs when $\cos\theta = -\frac{1}{2}$. Thus the solutions are $\theta = 0°, 120°, 180°, 240°$.

**5**. Since $\sec\theta = \frac{1}{\cos\theta}$, the equation becomes:

$$4\cos\theta - \frac{2}{\cos\theta} = 0$$
$$4\cos^2\theta - 2 = 0$$
$$4\cos^2\theta = 2$$
$$\cos^2\theta = \frac{1}{2}$$
$$\cos\theta = \pm\frac{1}{\sqrt{2}}$$
$$\theta = 45°, 135°, 225°, 315°$$

**6**. Since $\csc\theta = \frac{1}{\sin\theta}$, the equation becomes:

$$2\sin\theta - \frac{1}{\sin\theta} = 1$$
$$2\sin^2\theta - 1 = \sin\theta$$
$$2\sin^2\theta - \sin\theta - 1 = 0$$
$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$
$$\sin\theta = -\frac{1}{2}, 1$$
$$\theta = 90°, 210°, 330°$$

**7**. Isolating $\sin\frac{\theta}{2}$ and then squaring:

$$\sin\frac{\theta}{2} + \cos\theta = 0$$
$$\sin\frac{\theta}{2} = -\cos\theta$$
$$\sin^2\frac{\theta}{2} = (-\cos\theta)^2$$
$$\frac{1 - \cos\theta}{2} = \cos^2\theta$$
$$1 - \cos\theta = 2\cos^2\theta$$
$$2\cos^2\theta + \cos\theta - 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$
$$\cos\theta = \frac{1}{2}, -1$$
$$\theta = 60°, 180°, 300°$$



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Since this equation was solved by squaring, each solution must be checked:

$\sin\frac{60°}{2}+\cos 60° = \sin 30°+\cos 60° = \frac{1}{2}+\frac{1}{2} = 1$ (doesn't check)

$\sin\frac{180°}{2}+\cos 180° = \sin 90°+\cos 180° = 1+(-1) = 0$ (checks)

$\sin\frac{300°}{2}+\cos 300° = \sin 150°+\cos 300° = \frac{1}{2}+\frac{1}{2} = 1$ (doesn't check)

Thus $\theta = 180°$ is the only solution.

**8**. Using the double-angle formula for cosine:

$$4\cos 2\theta + 2\sin\theta = 1$$
$$4\left(1-2\sin^2\theta\right)+2\sin\theta = 1$$
$$4-8\sin^2\theta+2\sin\theta = 1$$
$$-8\sin^2\theta+2\sin\theta+3 = 0$$
$$8\sin^2\theta-2\sin\theta-3 = 0$$
$$(4\sin\theta-3)(2\sin\theta+1) = 0$$
$$\sin\theta = -\frac{1}{2}, \frac{3}{4}$$

If $\sin\theta = -\frac{1}{2}$, $\theta = 210°, 330°$, and if $\sin\theta = \frac{3}{4}$, $\theta \approx 48.6°, 131.4°$.

**9**. For $\sin(3\theta-45°) = -\frac{\sqrt{3}}{2}$, all values will take the form:

$3\theta-45° = 240°+360°k$ $\qquad$ $3\theta-45° = 300°+360°k$

$3\theta = 285°+360°k$ $\qquad$ $3\theta = 345°+360°k$

$\theta = 95°+120°k$ $\qquad$ $\theta = 115°+120°k$

Substituting $k = 0$, 1, and 2 results in the solutions $\theta = 95°, 115°, 215°, 235°, 335°, 355°$.

**10**. Squaring each side of the equation:

$$(\sin\theta+\cos\theta)^2 = 1$$
$$\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta = 1$$
$$1+\sin 2\theta = 1$$
$$\sin 2\theta = 0$$

All values will take the form:

$2\theta = 0°+360°k$ $\qquad$ $2\theta = 180°+360°k$

$\theta = 180°k$ $\qquad$ $\theta = 90°+180°k$

Substituting $k = 0$ and $k = 1$ results in the possible solutions $\theta = 0°, 90°, 180°, 270°$. Since this equation was solved by squaring, each solution must be checked:

$\sin 0°+\cos 0° = 0+1 = 1$ (checks)

$\sin 90°+\cos 90° = 1+0 = 1$ (checks)

$\sin 180°+\cos 180° = 0-1 = -1$ (doesn't check)

$\sin 270°+\cos 270° = -1+0 = -1$ (doesn't check)

Thus $\theta = 0°, 90°$ are the solutions.

**11**. All values will take the form:

$3\theta = 120°+360°k$ $\qquad$ $3\theta = 240°+360°k$

$\theta = 40°+120°k$ $\qquad$ $\theta = 80°+120°k$

Substituting $k = 0$, $k = 1$, and $k = 2$ results in the solutions $\theta = 40°, 80°, 160°, 200°, 280°, 320°$.



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**12**. All values will take the form:

$$2\theta = 45° + 180°k$$

$$\theta = 22.5° + 90°k$$

Substituting $k = 0$, $k = 1$, $k = 2$, and $k = 3$ results in the solutions $\theta = 22.5°, 112.5°, 202.5°, 292.5°$.

**13**. Solving the equation:

$$-3 + 10\sin(x-1) = 5$$

$$10\sin(x-1) = 8$$

$$\sin(x-1) = \frac{4}{5}$$

$$x - 1 = \sin^{-1}\left(\frac{4}{5}\right), \pi - \sin^{-1}\left(\frac{4}{5}\right)$$

$$x - 1 \approx 0.93, 2.21$$

$$x \approx 1.93, 3.21$$

**14**. Solving the equation:

$$7 - 4\tan(3x+1) = 19$$

$$-4\tan(3x+1) = 12$$

$$\tan(3x+1) = -3$$

$$3x + 1 = \left(\pi - \tan^{-1}(3)\right) + k\pi$$

$$3x + 1 \approx 1.89 + k\pi$$

$$3x \approx 0.89 + k\pi$$

$$x \approx 0.30 + \frac{k\pi}{3}$$

Substituting $k = 0$, $k = 1$, $k = 2$, $k = 3$, $k = 4$, and $k = 5$ results in the solutions $x \approx 0.30, 1.35, 2.39, 3.44, 4.49, 5.54$.

**15**. Using the double-angle formula for cosine:

$$\cos 2x - 3\cos x = -2$$

$$\left(2\cos^2 x - 1\right) - 3\cos x = -2$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$$\cos x = \frac{1}{2}, 1$$

All solutions will take the form $x = 2k\pi$, $x = \frac{\pi}{3} + 2k\pi$, or $x = \frac{5\pi}{3} + 2k\pi$.

**16**. Solving the equation:

$$\sqrt{3}\sin x - \cos x = 0$$

$$\sqrt{3}\sin x = \cos x$$

$$\frac{\sqrt{3}\sin x}{\cos x} = \frac{\cos x}{\cos x}$$

$$\sqrt{3}\tan x = 1$$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}$$

All solutions will take the form $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{7\pi}{6} + 2k\pi$. Note that these solutions combine as $x = \frac{\pi}{6} + k\pi$.



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**17**. Since $\sin 2x\cos x+\cos 2x\sin x=\sin(2x+x)=\sin 3x$, the equation is $\sin 3x=-1$. All solutions will take the form:

$$3x=\frac{3\pi}{2}+2k\pi$$

$$x=\frac{\pi}{2}+\frac{2k\pi}{3}$$

**18**. Solving by taking cube roots:

$$\sin^3 4x=1$$

$$\sin 4x=1$$

$$4x=\frac{\pi}{2}+2\pi k$$

$$x=\frac{\pi}{8}+\frac{k\pi}{2}$$

**19**. Solving by factoring:

$$5\sin^2\theta-3\sin\theta=2$$

$$5\sin^2\theta-3\sin\theta-2=0$$

$$(5\sin\theta+2)(\sin\theta-1)=0$$

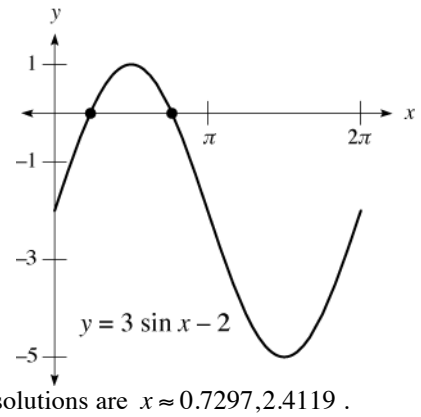
$$\sin\theta=-\frac{2}{5},1$$

For $\sin\theta=1$, $\theta=90°$. For $\sin\theta=-\frac{2}{5}$, $\theta=203.6°,336.4°$.

**20**. Write the equation as $4\cos^2\theta-4\cos\theta-2=0$, or $2\cos^2\theta-2\cos\theta-1=0$. Using $a=2$, $b=-2$, and $c=-1$ in the quadratic formula: $\cos\theta=\frac{-(-2)\pm\sqrt{(-2)^2-4(2)(-1)}}{2(2)}=\frac{2\pm\sqrt{4+8}}{4}=\frac{2\pm\sqrt{12}}{4}=\frac{2\pm2\sqrt{3}}{4}=\frac{1\pm\sqrt{3}}{2}$

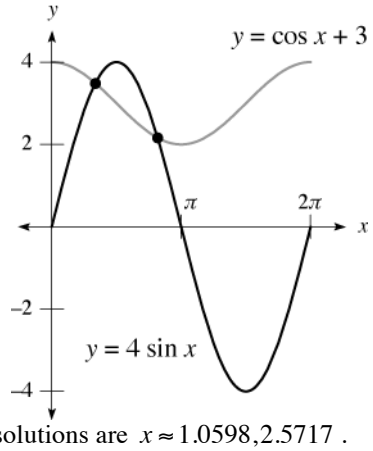
Either $\cos\theta=\frac{1+\sqrt{3}}{2}\approx 1.37$, which is impossible, or $\cos\theta=\frac{1-\sqrt{3}}{2}\approx -0.367$, which occurs when $\theta\approx 111.5°,248.5°$.

**21**. Sketching the graph:




The solutions are $x\approx 0.7297,2.4119$.

**22**. Sketching the graph:




The solutions are $x\approx 1.0598,2.5717$.



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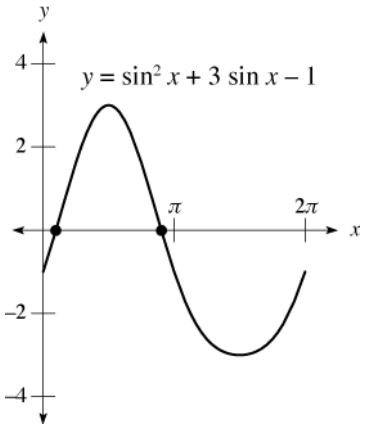
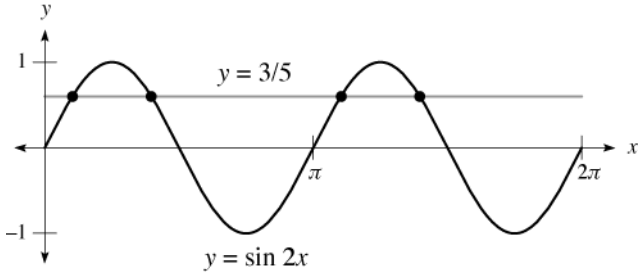
**23**. Sketching the graph:




The solutions are $x \approx 0.3076, 2.8340$ .

**24**. Sketching the graph:




The solutions are $x \approx 0.3218, 1.2490, 3.4633, 4.3906$ .

**25**. Solving the equation:

$$139 - 125\cos\frac{\pi}{10}t = 150$$

$$-125\cos\frac{\pi}{10}t = 11$$

$$\cos\frac{\pi}{10}t = -\frac{11}{125}$$

Then either:

$$\frac{\pi}{10}t = \cos^{-1}\left(-\frac{11}{125}\right) \qquad \frac{\pi}{10}t = \pi + \cos^{-1}\left(\frac{11}{125}\right)$$

$$t = \frac{10}{\pi}\cos^{-1}\left(-\frac{11}{125}\right) \approx 5.3 \qquad t = \frac{10}{\pi}\left(\pi + \cos^{-1}\left(\frac{11}{125}\right)\right) \approx 14.7$$

The time will be either 5.3 minutes or 14.7 minutes.



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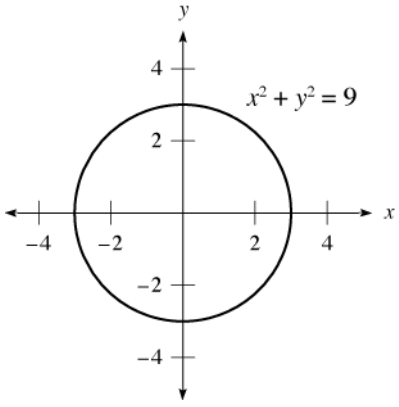
**26**. Since $\cos t = \frac{x}{3}$ and $\sin t = \frac{y}{3}$, the equation becomes:

$$\cos^2 t + \sin^2 t = 1$$
$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$
$$\frac{x^2}{9} + \frac{y^2}{9} = 1$$
$$x^2 + y^2 = 9$$

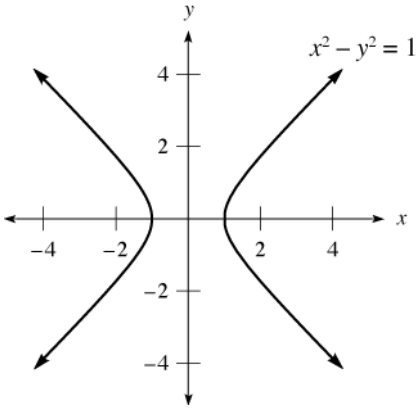
The graph is a circle with center = (0,0) and radius = 3:




**27**. Since $\sec t = x$ and $\tan t = y$, the equation becomes:

$$1 + \tan^2 t = \sec^2 t$$
$$1 + y^2 = x^2$$
$$x^2 - y^2 = 1$$

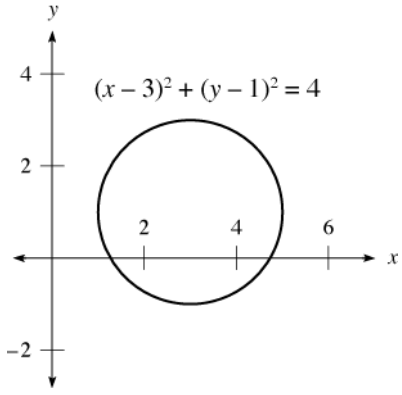
The graph is a hyperbola:




**28**. Since $x - 3 = 2\sin t, \sin t = \frac{x-3}{2}$. Since $y - 1 = 2\cos t, \cos t = \frac{y-1}{2}$. Therefore:

$$\sin^2 t + \cos^2 t = 1$$
$$\left(\frac{x-3}{2}\right)^2 + \left(\frac{y-1}{2}\right)^2 = 1$$
$$\frac{(x-3)^2}{4} + \frac{(y-1)^2}{4} = 1$$
$$(x-3)^2 + (y-1)^2 = 4$$

The graph is a circle with center = (3,1) and radius = 2:



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**29**. Since $x+3=3\cos t, \cos t=\dfrac{x+3}{3}$. Since $y-1=3\sin t, \sin t=\dfrac{y-1}{3}$. Therefore:

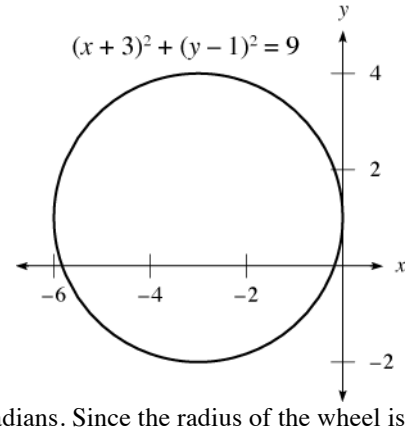
$$\sin^2 t+\cos^2 t=1$$

$$\left(\frac{y-1}{3}\right)^2+\left(\frac{x+3}{3}\right)^2=1$$

$$\frac{(x+3)^2}{9}+\frac{(y-1)^2}{9}=1$$

$$(x+3)^2+(y-1)^2=9$$

The graph is a circle with center = (–3,1) and radius = 3:




**30**. Let $t$ represent the central angle in radians. Since the radius of the wheel is 90 feet, we have:

$$x=90\cos t$$

$$y=90\sin t$$

To shift the wheel 98 feet up (90 + 8 = 98), the parametric equations are:

$$x=90\cos t$$

$$y=98+90\sin t$$

To shift the wheel so that the rider starts at the bottom, subtract $\dfrac{\pi}{2}$ from $t$ to obtain the equations:

$$x=90\cos\left(t-\frac{\pi}{2}\right)$$

$$y=98+90\sin\left(t-\frac{\pi}{2}\right)$$

We now need to find a connection between $t$ and the parameter $T$. Since it takes 3 min to make 1 revolution, then:

$$\frac{t}{T}=\frac{2\pi}{3}$$

$$t=\frac{2\pi}{3}T$$

Therefore the parametric equations are:

$$x=90\cos\left(\frac{2\pi}{3}T-\frac{\pi}{2}\right)$$

$$y=98+90\sin\left(\frac{2\pi}{3}T-\frac{\pi}{2}\right)$$



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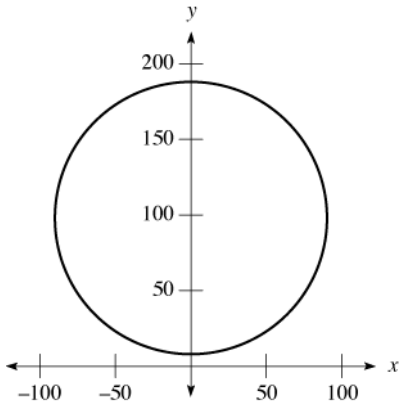
Graphing these equations:

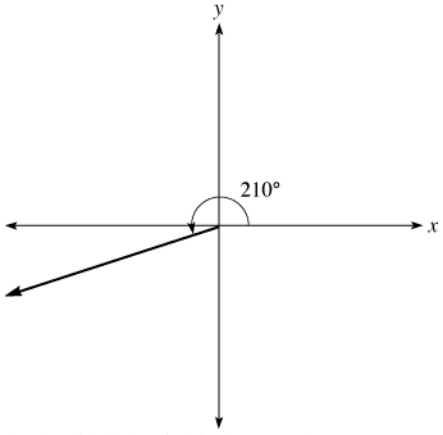



## Cumulative Test Chapters 1–6

**1**. Using the Pythagorean Theorem:

$$4^2 + x^2 = (x+2)^2$$
$$16 + x^2 = x^2 + 4x + 4$$
$$16 = 4x + 4$$
$$12 = 4x$$
$$x = 3$$

**2**. Drawing 210° in standard position:




A positive angle which is coterminal with 210° is 570°. A negative angle which is coterminal with 210° is –150°.

**3**. If $\sin\theta = \frac{24}{25}$, we can choose $y = 24$ and $r = 25$. To find $x$, use $x^2 + y^2 = r^2$:

$$x^2 + 24^2 = 25^2$$
$$x^2 + 576 = 625$$
$$x^2 = 49$$
$$x = \pm 7$$

Since $\theta$ terminates in quadrant II, $x < 0$ and thus $x = -7$. Using $x = -7$, $y = 24$, and $r = 25$ in the definitions:

$$\cos\theta = \frac{x}{r} = -\frac{7}{25} \qquad \sec\theta = \frac{r}{x} = -\frac{25}{7} \qquad \csc\theta = \frac{r}{y} = \frac{25}{24}$$
$$\tan\theta = \frac{y}{x} = -\frac{24}{7} \qquad \cot\theta = \frac{x}{y} = -\frac{7}{24}$$



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**4.** If $\sin\theta = \frac{1}{a}$, we can choose $y = 1$ and $r = a$. To find $x$, use $x^2 + 1^2 = a^2$:

$$x^2 + 1^2 = a^2$$
$$x^2 + 1 = a^2$$
$$x^2 = a^2 - 1$$
$$x = \pm\sqrt{a^2 - 1}$$

Since $\theta$ terminates in quadrant I, $x > 0$ and thus $x = \sqrt{a^2 - 1}$. Using $x = \sqrt{a^2 - 1}$, $y = 1$, and $r = a$ in the definitions:

$$\cos\theta = \frac{x}{r} = \frac{\sqrt{a^2 - 1}}{a} \qquad \csc\theta = \frac{r}{y} = \frac{a}{1} = a \qquad \cot\theta = \frac{x}{y} = \frac{\sqrt{a^2 - 1}}{1} = \sqrt{a^2 - 1}$$

**5.** Working from the right side: $\sin\theta\sec\theta = \sin\theta \bullet \frac{1}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

**6.** Simplifying the expression: $\cos^2 30° - \sin^2 45° = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

**7.** Converting to degrees and minutes: $14.65° = 14° + 0.65° = 14° + 0.65(60') = 14°39'$

**8.** Using triangle *DBC*:

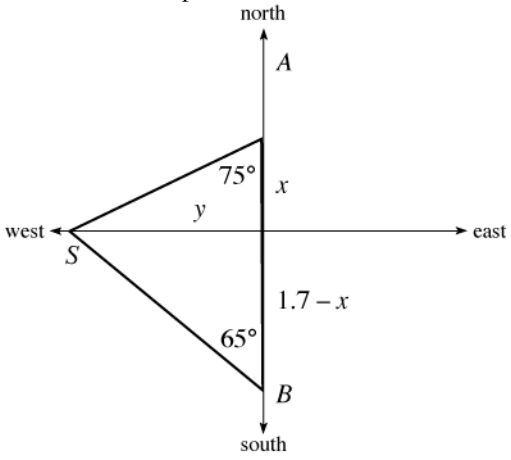
$$\cos(\angle BDC) = \frac{x}{DB}$$
$$\cos 55° = \frac{x}{29}$$
$$x = 29\cos 55° \approx 16.6$$

Using triangle *BAC*:

$$\cos(\angle A) = \frac{x + y}{AB}$$
$$\cos 38° = \frac{x + y}{41}$$
$$x + y = 41\cos 38°$$
$$y = 41\cos 38° - 29\cos 55° \approx 15.7$$

Using two significant figures, the answers are $x \approx 17$ and $y \approx 16$.

**9.** Let *A* and *B* represent the two observers, and S represent the sailboat. Construct the figure:




Therefore:

$$\tan 75° = \frac{y}{x} \qquad \tan 65° = \frac{y}{1.5 - x}$$
$$y = x\tan 75° \qquad y = (1.7 - x)\tan 65°$$



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Setting these two expressions equal:

$$(1.7 - x)\tan 65° = x \tan 75°$$

$$1.7 \tan 65° - x \tan 65° = x \tan 75°$$

$$1.7 \tan 65° = x \tan 75° + x \tan 65°$$

$$1.7 \tan 65° = x(\tan 75° + \tan 65°)$$

$$x = \frac{1.7 \tan 65°}{\tan 75° + \tan 65°} \approx 0.62$$

Therefore:

$$\tan 75° = \frac{y}{x}$$

$$\tan 75° = \frac{y}{0.62}$$

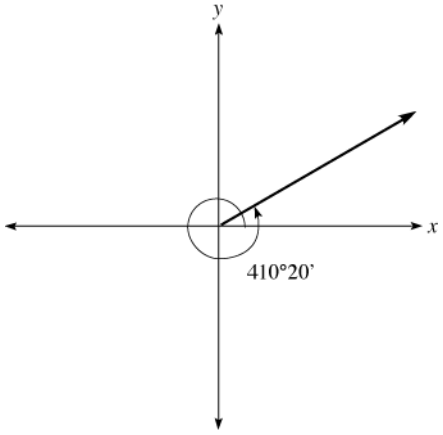
$$t = 0.62 \tan 75° \approx 2.32 \text{ mi}$$

The sailboat is approximately 2.3 miles from the shore.

**10**. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$|\mathbf{V}_x| = 48 \cos 33° \approx 40 \text{ ft/sec} \qquad |\mathbf{V}_y| = 48 \sin 33° \approx 26 \text{ ft/sec}$$

**11**. Drawing the angle in standard position:




The reference angle is 50°20'.

**12**. Computing the value: $4 \sin\left(2 \cdot \frac{\pi}{6} + \frac{\pi}{2}\right) = 4 \sin \frac{5\pi}{6} = 4 \cdot \frac{1}{2} = 2$

**13**. Finding the trigonometric values:

$$\sin\theta = y = -0.9673 \qquad \cos\theta = x = 0.2537 \qquad \tan\theta = \frac{y}{x} = \frac{-0.9673}{0.2537} \approx -3.8128$$

**14**. Since $35 \text{ minutes} = \frac{35}{60} \cdot 2\pi = \frac{7\pi}{6}$ radians, the arc length is given by $s = r\theta = 2 \cdot \frac{7\pi}{6} = \frac{7\pi}{3}$ cm.

**15**. Finding the angular velocity: $\omega = \frac{v}{r} = \frac{5}{3} \frac{\text{cm}/\text{sec}}{\text{cm}} = \frac{5}{3} \frac{\text{radians}}{\text{sec}}$



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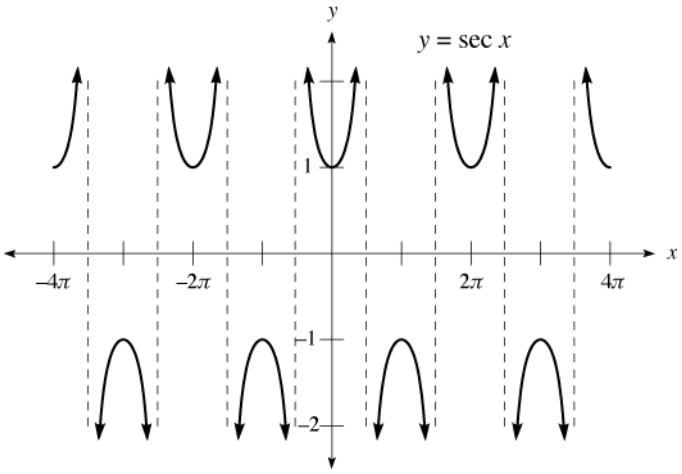
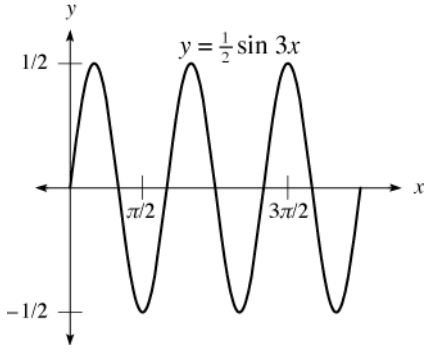
**16**. Sketching the graph:

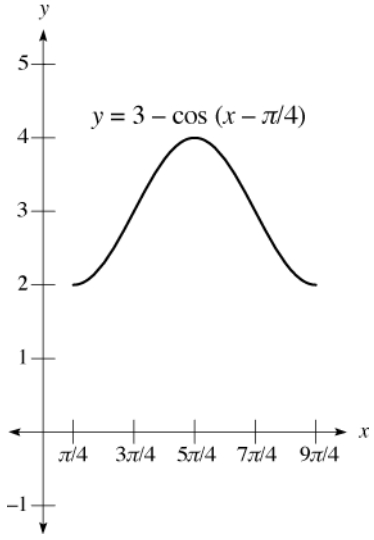



Thus sec $x$ is undefined when $x = -\frac{7\pi}{2}, -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$.

**17**. The amplitude is $\frac{1}{2}$ and the period is $\frac{2\pi}{3}$. Sketching the graph:




**18**. The amplitude is 1, the period is $2\pi$, the horizontal shift is $\frac{\pi}{4}$, and the vertical shift is 3. Note this is a reflection of $y = \cos\left(x - \frac{\pi}{4}\right)$ across the $x$-axis:



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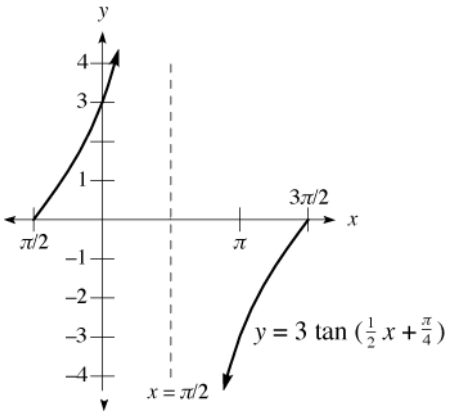
**19**. The period is $\frac{\pi}{1/2} = 2\pi$ and the horizontal shift is $\frac{-\pi/4}{1/2} = -\frac{\pi}{2}$. Sketching the graph:




**20**. This is a cosine curve with amplitude = 2 and period = $2$, therefore:

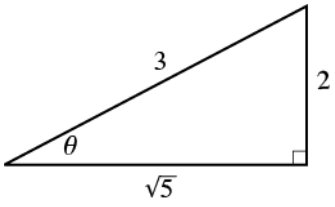
$$\frac{2\pi}{B} = 2$$
$$B = \pi$$

The horizontal translation is $\frac{1}{2}$, therefore:

$$-\frac{C}{B} = \frac{1}{2}$$
$$-\frac{C}{\pi} = \frac{1}{2}$$
$$C = -\frac{\pi}{2}$$

The curve is a reflection across the $x$-axis with a vertical translation of 3 units, so its equation is $y = 3 - 2\cos\left(\pi x - \frac{\pi}{2}\right)$.

**21**. Let $\theta = \sin^{-1}\frac{2}{3}$, so $\sin\theta = \frac{2}{3}$. Drawing the triangle:




The missing side was found using the Pythagorean Theorem: $b = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$.

Therefore: $\cos\left(\sin^{-1}\frac{2}{3}\right) = \cos\theta = \frac{\sqrt{5}}{3}$



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**22**. Working from the left side:

$$\begin{aligned}(1+\sec x)(1-\cos x) &= \left(1+\frac{1}{\cos x}\right)(1-\cos x)\\ &= 1-\cos x+\frac{1}{\cos x}-1\\ &= \frac{1}{\cos x}-\frac{\cos x}{1}\bullet\frac{\cos x}{\cos x}\\ &= \frac{1-\cos^2 x}{\cos x}\\ &= \frac{\sin^2 x}{\cos x}\\ &= \frac{\sin^2 x}{\cos^2 x}\bullet\cos x\\ &= \tan^2 x\cos x\end{aligned}$$

**23**. Using the difference formula for sine: $\sin(\theta-90°)=\sin\theta\cos 90°-\cos\theta\sin 90°=\sin\theta\bullet 0-\cos\theta\bullet 1=-\cos\theta$

**24**. Since $\sin A=-\frac{3}{5}$ and $A$ terminates in quadrant IV, $\cos A>0$ and thus:

$$\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\left(-\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

Since $\sin B=\frac{12}{13}$ and $B$ terminates in quadrant II, $\cos B<0$ and thus:

$$\cos B=-\sqrt{1-\sin^2 B}=-\sqrt{1-\left(\frac{12}{13}\right)^2}=-\sqrt{1-\frac{144}{169}}=-\sqrt{\frac{25}{169}}=-\frac{5}{13}$$

Using the addition formula for sine: $\sin(A+B)=\sin A\cos B+\cos A\sin B=\left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right)+\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)=\frac{15}{65}+\frac{48}{65}=\frac{63}{65}$

**25**. Since $270°<A<360°$, $135°<\frac{A}{2}<180°$. Thus $\frac{A}{2}$ terminates in quadrant II, so $\cos\frac{A}{2}<0$ and therefore:

$$\cos\frac{A}{2}=-\sqrt{\frac{1+\cos A}{2}}=-\sqrt{\frac{1+\frac{4}{5}}{2}}=-\sqrt{\frac{9}{10}}=-\frac{3}{\sqrt{10}}=-\frac{3\sqrt{10}}{10}$$

**26**. Using the product-to-sum formula: $4\sin 7x\cos 3x=4\bullet\frac{1}{2}\left[\sin(7x+3x)+\sin(7x-3x)\right]=2(\sin 10x+\sin 4x)$

**27**. Factoring the equation:

$$2\cos^2\theta-\cos\theta-1=0$$
$$(2\cos\theta+1)(\cos\theta-1)=0$$

Thus either $2\cos\theta+1=0$, so $\cos\theta=-\frac{1}{2}$, or $\cos\theta-1=0$, so $\cos\theta=1$. Thus the solutions are $x=0°,120°,240°$.

**28**. Using the double-angle formula:

$$\sin 2x-\sin x=0$$
$$2\sin x\cos x-\sin x=0$$
$$\sin x(2\cos x-1)=0$$

Thus either $\sin x=0$ or $\cos x=\frac{1}{2}$. Thus the solutions are $x=0,\frac{\pi}{3},\pi,\frac{5\pi}{3}$.



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**29**. Since $\cos 4x\cos x+\sin 4x\sin x=\cos(4x-x)=\cos 3x$, solving $\cos 3x=-\frac{1}{2}$ results in all values of the form:

$3x=120°+360°k$ $\qquad$ $3x=240°+360°k$

$x=40°+120°k$ $\qquad$ $x=80°+120°k$

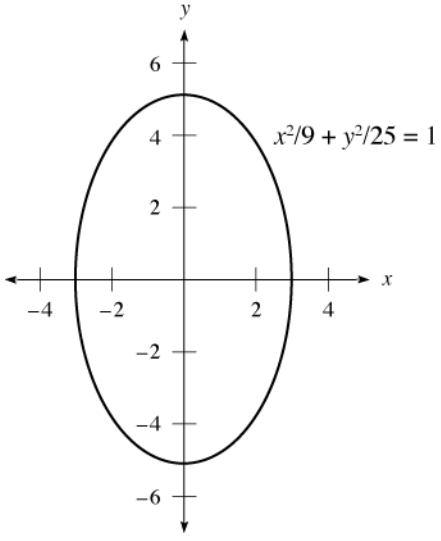
**30**. Since $\cos t=\frac{x}{3}$ and $\sin t=\frac{y}{5}$:

$$\sin^2 t+\cos^2 t=1$$

$$\left(\frac{x}{3}\right)^2+\left(\frac{y}{5}\right)^2=1$$

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

The graph is an ellipse:



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# Chapter 7
# Triangles

## 7.1 The Law of Sines

**EVEN SOLUTIONS**

**2**. We use the law of sides to find the missing parts of triangles for which we are given two angles and one side. In either case, a unique triangle is determined.

**4**. To solve an oblique triangle given the case ASA, the first step is to find the missing angle so that the law of sines can be used.

**6**. Using the law of sines:

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$
$$\frac{12}{\sin 45^\circ}=\frac{b}{\sin 60^\circ}$$
$$b=\frac{12\sin 60^\circ}{\sin 45^\circ}\approx 15 \text{ cm}$$

**8**. Using the law of sines:

$$\frac{c}{\sin C}=\frac{b}{\sin B}$$
$$\frac{c}{\sin 40^\circ}=\frac{18}{\sin 100^\circ}$$
$$c=\frac{18\sin 40^\circ}{\sin 100^\circ}\approx 12 \text{ inches}$$

**10**. Using the law of sines:

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$
$$\frac{24}{\sin 10^\circ}=\frac{c}{\sin 150^\circ}$$
$$c=\frac{24\sin 150^\circ}{\sin 10^\circ}\approx 69 \text{ yd}$$

**12**. First note that $A=180^\circ-(B+C)=180^\circ-(40^\circ+70^\circ)=70^\circ$. Using the law of sines:

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$
$$\frac{a}{\sin 70^\circ}=\frac{82}{\sin 70^\circ}$$
$$a=\frac{82\sin 70^\circ}{\sin 70^\circ}=82 \text{ km}$$

**14**. First note that $B=180^\circ-(A+C)=180^\circ-(33^\circ+82^\circ)=65^\circ$. Using the law of sines:

$$\frac{c}{\sin C}=\frac{b}{\sin B}$$
$$\frac{c}{\sin 82^\circ}=\frac{44}{\sin 65^\circ}$$
$$c=\frac{44\sin 82^\circ}{\sin 65^\circ}\approx 48 \text{ cm}$$



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**16**. Note that $B = 180° - (A + C) = 180° - (110.4° + 21.8°) = 47.8°$. Find $a$ using the law of sines:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 110.4°} = \frac{246}{\sin 21.8°}$$
$$a = \frac{246 \sin 110.4°}{\sin 21.8°} \approx 621 \text{ inches}$$

Find $b$ using the law of sines:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 47.8°} = \frac{246}{\sin 21.8°}$$
$$b = \frac{246 \sin 47.8°}{\sin 21.8°} \approx 491 \text{ inches}$$

**18**. Note that $C = 180° - (A + B) = 180° - (46° + 95°) = 39°$. Find $a$ using the law of sines:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 46°} = \frac{6.8}{\sin 39°}$$
$$a = \frac{6.8 \sin 46°}{\sin 39°} \approx 7.8 \text{ m}$$

Find $b$ using the law of sines:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 95°} = \frac{6.8}{\sin 39°}$$
$$b = \frac{6.8 \sin 95°}{\sin 39°} \approx 11 \text{ m}$$

**20**. Note that $A = 180° - (B + C) = 180° - (14°20' + 75°40') = 90°$. Find $a$ using the law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 90°} = \frac{2.72}{\sin 14°20'}$$
$$a = \frac{2.72 \sin 90°}{\sin 14°20'} \approx 11.0 \text{ ft}$$

Find $c$ using the law of sines:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 75°40'} = \frac{2.72}{\sin 14°20'}$$
$$c = \frac{2.72 \sin 75°40'}{\sin 14°20'} \approx 10.6 \text{ ft}$$

**22**. Note that $C = 180° - (A + B) = 180° - (105° + 45°) = 30°$. Finding $a$ using the law of sines:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 105°} = \frac{630}{\sin 30°}$$
$$a = \frac{630 \sin 105°}{\sin 30°} \approx 1{,}200 \text{ cm}$$



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Finding $b$ using the law of sines:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 45°} = \frac{630}{\sin 30°}$$

$$b = \frac{630 \sin 45°}{\sin 30°} \approx 890 \text{ cm}$$

**24**. Using the law of sines:

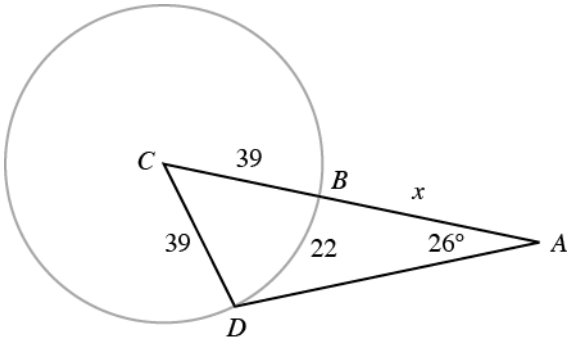
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{19}{\sin B} = \frac{18}{\sin 40°}$$

$$\sin B = \frac{19 \sin 40°}{18} \approx 0.6785$$

$$B \approx 43°, 137°$$

**26**. Re-drawing the figure:




First find $C$ using the arc length: $C = \frac{s}{r} = \frac{22}{39} \approx 0.56$ radians

Converting to degrees: $0.56 \text{ radians} = 0.56 \cdot \frac{180°}{\pi} \approx 32.3°$

Thus $D = 180° - (A + C) = 180° - (26° + 32.3°) = 121.7°$ . Using the law of sines:

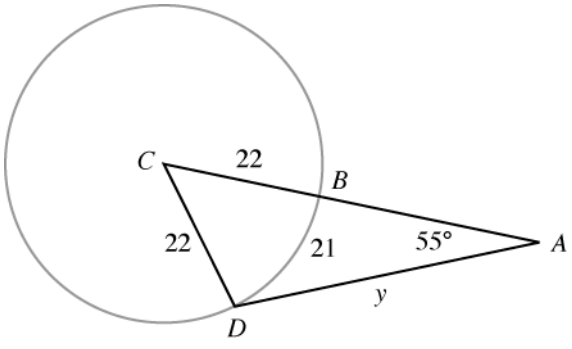
$$\frac{CA}{\sin D} = \frac{CD}{\sin A}$$

$$\frac{39 + x}{\sin 121.7°} = \frac{39}{\sin 26°}$$

$$39 + x = \frac{39 \sin 121.7°}{\sin 26°}$$

$$x = \frac{39 \sin 121.7°}{\sin 26°} - 39 \approx 36.7 \approx 37$$

**28**. Re-drawing the figure:



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First find $C$ using the arc length: $C = \frac{s}{r} = \frac{21}{22}$ radians

Converting to degrees: $\frac{21}{22}$ radians $= \frac{21}{22} \cdot \frac{180°}{\pi} \approx 54.7°$

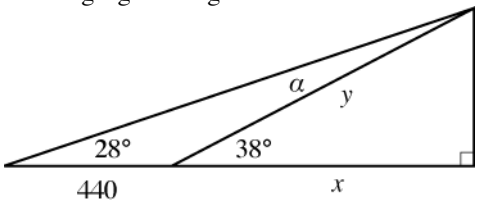
Using the law of sines:

$$\frac{DA}{\sin C} = \frac{DC}{\sin A}$$

$$\frac{y}{\sin 54.7°} = \frac{22}{\sin 55°}$$

$$y = \frac{22 \sin 54.7°}{\sin 55°} \approx 22$$

**30**. Note that this problem can be solved using right triangles. To use the law of sines, first label missing information:




Note that $\alpha = 180° - (28° + 142°) = 10°$. Find $y$ using the law of sines:

$$\frac{y}{\sin 28°} = \frac{440}{\sin 10°}$$

$$y = \frac{440 \sin 28°}{\sin 10°} \approx 1190 \text{ feet}$$

Therefore (using the smaller right triangle):

$$\cos 38° = \frac{x}{1190}$$

$$x = 1190 \cos 38° \approx 937 \text{ feet}$$

The distance from the building on the second observation is therefore 937 + 440 ≈ 1,400 feet.

**32**. Let $x$ represent the distance from the person to the top of the hill. Using the law of sines:

$$\frac{x}{\sin 63.5°} = \frac{110}{\sin 1.5°}$$

$$x = \frac{110 \sin 63.5°}{\sin 1.5°} \approx 3{,}761 \text{ feet}$$

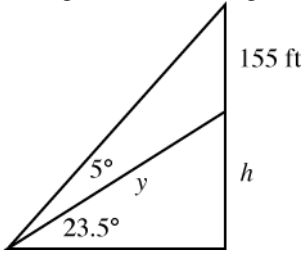
Then using the right triangle:

$$\sin 25° = \frac{h}{3{,}761}$$

$$h = 3{,}761 \sin 25° \approx 1{,}600 \text{ feet}$$

The hill is 1,600 feet tall.

**34**. First draw the figure, where $h$ represents the height of the building:



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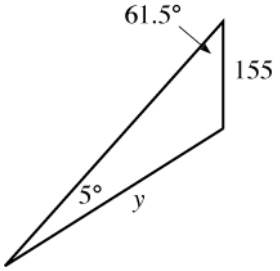
The upper triangle can be drawn, noting 28.5° – 23.5° = 5° and 90° – 28.5° = 61.5°:




Using the law of sines:

$$\frac{y}{\sin 61.5^\circ} = \frac{155}{\sin 5^\circ}$$

$$y = \frac{155 \sin 61.5^\circ}{\sin 5^\circ} \approx 1{,}563 \text{ feet}$$

Therefore (using the smaller right triangle):

$$\sin 23.5^\circ = \frac{h}{1{,}563}$$

$$h = 1{,}563 \sin 23.5^\circ \approx 623 \text{ feet}$$

The building is approximately 623 feet tall.

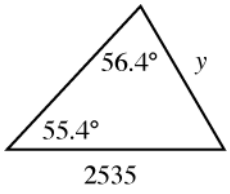
**36**. First find *CB* using right triangle *CBD* (let *x* represent length *CB*):

$$\tan 22.5^\circ = \frac{1{,}050}{x}$$

$$x \tan 22.5^\circ = 1{,}050$$

$$x = \frac{1{,}050}{\tan 22.5^\circ} \approx 2{,}535 \text{ feet}$$

Now consider triangle *ABC*, where *y* is the desired distance:




Using the law of sines:

$$\frac{y}{\sin 55.4^\circ} = \frac{2{,}535}{\sin 56.4^\circ}$$

$$y = \frac{2{,}535 \sin 55.4^\circ}{\sin 56.4^\circ} \approx 2{,}510 \text{ feet}$$

The rescue boat will need to travel approximately 2,510 feet.



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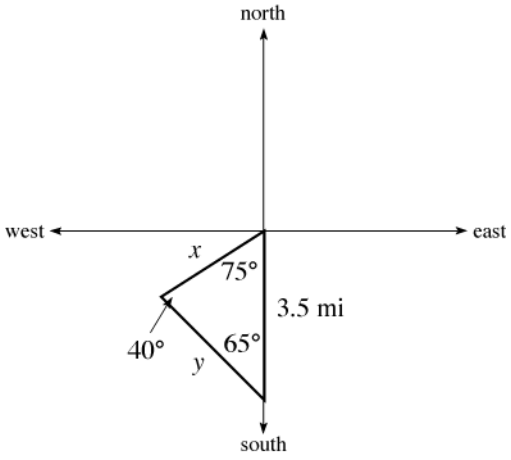
**38.** First draw the figure, where $x$ and $y$ represent the required distances. Note that the angle at the rocket is $180° - (75° + 65°) = 40°$:




Find $x$ using the law of sines:

$$\frac{x}{\sin 65°} = \frac{3.5}{\sin 40°}$$

$$x = \frac{3.5 \sin 65°}{\sin 40°} \approx 4.9 \text{ mi}$$

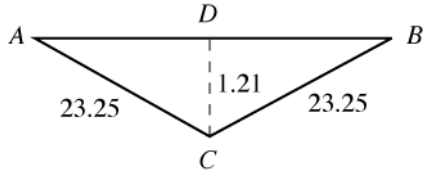
Find $y$ using the law of sines:

$$\frac{y}{\sin 75°} = \frac{3.5}{\sin 40°}$$

$$y = \frac{3.5 \sin 75°}{\sin 40°} \approx 5.3 \text{ mi}$$

Tom is approximately 4.9 mi from the rocket, and Fred is approximately 5.3 mi from the rocket.

**40.** First note that 14.5 inches ≈ 1.21 feet. Since the tightrope walker is standing in the center of the rope, we can construct the figure:

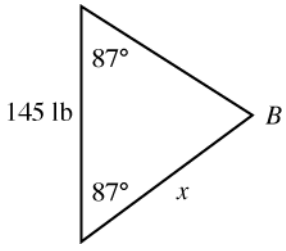



Finding $\angle DCB$ using the right triangle:

$$\cos \angle DCB = \frac{1.21}{23.25}$$

$$\angle DCB = \cos^{-1}\left(\frac{1.21}{23.25}\right) \approx 87.0°$$

Now re-drawing the tension vectors:




Note that $B = 180° - (87° + 87°) = 6°$. Using the law of sines:

$$\frac{x}{\sin 87°} = \frac{145}{\sin 6°}$$

$$x = \frac{145 \sin 87°}{\sin 6°} \approx 1{,}390 \text{ pounds}$$

The tension is approximately 1,390 pounds.



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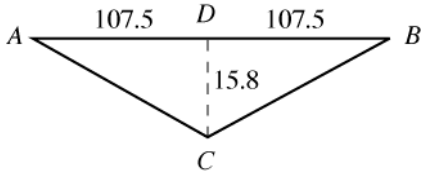
**42**. Since the chair lift is in the center of the cable, construct the figure:

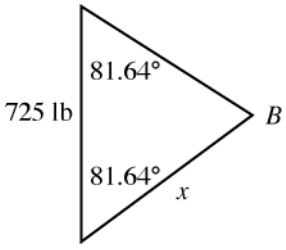



Finding $\angle DCB$ using the right triangle:

$$\tan\angle DCB = \frac{107.5}{15.8}$$

$$\angle DCB = \tan^{-1}\left(\frac{107.5}{15.8}\right) \approx 81.64°$$

Now re-drawing the tension vectors:




Note that $B = 180° - (81.64° + 81.64°) = 16.72°$. Using the law of sines:

$$\frac{x}{\sin 81.64°} = \frac{725}{\sin 16.72°}$$

$$x = \frac{725 \sin 81.64°}{\sin 16.72°} \approx 2{,}490 \text{ pounds}$$

The tension is approximately 2,490 pounds.

**44**. Solving the equation:

$$5\tan\theta - 3 = 0$$

$$5\tan\theta = 3$$

$$\tan\theta = \frac{3}{5}$$

$$\theta = \tan^{-1}\left(\frac{3}{5}\right) + 180°k$$

$$\theta \approx 31.0°, 211.0°$$

**46**. Solving by factoring:

$$3\sin\theta + 2\sin\theta\cos\theta = 0$$

$$\sin\theta(3 + 2\cos\theta) = 0$$

Either $\sin\theta = 0$ or $\cos\theta = -\frac{3}{2}$, which is impossible. The solutions are $\theta = 0°, 180°$.

**48**. Solving by factoring:

$$10\cos^2\theta + \cos\theta - 3 = 0$$

$$(5\cos\theta + 3)(2\cos\theta - 1) = 0$$

$$\cos\theta = -\frac{3}{5}, \frac{1}{2}$$

If $\cos\theta = -\frac{3}{5}$, $\theta = 180° - 53.1° = 126.9°$ or $\theta = 180° + 53.1° = 233.1°$. If $\cos\theta = \frac{1}{2}$, $\theta = 60°$ or $\theta = 300°$.

The solutions are $\theta = 60°, 126.9°, 233.1°, 300°$.

**50**. The reference angle is $\hat{\theta} = \cos^{-1}(0.2351) \approx 76.4°$. Thus either $\theta = 76.4°$ or $\theta = 360° - 76.4° = 283.6°$.



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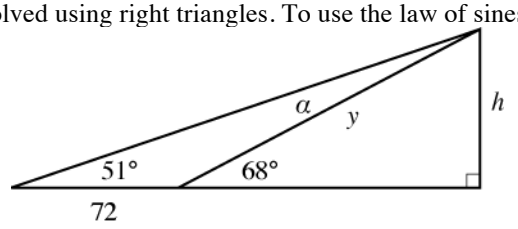
**52**. Note that this problem can be solved using right triangles. To use the law of sines, first label missing information:




Note that $\alpha = 180° - (51° + 112°) = 17°$. Find $y$ using the law of sines:

$$\frac{y}{\sin 51°} = \frac{72}{\sin 17°}$$

$$y = \frac{72 \sin 51°}{\sin 17°} \approx 191.4 \text{ feet}$$

Therefore (using the smaller right triangle):

$$\sin 68° = \frac{h}{191.4}$$

$$x = 191.4 \sin 68° \approx 180 \text{ feet}$$

The height of the building is 180 feet. The correct answer is d.

**ODD SOLUTIONS**

**1.** oblique

**3.** sine, opposite, constant

**5.**

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$ Law of Sines

$$\frac{a}{\sin 80^\circ} = \frac{14}{\sin 30^\circ}$$ Substitute known values

$$a = \frac{14 \sin 80^\circ}{\sin 30^\circ}$$ Multiply both sides by sin 80°

$$= 28 \text{ cm}$$ Round to 2 significant digits

**7.**

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$ Law of Sines

$$\frac{b}{\sin 120^\circ} = \frac{28}{\sin 20^\circ}$$ Substitute known values

$$b = \frac{28 \sin 120^\circ}{\sin 20^\circ}$$ Multiply both sides by sin 120°

$$= 71 \text{ inches}$$ Round to 2 significant digits

**9.**

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$ Law of Sines

$$\frac{a}{\sin 5^\circ} = \frac{510}{\sin 125^\circ}$$ Substitute known values

$$a = \frac{510 \sin 5^\circ}{\sin 125^\circ}$$ Multiply both sides by sin 5°

$$= 54 \text{ yards}$$ Round to 2 significant digits



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**11.** $C = 180^\circ - (A + B)$
$= 180^\circ - (50^\circ + 60^\circ)$
$= 180^\circ - 110^\circ = 70^\circ$

$\frac{c}{\sin C} = \frac{a}{\sin A}$ Law of Sines

$\frac{c}{\sin 70^\circ} = \frac{36}{\sin 50^\circ}$ Substitute known values

$c = \frac{36 \sin 70^\circ}{\sin 50^\circ}$ Multiply both sides by $\sin 70^\circ$

$= 44 \text{ km}$ Round to 2 significant digits

**13.** $C = 180^\circ - (A + B)$
$= 180^\circ - (52^\circ + 48^\circ)$
$= 180^\circ - 100^\circ = 80^\circ$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ Law of Sines

$\frac{a}{\sin 52^\circ} = \frac{14}{\sin 80^\circ}$ Substitute known values

$a = \frac{14 \sin 52^\circ}{\sin 80^\circ}$ Multiply both sides by $\sin 52^\circ$

$= 11 \text{ cm}$ Round to 2 significant digits

**15.** $C = 180^\circ - (A + B)$
$= 180^\circ - (42.5^\circ + 71.4^\circ)$
$= 180^\circ - 113.9^\circ = 66.1^\circ$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 66.1^\circ} = \frac{215}{\sin 42.5^\circ}$

$c = \frac{215 \sin 66.1^\circ}{\sin 42.5^\circ}$

$= 291 \text{ inches}$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 71.4^\circ} = \frac{215}{\sin 42.5^\circ}$

$b = \frac{215 \sin 71.4^\circ}{\sin 42.5^\circ}$

$= 302 \text{ inches}$

**17.** $A = 180^\circ - (B + C)$
$= 180^\circ - (57^\circ + 31^\circ) = 92^\circ$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 57^\circ} = \frac{7.3}{\sin 92^\circ}$

$b = \frac{7.3 \sin 57^\circ}{\sin 92^\circ}$

$= 6.1 \text{ meters}$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 31^\circ} = \frac{7.3}{\sin 92^\circ}$

$c = \frac{7.3 \sin 31^\circ}{\sin 92^\circ}$

$= 3.8 \text{ meters}$

**19.** $B = 180^\circ - (A + C)$
$= 180^\circ - (43.5^\circ + 120.5^\circ)$
$= 180^\circ - 164^\circ = 16^\circ$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 16^\circ} = \frac{3.48}{\sin 43.5^\circ}$

$b = \frac{3.48 \sin 16^\circ}{\sin 43.5^\circ} = 1.39 \text{ ft}$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 120.5^\circ} = \frac{3.48}{\sin 43.5^\circ}$

$c = \frac{3.48 \sin 120.5^\circ}{\sin 43.5^\circ} = 4.36 \text{ ft}$



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**21.** $A = 180^\circ - (B + C)$
$= 180^\circ - (13.4^\circ + 24.8^\circ)$
$= 180^\circ - 38.2^\circ = 141.8^\circ$

$$\frac{b}{\sin B} = \frac{a}{\sin A} \qquad \frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{b}{\sin 13.4^\circ} = \frac{315}{\sin 141.8^\circ} \qquad \frac{c}{\sin 24.8^\circ} = \frac{315}{\sin 141.8^\circ}$$

$$b = \frac{315 \sin 13.4^\circ}{\sin 141.8^\circ} \qquad c = \frac{315 \sin 24.8^\circ}{\sin 141.8^\circ}$$

$= 118$ cm $\qquad = 214$ cm

**23.** $\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$

$$\frac{\sin B}{20} = \frac{\sin 30^\circ}{2}$$

$$\sin B = \frac{20 \sin 30^\circ}{2} = 5$$

This is impossible because the sine function must be between – 1 and 1.

**25.** $s = r\theta \quad (\theta \text{ is } \angle C)$ Arc length formula

$11 = 12 \cdot \theta$ Substitute known values

$\theta = \dfrac{11}{12}$ Divide both sides by 12

$\angle C = \dfrac{11}{12}$ radians. Converting this to degrees, we get:

$$\angle C = \left(\frac{11}{12} \cdot \frac{180}{\pi}\right)^\circ = 53^\circ$$

Also: $D = 180^\circ - (C + A)$

$= 180^\circ - (53^\circ + 31^\circ)$

$= 180^\circ - 84^\circ = 96^\circ$

Using the Law of Sines, we find $x$:

$$\frac{x + r}{\sin D} = \frac{r}{\sin A} \qquad x + 12 = 23$$

$$\frac{x + 12}{\sin 96^\circ} = \frac{12}{\sin 31^\circ} \qquad x = 11$$

$$x + 12 = \frac{12 \sin 96^\circ}{\sin 31^\circ}$$

**27.** $s = r\theta \quad (\theta \text{ is } \angle C)$ Arc length formula

$18 = 15 \cdot \theta$ Substitute known values

$\theta = \dfrac{18}{15} = \dfrac{6}{5}$ Divide both sides by 15

$\angle C = \dfrac{6}{5}$ radians. Converting this to degrees, we get:

$$\angle C = \left(\frac{6}{5} \cdot \frac{180}{\pi}\right)^\circ = 69^\circ$$

Also: $D = 180^\circ - (C + A)$

$= 180^\circ - (69^\circ + 45^\circ)$

$= 180^\circ - 114^\circ = 66^\circ$



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Using the Law of Sines, we find $y$:

$$\frac{y}{\sin C} = \frac{r}{\sin A} \qquad y = \frac{15\sin 69^\circ}{\sin 45^\circ}$$

$$\frac{y}{\sin 69^\circ} = \frac{15}{\sin 45^\circ} \qquad y = 20$$

**29.** We find the missing angles first:

$\angle ABD = 180^\circ - 64^\circ = 116^\circ$

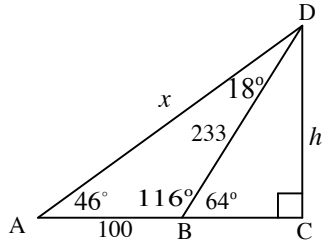
$\angle ADB = 180^\circ - (46^\circ + 116^\circ) = 18^\circ$

Now we find $\overline{BD}$ using the Law of Sines:

$$\frac{\overline{BD}}{\sin A} = \frac{\overline{AB}}{\sin \angle ADB}$$

$$\frac{\overline{BD}}{\sin 46^\circ} = \frac{100}{\sin 18^\circ}$$

$$\overline{BD} = \frac{100\sin 46^\circ}{\sin 18^\circ} = 233$$




Then we find $h$, using the sine ratio:

$$\sin 64^\circ = \frac{h}{233}$$

$$h = 233\sin 64^\circ = 209 \text{ feet}$$

**31.** First, we find $C$:

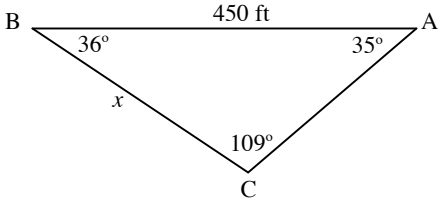
$$C = 180^\circ - (36^\circ + 35^\circ)$$

$$= 180^\circ - 71^\circ = 109^\circ$$

Next, we find the distance that the man travels from $A$ to $B$:

$$d = r \cdot t$$

$$= 5\frac{\text{ft}}{\text{sec}} \cdot 90 \text{ sec} = 450 \text{ ft}$$




Now we use the Law of Sines to find $x$:

$$\frac{x}{\sin 35^\circ} = \frac{450}{\sin 109^\circ}$$

$$x = \frac{450\sin 35^\circ}{\sin 109^\circ} = 273 \text{ ft}$$

**33.** We find the missing angles first:

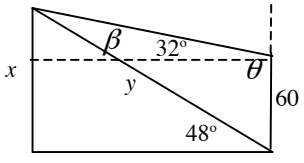
$\theta = 90^\circ + 32^\circ = 122^\circ$

$\beta = 180^\circ - (122^\circ + 42^\circ) = 16^\circ$

Now we find $y$ using the Law of Sines:

$$\frac{y}{\sin 122^\circ} = \frac{60}{\sin 16^\circ}$$

$$y = \frac{60\sin 122^\circ}{\sin 16^\circ} = 184.60$$




Then we find $x$, using the sine ratio:

$$\sin 48^\circ = \frac{x}{184.60}$$

$$x = 184.60\sin 48^\circ = 137 \text{ ft}$$



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**35.** Using triangle ABC on the ground, $C = 180^\circ - (105^\circ + 44^\circ) = 31^\circ$.

Now we can use the Law of Sines to find *b*:

$$\frac{b}{\sin 44^\circ} = \frac{25}{\sin 31^\circ}$$

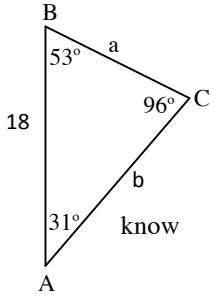
$$b = \frac{25 \sin 44^\circ}{\sin 31^\circ} = 34$$

Then we can find h using the tangent ratio:

$$\tan 51^\circ = \frac{h}{34}$$

$$h = 34 \tan 51^\circ = 42 \text{ ft}$$

**37.** We find the missing angle first: $\angle C = 180^\circ - (53^\circ + 31^\circ) = 96^\circ$

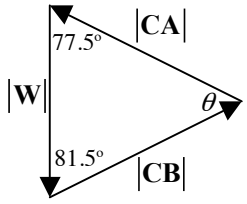



Then we find the missing sides using the Law of Sines:

$$\frac{18}{\sin 96^\circ} = \frac{a}{\sin 31^\circ} \qquad \frac{18}{\sin 96^\circ} = \frac{b}{\sin 53^\circ}$$

$$a = \frac{18 \sin 31^\circ}{\sin 96^\circ} = 9.3 \text{ miles} \qquad b = \frac{18 \sin 53^\circ}{\sin 96^\circ} = 14 \text{ miles}$$

**39.** We can redraw the two tension vectors **CA** and **CB** and the vector **W** due to gravity. We know that the magnitude of **W** is 125 pounds.




First, we find the missing angle:

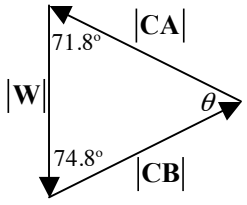
$\theta = 180^\circ - (77.5^\circ + 81.5^\circ) = 21^\circ$

Then, we find $|\mathbf{CA}|$ and $|\mathbf{CB}|$ using the Law of Sines:

$$\frac{|\mathbf{CA}|}{\sin 81.5^\circ} = \frac{125}{\sin 21^\circ} \qquad \frac{|\mathbf{CB}|}{\sin 77.5^\circ} = \frac{125}{\sin 21^\circ}$$

$$|\mathbf{CA}| = \frac{125 \sin 81.5^\circ}{\sin 21^\circ} = 345 \text{ lb} \qquad |\mathbf{CB}| = \frac{125 \sin 77.5^\circ}{\sin 21^\circ} = 341 \text{ lb}$$

**41.** We can redraw the two tension vectors **CA** and **CB** and the vector **W** due to gravity. We know that the magnitude of **W** is 1850 pounds.




First, we find the missing angle:

$\theta = 180^\circ - (71.8^\circ + 74.8^\circ) = 33.4^\circ$

Then, we find $|\mathbf{CA}|$ and $|\mathbf{CB}|$ using the Law of Sines:

$$\frac{|\mathbf{CA}|}{\sin 71.8^\circ} = \frac{1850}{\sin 33.4^\circ} \qquad \frac{|\mathbf{CB}|}{\sin 74.8^\circ} = \frac{1850}{\sin 33.4^\circ}$$

$$|\mathbf{CA}| = \frac{1850 \sin 74.8^\circ}{\sin 33.4^\circ} = 3240 \text{ lb} \qquad |\mathbf{CB}| = \frac{1850 \sin 71.8^\circ}{\sin 33.4^\circ} = 3190 \text{ lb}$$

**43.**

$$2\sin\theta - \sqrt{2} = 0$$

$$2\sin\theta = \sqrt{2}$$

$$\sin\theta = \frac{\sqrt{2}}{2}$$

$$\hat{\theta} = 45^\circ$$

$$\theta = 45^\circ \text{ or } 135^\circ$$

**45.**

$$\sin\theta\cos\theta - 2\cos\theta = 0$$

$$\cos\theta(\sin\theta - 2) = 0$$

$$\sin\theta = 2 \text{ or } \cos\theta = 0$$

No solution $\qquad \theta = 90^\circ \text{ or } 270^\circ$

**47.**

$$2\sin^2\theta - 3\sin\theta + 1 = 0$$

$$(2\sin\theta - 1)(\sin\theta - 1) = 0$$

$$2\sin\theta - 1 = 0 \text{ or } \sin\theta - 1 = 0$$

$$\sin\theta = \frac{1}{2} \qquad \sin\theta = 1$$

$$\theta = 30^\circ \text{ or } 150^\circ \qquad \theta = 90^\circ$$



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## 7.2 The Law of Cosines

**EVEN SOLUTIONS**

**2**. An oblique triangle can have at most one obtuse angle, which must be opposite the longest side.

**4**. Given all three sides, first use the law of cosines to find the largest angle.

**6**. The difference between heading and true course is that true course is measured to the vector representing the actual path of the object.

**8**. Ground speed is the speed is the speed of the object relative to the ground, and is the magnitude of the vector representing the velocity of the object in the direction of the true course.

**10**. Completing the formula:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 - b^2 - c^2 = -2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

**12**. Using the law of cosines to find $c$:

$$c^2 = a^2 + b^2 - 2ab\cos C = 48^2 + 84^2 - 2(48)(84)\cos 120° = 13{,}392$$

$$c = \sqrt{13{,}392} \approx 115.7 \text{ inches} \approx 120 \text{ inches}$$

**14**. Using the law of cosines to find $C$:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$29^2 = 22^2 + 24^2 - 2(22)(24)\cos C$$

$$841 = 1060 - 1056\cos C$$

$$-219 = -1056\cos C$$

$$\cos C \approx 0.2074$$

$$C \approx 78°$$

**16**. Using the law of cosines to find $b$:

$$b^2 = a^2 + c^2 - 2ac\cos B = 3.7^2 + 6.4^2 - 2(3.7)(6.4)\cos 33° \approx 14.9306$$

$$b = \sqrt{14.9306} \approx 3.9 \text{ m}$$

**18**. Using the law of cosines to find $A$:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$51^2 = 24^2 + 41^2 - 2(24)(41)\cos A$$

$$2601 = 2257 - 1968\cos A$$

$$344 = -1968\cos A$$

$$\cos A \approx -0.1748$$

$$A \approx 100°$$

**20**. First find $b$ using the law of cosines:

$$b^2 = a^2 + c^2 - 2ac\cos B = 76.3^2 + 42.8^2 - 2(76.3)(42.8)\cos 16.3° \approx 1384.77$$

$$b = \sqrt{1384.77} \approx 37.2 \text{ m}$$

Now find $C$ using the law of sines:

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{42.8} = \frac{\sin 16.3°}{37.2}$$

$$\sin C = \frac{42.8\sin 16.3°}{37.2} \approx 0.3229$$

$$C \approx 18.8°, 161.2°$$

Note that $C \approx 161.2°$ is impossible, since $a$ is the largest side, thus $A$ must be the largest angle. Finally, $A = 180° - (16.3° + 18.8°) = 144.9°$.



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**22**. First find $B$ using the law of cosines:

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$75^2 = 48^2 + 63^2 - 2(48)(63)\cos B$$
$$5625 = 6273 - 6048\cos B$$
$$-648 = -6048\cos B$$
$$\cos B \approx 0.1071$$
$$B \approx 84°$$

Now find $A$ using the law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin A}{48} = \frac{\sin 84°}{75}$$
$$\sin A = \frac{48\sin 84°}{75} \approx 0.6365$$
$$A \approx 39°$$

Finally $C = 180° - (39° + 84°) = 57°$.

**24**. First find $a$ using the law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A = 63.4^2 + 75.2^2 - 2(63.4)(75.2)\cos 124°40' \approx 15098.32$$
$$a = \sqrt{15098.32} \approx 123 \text{ km}$$

Now find $C$ using the law of sines:

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin C}{75.2} = \frac{\sin 124°40'}{123}$$
$$\sin C = \frac{75.2\sin 124°40'}{123} \approx 0.5028$$
$$C \approx 30.19° = 30°10'$$

Finally $B = 180° - (124°40' + 30°10') = 25°10'$.

**26**. First find $A$ using the law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$832^2 = 623^2 + 345^2 - 2(623)(345)\cos A$$
$$692224 = 507154 - 429870\cos A$$
$$185070 = -429870\cos A$$
$$\cos A \approx -0.4305$$
$$A \approx 115.5°$$

Now find $B$ using the law of sines:

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{623} = \frac{\sin 115.5°}{832}$$
$$\sin B = \frac{623\sin 115.5°}{832} \approx 0.6759$$
$$B \approx 42.5°$$

Finally $C = 180° - (115.5° + 42.5°) = 22.0°$.



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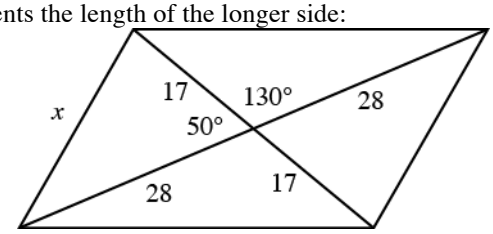
**28**. Using the law of cosines and substituting $b^2 + c^2$ for $a^2$:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 + c^2 = b^2 + c^2 - 2bc\cos A$$
$$0 = -2bc\cos A$$
$$\cos A = 0$$
$$A = 90°$$

**30**. Draw the figure, where $x$ represents the length of the longer side:

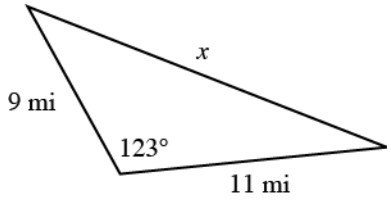



Using the law of cosines:

$$x^2 = 17^2 + 28^2 - 2(17)(28)\cos 50° \approx 461$$
$$x = \sqrt{461} \approx 21 \text{ in.}$$

The length of the shorter side is 21 inches.

**32**. After 30 minutes (1/2 hour) the ships have traveled 9 mi and 11 mi, respectively. Draw the figure, where $x$ represents their distance apart after 30 minutes:

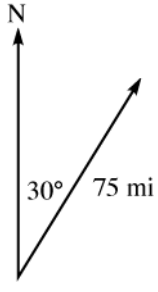



Using the law of cosines:

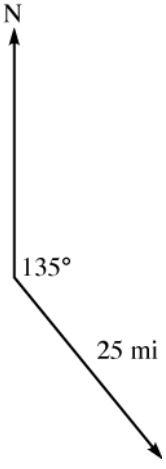
$$x^2 = 9^2 + 11^2 - 2(9)(11)\cos 123° \approx 309.84$$
$$x = \sqrt{309.84} \approx 18 \text{ mi}$$

The two ships are approximately 18 miles apart.

**34**. Drawing the vector:




**36**. Drawing the vector:



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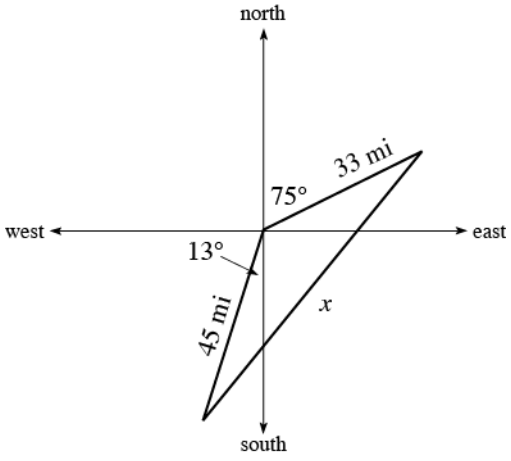
**38**. Draw the figure (using 45 mi and 33 mi for the ship distances), where $x$ represents the distance between the two ships after 3 hours:

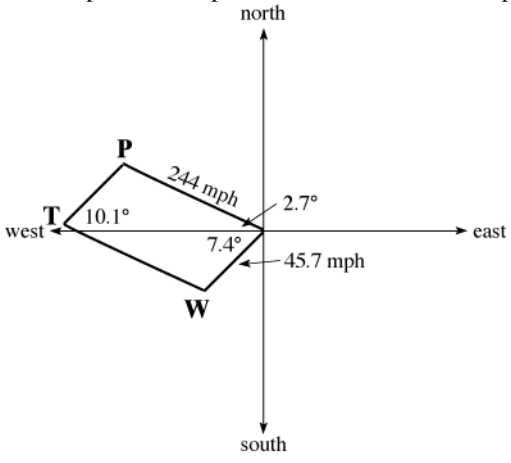



Note that the angle opposite $x$ is 13° + 90° + 15° = 118°. Using the law of cosines:

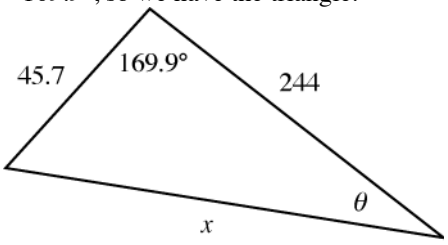
$$x^2 = 45^2 + 33^2 - 2(45)(33)\cos 118° \approx 4508.33$$

$$x = \sqrt{4508.33} \approx 67 \text{ mi}$$

**40**. Draw the figure, where **P** represents the plane, **W** represents the wind, and **T** represents the true course:




Note the angle at **P** is 180° – 10.1° = 169.9°, so we have the triangle:




Find $x$ using the law of cosines:

$$x^2 = 45.7^2 + 244^2 - 2(45.7)(244)\cos 169.9° \approx 83580.49$$

$$x = \sqrt{83580.49} \approx 289 \text{ mph}$$

Find $\theta$ using the law of sines:

$$\frac{\sin\theta}{45.7} = \frac{\sin 169.9°}{289}$$

$$\sin\theta = \frac{45.7 \sin 169.9°}{289} \approx 0.0277$$

$$\theta \approx 1.6°$$

The ground speed is approximately 289 mph, and the true course is 272.7° – 1.6° = 271.1° from due north.



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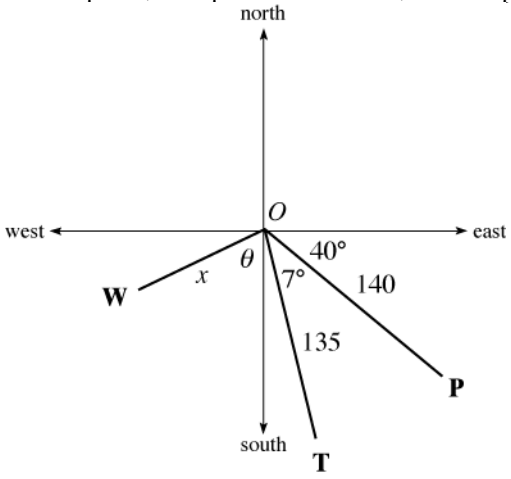
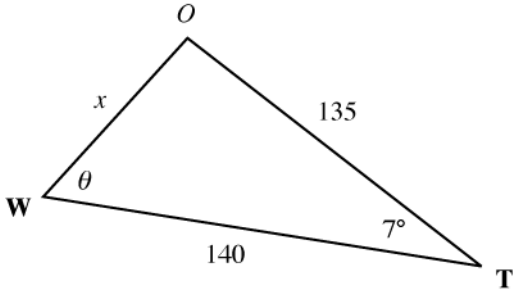
**42**. Draw the figure, where **P** represents the plane, **W** represents the wind, and **T** represents the true course:




We have the triangle:




Find $x$ using the law of cosines:

$$x^2 = 135^2 + 140^2 - 2(135)(140)\cos 7° \approx 306.76$$

$$x = \sqrt{306.76} \approx 17.5 \text{ mph}$$

Find $\theta$ using the law of sines:

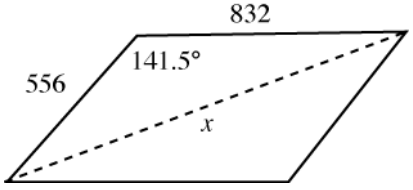
$$\frac{\sin\theta}{135} = \frac{\sin 7°}{17.5}$$

$$\sin\theta = \frac{135\sin 7°}{17.5} \approx 0.9401$$

$$\theta \approx 70°$$

The wind speed is approximately 18 mph, and the wind direction is 270° – 70° + 40° = 240° from due north.

**44**. First sketch the figure:




Using the law of cosines:

$$x^2 = 556^2 + 832^2 - 2(556)(832)\cos 141.5° \approx 1{,}725{,}416$$

$$x = \sqrt{1{,}725{,}416} \approx 1{,}310 \text{ lb}$$



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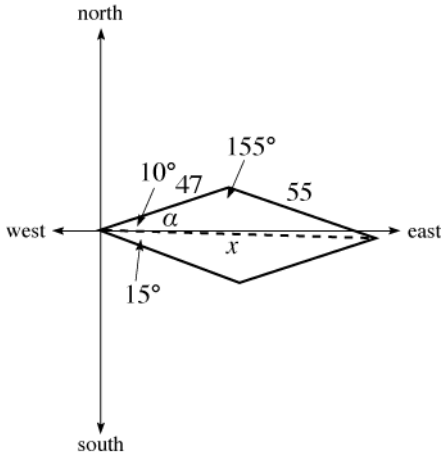
**46**. First sketch the figure:




Using the law of cosines:

$$x^2 = 47^2 + 55^2 - 2(47)(55)\cos 155° \approx 9{,}920$$

$$x = \sqrt{9{,}920} \approx 100 \text{ lb}$$

Using the law of sines:

$$\frac{\sin\alpha}{55} = \frac{\sin 155°}{100}$$

$$\sin\alpha = \frac{55\sin 155°}{100} \approx 0.2334$$

$$\alpha = \sin^{-1}(0.2334) \approx 13°$$

So the direction is 80° + 13° = 93°.

**48**. Let $x$ represent the length of the down tube. Using the law of cosines:

$$x^2 = 56.9^2 + 57.0^2 - 2(56.9)(57.0)\cos 73.0° \approx 4590.11$$

$$x = \sqrt{4590.11} \approx 67.8$$

The down tube is approximately 67.8 cm long. Let $\theta$ represent the angle between the seat tube and the down tube. Using the law of sines:

$$\frac{\sin\theta}{56.9} = \frac{\sin 73.0°}{67.8}$$

$$\sin\theta = \frac{56.9\sin 73.0°}{67.8} \approx 0.8026$$

$$\theta = \sin^{-1}(0.8026) \approx 53.4°$$

The angle between the seat tube and the down tube is approximately 53.4°.

**50**. **a**. Note that $\angle ABC = 180° - 53° - 69° = 58°$ . Using the law of sines:

$$\frac{\sin 58°}{AC} = \frac{\sin 53°}{49}$$

$$AC = \frac{49\sin 58°}{\sin 53°} \approx 52 \text{ cm}$$



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**b**. Using the law of cosines:

$$CD^2 = 49^2 + 59^2 - 2(49)(59)\cos 88° \approx 5{,}680.2$$

$$CD = \sqrt{5{,}680.2} \approx 75.4$$

Now using the law of sines:

$$\frac{\sin 88°}{75.4} = \frac{\sin \angle BDC}{49}$$

$$\sin \angle BDC = \frac{49 \sin 88°}{75.4} \approx 0.6498$$

$$\angle BDC = \sin^{-1}(0.6498) \approx 41°$$

**52**. Let $x$ represent the distance between Cayucos and Atascadero. Using the law of cosines:

$$x^2 = 12.9^2 + 4.61^2 - 2(12.9)(4.61)\cos 79.2° \approx 165.4$$

$$x = \sqrt{165.4} \approx 12.9$$

Let $y$ represent the distance between Cambria and Templeton. Using the law of sines:

$$\frac{\sin 36.1°}{12.9} = \frac{\sin 106.1°}{y}$$

$$y = \frac{12.9 \sin 106.1°}{\sin 36.1°} \approx 21$$

Let $z$ represent the distance between Cambria and Paso Robles. Using the law of sines:

$$\frac{\sin 109.8°}{z} = \frac{\sin 56.1°}{21}$$

$$z = \frac{21 \sin 109.8°}{\sin 56.1°} \approx 23.8$$

The distance is 23.8 miles.

**54**. The radian solutions are:

$$4x = \frac{2\pi}{3} + 2k\pi \qquad 4x = \frac{4\pi}{3} + 2k\pi$$

$$x = \frac{\pi}{6} + \frac{k\pi}{2} \qquad x = \frac{\pi}{3} + \frac{k\pi}{2}$$

**56**. Since $\tan^2 4x = 3$, $\tan 4x = \pm\sqrt{3}$. The radian solutions are:

$$4x = \frac{\pi}{3} + k\pi \qquad 4x = \frac{2\pi}{3} + k\pi$$

$$x = \frac{\pi}{12} + \frac{k\pi}{4} \qquad x = \frac{\pi}{6} + \frac{k\pi}{4}$$

**58**. Factoring the equation:

$$3\sin^2 2\theta - 2\sin 2\theta - 5 = 0$$

$$(3\sin 2\theta - 5)(\sin 2\theta + 1) = 0$$

$$\sin 2\theta = \frac{5}{3}, -1$$

Note that $\sin 2\theta = \frac{5}{3}$ has no solutions. For $\sin 2\theta = -1$:

$$2\theta = 270° + 360°k$$

$$\theta = 135° + 180°k$$

**60**. Using the addition formula for cosine:

$$\cos 3\theta \cos 2\theta - \sin 3\theta \sin 2\theta = -1$$

$$\cos(3\theta + 2\theta) = -1$$

$$\cos 5\theta = -1$$

$$5\theta = 180° + 360°k$$

$$\theta = 36° + 72°k$$



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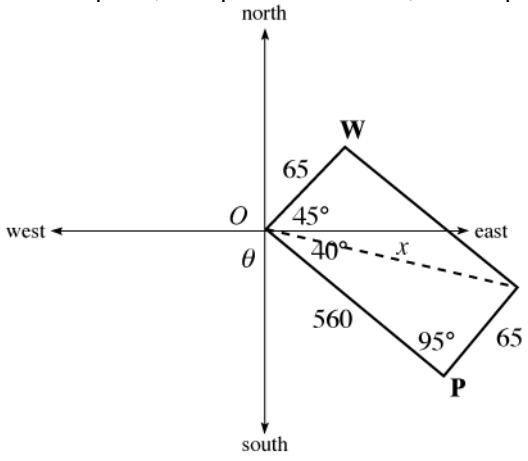
**62.** If $\sin\theta - \cos\theta = 0$, $\sin\theta = \cos\theta$ and thus $\tan\theta = 1$. Thus $\theta = 45°, 225°$.

**64.** Using the law of cosines:

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$22.3^2 = 13.8^2 + 9.5^2 - 2(13.8)(9.5)\cos B$$
$$497.29 = 190.44 + 90.25 - 262.2\cos B$$
$$216.6 = -262.2\cos B$$
$$\cos B \approx -0.8261$$
$$B \approx 145.7°$$

The correct answer is b.

**66.** Draw the figure, where **P** represents the plane, **W** represents the wind, and $x$ represents the ground speed:




Find $x$ using the law of cosines:

$$x^2 = 65^2 + 560^2 - 2(65)(560)\cos 95° \approx 324{,}170$$
$$x = \sqrt{324{,}170} \approx 569 \text{ mph}$$

The ground speed is approximately 569 mph. The correct answer is b.

**ODD SOLUTIONS**

**1.** sides, angle, sides, unique

**3.** side, cosines, sines, smaller

**5.** clockwise, north, intended

**7.** air, heading

**9.** $b^2 + c^2 - 2bc\cos A$

**11.**
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= (120)^2 + (66)^2 - 2(120)(66)\cos 60°$$
$$= 10{,}836$$
$$c = 100 \text{ inches (rounded to 2 significant digits)}$$

**13.**
$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$
$$= \frac{15^2 - 13^2 - 14^2}{-2(13)(14)}$$
$$= 0.3846$$
$C = 67°$ (The largest angle is opposite the longest side)

**15.**
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= (4.2)^2 + (6.8)^2 - 2(4.2)(6.8)\cos 116°$$
$$= 88.92$$
$$a = 9.4 \text{ meters}$$

**17.**
$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$
$$= \frac{38^2 - 10^2 - 31^2}{-2(10)(31)}$$
$$= -0.6177$$
$$A = 128°$$



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**19.** $b^2 = a^2 + c^2 - 2ac\cos B$

$= (412)^2 + (342)^2 - 2(412)(342)\cos 151.5^\circ$

$= 534{,}365.6929$

$b = 731$ meters

$\sin A = \dfrac{a\sin B}{b}$

$= \dfrac{412\sin 151.5^\circ}{731}$

$= 0.2689$

$A = 15.6^\circ$

$C = 180^\circ - (15.6^\circ + 151.5^\circ) = 12.9^\circ$

**21.** $\cos B = \dfrac{b^2 - a^2 - c^2}{-2ac}$

$= \dfrac{75^2 - 48^2 - 63^2}{-2(48)(63)}$

$= 0.1071$

$B = 84^\circ$

$\sin C = \dfrac{c\sin B}{b}$

$= \dfrac{63\sin 84^\circ}{75}$

$= 0.8354$

$C = 57^\circ$

$A = 180^\circ - (57^\circ + 84^\circ) = 39^\circ$

**23.** $a^2 = b^2 + c^2 - 2bc\cos A$

$= (0.923)^2 + (0.387)^2 - 2(0.923)(0.387)\cos 43^\circ 20'$

$= 0.4821$

$a = 0.694$ kilometers

$\sin C = \dfrac{c\sin A}{a}$

$= \dfrac{0.387\sin 43^\circ 20'}{0.694} = 0.3827$

$C = 22^\circ 30'$

$B = 180^\circ - (22^\circ 30' + 43^\circ 20') = 114^\circ 10'$

***Note**: Answers may differ depending on the order in which the angles are found.*

**25.** $\cos C = \dfrac{c^2 - a^2 - b^2}{-2ab}$

$= \dfrac{5.22^2 - 4.38^2 - 3.79^2}{-2(4.38)(3.79)}$

$= 0.1898$

$C = 79.1^\circ$

$\sin B = \dfrac{b\sin C}{c}$

$= \dfrac{3.79\sin 79.1^\circ}{5.22}$

$= 0.7130$

$B = 45.5^\circ$

$A = 180^\circ - (79.1^\circ + 45.5^\circ) = 55.4^\circ$

***Note**: Answers may differ depending on the order in which the angles are found.*

**27.** $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = b^2 + c^2 - 2bc\cos 90^\circ$

$a^2 = b^2 + c^2 - 2bc(0)$

$a^2 = b^2 + c^2$

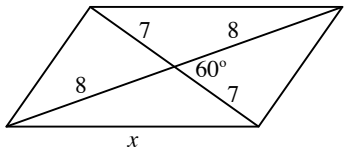
**29.** The diagonals of a parallelogram bisect each other.

The angle opposite side $x$ is $120^\circ$.

$x^2 = 7^2 + 8^2 - 2(7)(8)\cos 120^\circ$

$= 169$

$x = 13$ meters




**31.** $d_1 = r_1 t_1$ $= 130(1.5)$ $= 195$ miles

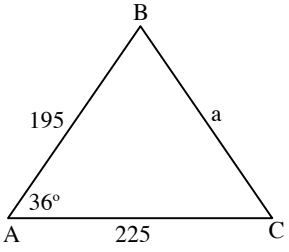
$d_2 = r_2 t_2$ $= 150(1.5)$ $= 225$ miles

$a^2 = b^2 + c^2 - 2bc\cos A$

$= (225)^2 + (195)^2 - 2(225)(195)\cos 36^\circ$

$= 17658.76$

$a = 130$ miles (rounded to 2 significant digits)




For problems 33 and 35, see textbook answer section for diagrams.



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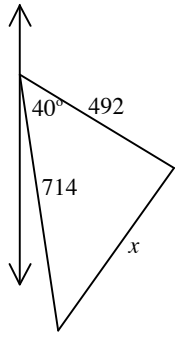
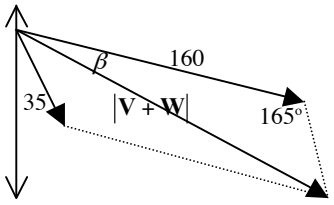
**37.** Distance of first plane is 246 ( 2 ) or 492 miles.
Distance of second plane is 357 ( 2 ) or 714 miles.
Angle between the two planes is 175º – 135º or 40º.
We will use the Law of Cosines to find $x$:

$$x^2 = (492)^2 + (714)^2 - 2(492)(714)\cos 40^\circ$$
$$= 213655.56$$
$$x = 462 \text{ miles (rounded to 3 significant digits)}$$




**39.** $|\mathbf{V}+\mathbf{W}|^2 = |\mathbf{V}|^2 + |\mathbf{W}|^2 - 2|\mathbf{V}||\mathbf{W}|\cos\theta$

$$= (35)^2 + (160)^2 - 2(35)(160)\cos 165^\circ$$
$$= 37{,}643$$
$$|\mathbf{V}+\mathbf{W}| = 190 \text{ mph (to 2 significant digits)}$$

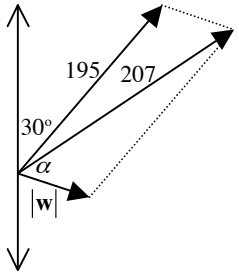



$$\sin\beta = \frac{35\sin 165^\circ}{190}$$
$$= 0.0477$$
$$\beta = 3^\circ \text{ (to the nearest degree)}$$

The true course is 150º + 3º = 153º. The ground speed of the plane is 190 mph.

**41.** The angle between the plane vector and its airspeed is 34.0º – 30.0º or 4.0º.
We will use the Law of Cosines to find the speed of the wind:

$$|\mathbf{w}|^2 = (195)^2 + (207)^2 - 2(195)(207)\cos 4.0^\circ$$
$$= 340.65$$
$$|\mathbf{w}| = 18.5 \text{ mph}$$




Next, we will use the Law of Sines to find $\alpha$:

$$\sin\alpha = \frac{195\sin 4.0^\circ}{18.5}$$
$$= 0.7353$$
$$\alpha = 47.3^\circ$$

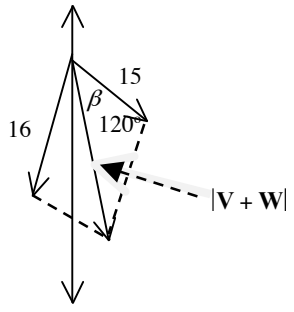
The wind is 18.5 mph at 34.0º + 47.3º or 81.3º from due north.

**43.** First we find the angle opposite the resultant: $\theta = 180^\circ - 26^\circ = 154^\circ$
We will use the Law of Cosines to find the magnitude of the resultant force.

$$|\mathbf{v}|^2 = (58)^2 + (73)^2 - 2(58)(73)\cos 154^\circ$$
$$= 16{,}303.98$$
$$|\mathbf{v}| = 130 \text{ pounds} \qquad \text{(rounded to 2 significant digits)}$$

**45.** First we find the angle opposite the resultant: $\theta = 180^\circ - 60^\circ = 120^\circ$
We will use the Law of Cosines to find the magnitude of the resultant vector.

$$|\mathbf{v}+\mathbf{w}|^2 = (15)^2 + (16)^2 - 2(15)(16)\cos 120^\circ$$
$$= 721$$
$$|\mathbf{v}+\mathbf{w}| = 27 \text{ knots} \qquad \text{(rounded to 2 significant digits)}$$




Next, we use the Law of Sines to find the direction of the resultant vector by finding $\beta$:

$$\sin\beta = \frac{16\sin 120^\circ}{27}$$
$$= 0.5132$$
$$\beta = 31^\circ$$

The direction of this vector is $130^\circ + 31^\circ = 161^\circ$.



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**47.** $x^2 = (52.3)^2 + (48.0)^2 - 2(52.3)(48.0)\cos 75.0^\circ$

$= 3740$

$x = 61.2$ cm

$\sin\theta = \dfrac{52.3\sin 75.0^\circ}{61.2}$

$= 0.8281$

$\theta = 55.6^\circ$

The length of the down tube is 61.2 cm and the angle between the seat tube and the down tube is $55.6^\circ$.

**49.** **a.** First we find $\angle ABC$: $\angle ABC = 180^\circ - (52^\circ + 65^\circ) = 63^\circ$

Then we use the Law of Sines to find $\overline{AC}$: $\overline{AC} = \dfrac{51\sin 52^\circ}{\sin 63^\circ}$

$= 45$ cm

**b.** We use the Law of Cosines to find $\angle BCD$: $\cos\angle BCD = \dfrac{61^2 - 51^2 - 78^2}{-2(51)(78)}$

$= 0.6239$

$\angle BCD = 51^\circ$

**51.** First we must find the distance from Cayucos to Templeton, $x$, using the Law of Cosines:

$x^2 = (12.9)^2 + (4.61)^2 - 2(12.9)(4.61)\cos 79.2^\circ$

$= 165.3753$

$x = 12.8598$ mi

We will call the distance from Cayucos to Templeton, $d$. Next we find the missing angle opposite side $d$:

$\angle = 180^\circ - (106.1^\circ + 37.8^\circ) = 36.1^\circ$

Now we can find $d$ using the Law of Sines: $d = \dfrac{12.8598\sin 37.8^\circ}{\sin 36.1^\circ} = 13.4$ mi

**53.** $\sin 3x = \dfrac{1}{2}$

$3x = \dfrac{\pi}{6} + 2k\pi$ or $3x = \dfrac{5\pi}{6} + 2k\pi$

$x = \dfrac{\pi}{18} + \dfrac{2k\pi}{3}$ $\quad$ $x = \dfrac{5\pi}{18} + \dfrac{2k\pi}{3}$

**55.** $\tan^2 3x = 1$

$\tan 3x = 1$ or $\tan 3x = -1$

$3x = \dfrac{\pi}{4} + k\pi$ $\quad$ $3x = \dfrac{3\pi}{4} + k\pi$

$x = \dfrac{\pi}{12} + \dfrac{k\pi}{3}$ $\quad$ $x = \dfrac{\pi}{4} + \dfrac{k\pi}{3}$

This can also be written as $x = \dfrac{\pi}{12} + \dfrac{k\pi}{6}$

**57.** $2\cos^2 3\theta - 9\cos 3\theta + 4 = 0$

$(2\cos 3\theta - 1)(\cos 3\theta - 4) = 0$

$\cos 3\theta - 4 = 0$ or $2\cos 3\theta - 1 = 0$

$\cos 3\theta = 4$ $\quad$ $\cos 3\theta = \dfrac{1}{2}$

No Solution $\quad$ $3\theta = 60^\circ + 360^\circ k$ or $3\theta = 300^\circ + 360^\circ k$

$\theta = 20^\circ + 120^\circ k$ $\quad$ $\theta = 100^\circ + 120^\circ k$



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**59.**
$$\sin 4\theta\cos 2\theta + \cos 4\theta\sin 2\theta = -1$$
$$\sin(4\theta + 2\theta) = -1$$
$$\sin 6\theta = -1$$
$$6\theta = 270^\circ + 360^\circ k$$
$$\theta = 45^\circ + 60^\circ k$$

**61.**
$$\sin\theta + \cos\theta = 1$$
$$(\sin\theta + \cos\theta)^2 = 1^2$$
$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1$$
$$1 + \sin 2\theta = 1$$
$$\sin 2\theta = 0$$
$$2\theta = 0^\circ \text{ or } 180^\circ$$
$$\theta = 0^\circ \text{ or } 90^\circ$$

Both answers check.

## 7.3 The Ambiguous Case

**EVEN SOLUTIONS**

**2**. With the ambiguous case, there may be zero, one, or two triangles that are determined.

**4**. When solving the ambiguous case using the law of cosines, the first step is to solve for the missing side. This will result in a quadratic equation, which can be solved using the quadratic formula.

**6**. Using the law of sines:
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{40} = \frac{\sin 30^\circ}{15}$$
$$\sin B = \frac{40\sin 30^\circ}{15} = \frac{40 \cdot \frac{1}{2}}{15} = \frac{4}{3}$$
Since $\sin B > 1$ is impossible, no such triangle exists.

**8**. Using the law of sines:
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{10} = \frac{\sin 30^\circ}{5}$$
$$\sin B = 2\sin 30^\circ = 2 \cdot \frac{1}{2} = 1$$
$$B = 90^\circ$$
Since there is only one solution ($B = 90^\circ$), one triangle exists.

**10**. Using the law of sines:
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{45} = \frac{\sin 20^\circ}{25}$$
$$\sin B = \frac{45\sin 20^\circ}{25} \approx 0.6156$$
$$B \approx 38^\circ, 142^\circ$$
Since $142^\circ + 20^\circ = 162^\circ < 180^\circ$, the third angle $C$ is possible. Two triangles exist.

**12**. Use the law of sines to find $B$:
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{37} = \frac{\sin 43^\circ}{31}$$
$$\sin B = \frac{37\sin 43^\circ}{31} \approx 0.8140$$
$$B \approx 54^\circ, 126^\circ$$



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If $B = 54°$, $C = 180° - (43° + 54°) = 83°$. Use the law of sines to find $c$:

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 83°}{c} = \frac{\sin 43°}{31}$$
$$c = \frac{31 \sin 83°}{\sin 43°} \approx 45 \text{ ft}$$

If $B' = 126°$, $C' = 180° - (43° + 126°) = 11°$. Use the law of sines to find $c'$:

$$\frac{\sin C'}{c'} = \frac{\sin A}{a}$$
$$\frac{\sin 11°}{c'} = \frac{\sin 43°}{31}$$
$$c' = \frac{31 \sin 11°}{\sin 43°} \approx 8.7 \text{ ft}$$

**14**. Use the law of sines to find $B$:

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{50.2} = \frac{\sin 132.4°}{27.3}$$
$$\sin B = \frac{50.2 \sin 132.4°}{27.3} \approx 1.36$$

Since $\sin B > 1$ is impossible, no such triangle exists.

**16**. Use the law of sines to find $B$:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin B}{821} = \frac{\sin 51°30'}{707}$$
$$\sin B = \frac{821 \sin 51°30'}{707} \approx 0.9088$$
$$B \approx 65.34° \approx 65°20'$$
$$B' \approx 114.66° \approx 114°40'$$

If $B = 65°20'$, $A = 180° - (65°20' + 51°30') = 63°10'$. Use the law of sines to find $a$:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 63°10'}{a} = \frac{\sin 51°30'}{707}$$
$$a = \frac{707 \sin 63°10'}{\sin 51°30'} \approx 806 \text{ m}$$

If $B' = 114°40'$, $A' = 180° - (114°40' + 51°30') = 13°50'$. Use the law of sines to find $a'$:

$$\frac{\sin A'}{a'} = \frac{\sin C}{c}$$
$$\frac{\sin 13°50'}{a'} = \frac{\sin 51°30'}{707}$$
$$a' = \frac{707 \sin 13°50'}{\sin 51°30'} \approx 216 \text{ m}$$



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**18**. Use the law of sines to find $C$:

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{1.12} = \frac{\sin 25°10'}{1.79}$$

$$\sin C = \frac{1.12 \sin 25°10'}{1.79} \approx 0.2661$$

$$C \approx 15.43° \approx 15°30'$$

$$C' \approx 164.57° \approx 164°30'$$

Note this second value of $C$ is impossible, since $B + C' \approx 189°40' > 180°$.

If $C = 15°30'$, $A = 180° - (25°10' + 15°30') = 139°20'$. Use the law of sines to find $a$:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 139°20'}{a} = \frac{\sin 25°10'}{1.79}$$

$$a = \frac{1.79 \sin 139°20'}{\sin 25°10'} \approx 2.74 \text{ inches}$$

**20**. Use the law of sines to find $A$:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{4.2} = \frac{\sin 34°}{4.2}$$

$$\sin A = \frac{4.2 \sin 34°}{4.2} \approx 0.5592$$

$$A = 34°$$

If $A = 34°$, $C = 180° - (34° + 34°) = 112°$. Use the law of sines to find $c$:

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 112°}{c} = \frac{\sin 34°}{4.2}$$

$$c = \frac{4.2 \sin 112°}{\sin 34°} \approx 7.0 \text{ cm}$$

**22**. Use the law of sines to find $B$:

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{7.6} = \frac{\sin 65°}{7.1}$$

$$\sin B = \frac{7.6 \sin 65°}{7.1} \approx 0.9701$$

$$B \approx 76°, 104°$$

If $B = 76°$, $C = 180° - (65° + 76°) = 39°$. Use the law of sines to find $c$:

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 39°}{c} = \frac{\sin 65°}{7.1}$$

$$c = \frac{7.1 \sin 39°}{\sin 65°} \approx 4.9 \text{ yd}$$



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If $B' = 104°$, $C' = 180° - (65° + 104°) = 11°$. Use the law of sines to find $c'$:

$$\frac{\sin C'}{c'} = \frac{\sin A}{a}$$

$$\frac{\sin 11°}{c'} = \frac{\sin 65°}{7.1}$$

$$c' = \frac{7.1 \sin 11°}{\sin 65°} \approx 1.5 \text{ yd}$$

**24**. Use the law of sines to find $B$:

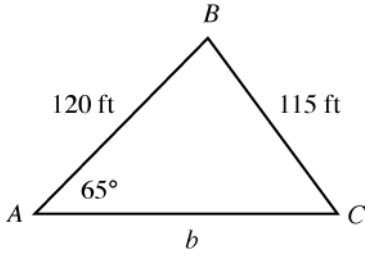
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin B}{94.2} = \frac{\sin 83.4°}{51.1}$$

$$\sin B = \frac{94.2 \sin 83.4°}{51.1} \approx 1.83$$

Since $\sin B > 1$ is impossible, no such triangle exists.

**26**. Drawing the figure, where $b$ represents the desired distance:




First find $C$ using the law of sines:

$$\frac{\sin C}{120} = \frac{\sin 65°}{115}$$

$$\sin C = \frac{120 \sin 65°}{115} \approx 0.9457$$

$$C \approx 71.0°, 109.0°$$

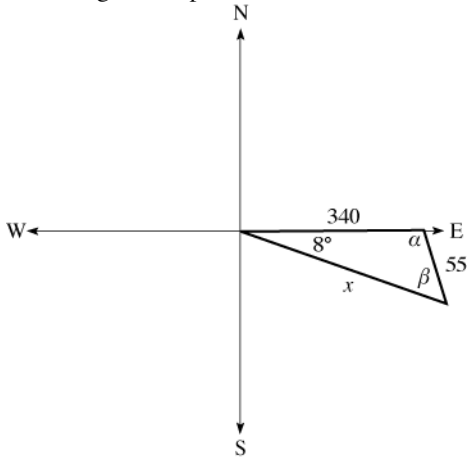
Thus $B = 180° - (65° + 71°) = 44°$ or $B = 180° - (65° + 109°) = 6°$. Now find $b$ using the law of sines:

$$\frac{\sin 44°}{b} = \frac{\sin 65°}{115} \qquad \frac{\sin 6°}{b} = \frac{\sin 65°}{115}$$

$$b = \frac{115 \sin 44°}{\sin 65°} \approx 88 \text{ ft} \qquad b = \frac{115 \sin 6°}{\sin 65°} \approx 13 \text{ ft}$$

The distance between the anchor points is either 88 feet or 13 feet.

**28**. Drawing the figure, where $x$ represents the ground speed:



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Using the law of sines:

$$\frac{\sin\beta}{340}=\frac{\sin 8°}{55}$$

$$\sin\beta=\frac{340\sin 8°}{55}$$

$$\sin\beta\approx 0.8603$$

$$\beta\approx 59.4°, 120.6°$$

$$\alpha\approx 112.6°, 51.4°$$

Now using the law of sines to find $x$ in each case:

$$\frac{\sin 51.4°}{x}=\frac{\sin 8°}{55}$$

$$x=\frac{55\sin 51.4°}{\sin 8°}$$

$$x\approx 310 \text{ mph}$$

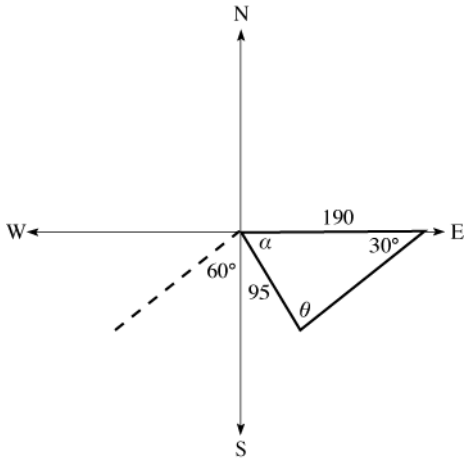
$$\frac{\sin 112.6°}{x}=\frac{\sin 8°}{55}$$

$$x=\frac{55\sin 112.6°}{\sin 8°}$$

$$x\approx 360 \text{ mph}$$

The ground speed (to two significant digits) is either 310 mph or 360 mph.

**30**. Drawing the figure:




Using the law of sines:

$$\frac{\sin\theta}{190}=\frac{\sin 30°}{95}$$

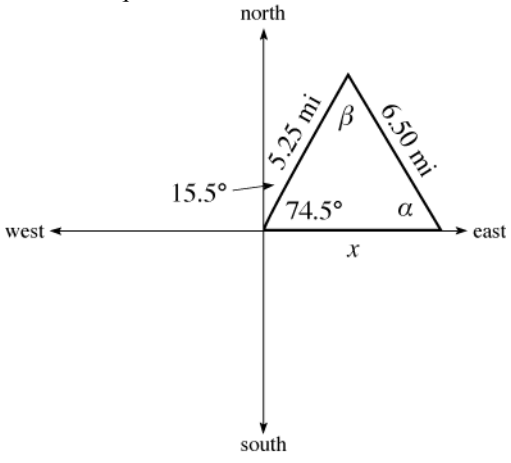
$$\sin\theta=2\sin 30°$$

$$\sin\theta=1$$

$$\theta=90°$$

Then $\alpha=60°$, so the true course is 150° from due north.

**32**. Drawing a figure, where $x$ represents the required distance:



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First find $\alpha$ using the law of sines:

$$\frac{\sin\alpha}{5.25}=\frac{\sin 74.5^\circ}{6.50}$$

$$\sin\alpha=\frac{5.25\sin 74.5^\circ}{6.50}\approx 0.7783$$

$$\alpha\approx 51.1^\circ$$

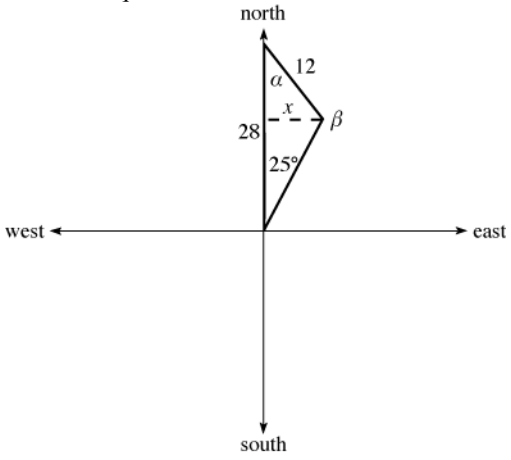
Thus $\beta=180^\circ-(74.5^\circ+51.1^\circ)=54.4^\circ$. Now find $x$ using the law of sines:

$$\frac{\sin 54.4^\circ}{x}=\frac{\sin 74.5^\circ}{6.50}$$

$$x=\frac{6.50\sin 54.4^\circ}{\sin 74.5^\circ}\approx 5.48 \text{ miles}$$

He will have to walk 5.48 miles back to the service station.

**34**. Drawing a figure, where $x$ represents the required distance:




First find $\beta$ using the law of sines:

$$\frac{\sin\beta}{28}=\frac{\sin 25^\circ}{12}$$

$$\sin\beta=\frac{28\sin 25^\circ}{12}\approx 0.9861$$

$$\beta\approx 80.4^\circ, 99.6^\circ$$

$$\alpha\approx 74.6^\circ, 55.4^\circ$$

If $\alpha\approx 74.6^\circ$, then:

$$\sin 74.6^\circ=\frac{x}{12}$$

$$x=12\sin 74.6^\circ\approx 11.6 \text{ mi}$$

If $\alpha\approx 55.4^\circ$, then:

$$\sin 55.4^\circ=\frac{x}{12}$$

$$x=12\sin 55.4^\circ\approx 9.9 \text{ mi}$$

The shortest distance is either 9.9 miles or 11.6 miles.

**36**. Since $\csc\theta=\frac{1}{\sin\theta}$, the equation becomes:

$$2\sin\theta-1=\frac{1}{\sin\theta}$$

$$2\sin^2\theta-\sin\theta=1$$

$$2\sin^2\theta-\sin\theta-1=0$$

$$(2\sin\theta+1)(\sin\theta-1)=0$$

$$\sin\theta=-\frac{1}{2},1$$

$$\theta=90^\circ, 210^\circ, 330^\circ$$



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**38**. Using the double-angle formula for cosine:

$$\cos 2\theta + 3\cos\theta - 2 = 0$$
$$2\cos^2\theta - 1 + 3\cos\theta - 2 = 0$$
$$2\cos^2\theta + 3\cos\theta - 3 = 0$$

Using the quadratic formula with $a = 2$, $b = 3$, and $c = -3$: $\cos\theta = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-3)}}{2(2)} = \dfrac{-3 \pm \sqrt{9+24}}{4} = \dfrac{-3 \pm \sqrt{33}}{4}$

So either $\cos\theta = \dfrac{-3-\sqrt{33}}{4} \approx -2.186$, which is impossible, or $\cos\theta = \dfrac{-3+\sqrt{33}}{4} \approx 0.686$, which occurs when $\theta \approx 46.7°, 313.3°$.

**40**. Using the double-angle formula for cosine:

$$7\sin^2\theta - 9\cos 2\theta = 0$$
$$7\sin^2\theta - 9\left(1 - 2\sin^2\theta\right) = 0$$
$$7\sin^2\theta - 9 + 18\sin^2\theta = 0$$
$$25\sin^2\theta = 9$$
$$\sin^2\theta = \frac{9}{25}$$
$$\sin\theta = \frac{3}{5}, -\frac{3}{5}$$

So either $\sin\theta = \dfrac{3}{5}$, which occurs when $\theta \approx 36.9°, 143.1°$, or $\sin\theta = -\dfrac{3}{5}$, which occurs when $\theta \approx 216.9°, 323.1°$.

**42**. Substituting $\cos^2 x = 1 - \sin^2 x$:

$$2\left(1 - \sin^2 x\right) - \sin x = 1$$
$$2 - 2\sin^2 x - \sin x = 1$$
$$-2\sin^2 x - \sin x + 1 = 0$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$\sin x = \frac{1}{2}, -1$$

For $\sin x = \dfrac{1}{2}$, $x = \dfrac{\pi}{6} + 2k\pi$ or $x = \dfrac{5\pi}{6} + 2k\pi$. For $\sin x = -1$, $x = \dfrac{3\pi}{2} + 2k\pi$.

**44**. Squaring each side:

$$(\sin x - \cos x)^2 = 1$$
$$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$$
$$1 - \sin 2x = 1$$
$$\sin 2x = 0$$
$$2x = 0, \pi, 2\pi, 3\pi$$
$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$



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Since this equation was solved by squaring, these possible solutions must be checked:

$x = 0$: $\sin 0 - \cos 0 = 0 - 1 = -1$ (doesn't check)

$x = \frac{\pi}{2}$: $\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = 1 - 0 = 1$ (checks)

$x = \pi$: $\sin\pi - \cos\pi = 0 - (-1) = 1$ (checks)

$x = \frac{3\pi}{2}$: $\sin\frac{3\pi}{2} - \cos\frac{3\pi}{2} = -1 - 0 = -1$ (doesn't check)

All solutions can be written in the form $x = \frac{\pi}{2} + 2k\pi$ or $x = \pi + 2k\pi$, where $k$ is any integer.

**46**. Using the law of cosines:

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$19^2 = 28^2 + c^2 - 2(28)c\cos 35°$$

$$c^2 - (56\cos 35°)c + 423 = 0$$

$$c^2 - 45.87c + 423 = 0$$

The correct answer is a.

**ODD SOLUTIONS**

**1.** two, opposite

**3.** angle, reference, I, II

**5.** $\sin B = \frac{b\sin A}{a}$

$= \frac{30\sin 150^\circ}{10} = 1.5$

Since sin $B$ can never be greater than 1, no triangle exists.

**7.** $\sin B = \frac{b\sin A}{a}$

$= \frac{20\sin 120^\circ}{30}$

$= 0.5774$

$B = 35^\circ$

Only one triangle is possible because there can only be one obtuse angle in a triangle.

**9.** $\sin B = \frac{b\sin A}{a}$

$= \frac{18\sin 60^\circ}{16}$

$= 0.9743$

$B = 77^\circ$ or $B' = 180^\circ - 77^\circ = 103^\circ$

Since $a < b$, there are 2 possible triangles.



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**11.** $\sin B = \dfrac{b \sin A}{a}$

$= \dfrac{54 \sin 38^\circ}{41}$

$= 0.8109$

$B = 54^\circ$ or $B' = 180^\circ - 54^\circ = 126^\circ$

Since $a < b$, there are 2 possible triangles.

$C = 180^\circ - (38^\circ + 54^\circ) = 88^\circ$

$c = \dfrac{a \sin C}{\sin A}$

$= \dfrac{41 \sin 88^\circ}{\sin 38^\circ} = 67$ ft

$C' = 180^\circ - (38^\circ + 126^\circ) = 16^\circ$

$c' = \dfrac{a \sin C'}{\sin A}$

$= \dfrac{41 \sin 16^\circ}{\sin 38^\circ} = 18$ ft

**13.** $\sin B = \dfrac{b \sin A}{a}$

$= \dfrac{22.3 \sin 112.2^\circ}{43.8}$

$= 0.4714$

$B = 28.1^\circ$

$C = 180^\circ - (112.2^\circ + 28.1^\circ)$

$= 180^\circ - 140.3^\circ = 39.7^\circ$

$c = \dfrac{a \sin C}{\sin A}$

$= \dfrac{43.8 \sin 39.7^\circ}{\sin 112.2^\circ}$

$= 30.2$ cm

There is only one triangle possible, because there can only be one obtuse angle in a triangle.

**15.** $\sin B = \dfrac{b \sin C}{c}$

$= \dfrac{425 \sin 27^\circ 50'}{347}$

$= 0.5719$

$B = 34^\circ 50'$ or $B' = 180^\circ - 34^\circ 50' = 145^\circ 10'$ There are two triangles possible because c < b.

$A = 180^\circ - (34^\circ 50' + 27^\circ 50')$

$= 180^\circ - 62^\circ 40' = 117^\circ 20'$

$a = \dfrac{c \sin A}{\sin C}$

$= \dfrac{347 \sin 117^\circ 20'}{\sin 27^\circ 50'}$

$= 660$ m

$A' = 180^\circ - (145^\circ 10' + 27^\circ 50')$

$= 180^\circ - 173^\circ = 7^\circ$

$a' = \dfrac{c \sin A'}{\sin C}$

$= \dfrac{347 \sin 7^\circ}{\sin 27^\circ 50'}$

$= 90.6$ m

**17.** $\sin C = \dfrac{c \sin B}{b}$

$= \dfrac{3.48 \sin 62^\circ 40'}{6.78}$

$= 0.4560$

$C = 27^\circ 10'$ or $C' = 180^\circ - 27^\circ 10'$

$C = 27^\circ 10'$ $C' = 152^\circ 50'$ (Rounded to the nearest 10 minutes)

This is impossible because $62^\circ 40' + 152^\circ 50' > 180^\circ$.

$A = 180^\circ - (62^\circ 40' + 27^\circ 10') = 90^\circ 10'$

$a = \dfrac{b \sin A}{\sin B}$

$= \dfrac{6.78 \sin 90^\circ 10'}{\sin 60^\circ 40'}$

$= 7.63$ in

**19.** $\sin A = \dfrac{a \sin B}{b}$

$= \dfrac{0.92 \sin 118^\circ}{0.68}$

$= 1.1946$

Since sin $A$ can never be greater than 1, no triangle exists.

**21.** $\sin B = \dfrac{b \sin A}{a}$

$= \dfrac{2.9 \sin 142^\circ}{1.4}$

$= 1.2753$

Since sin $B$ can never be greater than 1, no triangle exists.



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**23.** $\sin B = \dfrac{b\sin C}{c}$

$= \dfrac{36.8\sin 26.8^\circ}{36.8}$

$= 0.4509$ Only one triangle is possible because $b = c$.

$B = 26.8^\circ$ This is an isosceles triangle.

$A = 180^\circ - (26.8^\circ + 26.8^\circ) = 126.4^\circ$

$a = \dfrac{c\sin A}{\sin C} = \dfrac{36.8\sin 126.4^\circ}{\sin 26.8^\circ} = 65.7$ km

**25.** $\sin C = \dfrac{c\sin A}{a}$

$= \dfrac{51\sin 58^\circ}{44}$

$= 0.9637$

$C = 79^\circ$ or $C' = 101^\circ$

$B = 180^\circ - (58^\circ + 79^\circ)$

$= 180^\circ - 137^\circ = 43^\circ$

$b = \dfrac{44\sin 43^\circ}{\sin 58^\circ}$

$= 35$ feet

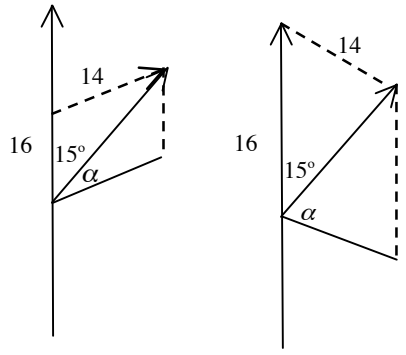
$B' = 180^\circ - (58^\circ + 101^\circ)$

$= 180^\circ - 159^\circ = 21^\circ$

$b' = \dfrac{44\sin 17^\circ}{\sin 58^\circ}$

$= 19$ feet

**27.** There are two possible drawings for this problem shown at the right. We will find $\alpha$ in each case:




$\sin\alpha = \dfrac{16\sin 15^\circ}{14}$

$= 0.2958$

$\alpha = 17^\circ$ or $\alpha' = 163^\circ$

The direction of the current is $\alpha + 15^\circ = 17^\circ + 15^\circ = 32^\circ$ or $\alpha' + 15^\circ = 163^\circ + 15^\circ = 178^\circ$ measured for due north.

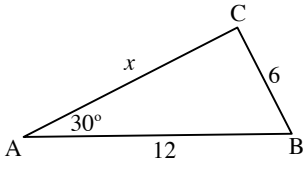
**29.** $\sin C = \dfrac{12\sin 30^\circ}{6}$

$= 1$

$C = 90^\circ$

This is a 30°-60°-90° right triangle.

Therefore, $x = 6\sqrt{3}$ or 10 mph.



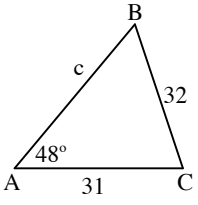

**31.** $\sin B = \dfrac{31\sin 48^\circ}{32}$

$= 0.7199$

$B = 46^\circ$ *or* $B' = 134^\circ$

This is impossible

$C = 180^\circ - (46^\circ + 44^\circ) = 86^\circ$




**33.** We must find $\theta$ and $\theta'$ as shown in the diagram at the right.

$\sin\theta = \dfrac{28\sin 25^\circ}{12}$

$= 0.9861$

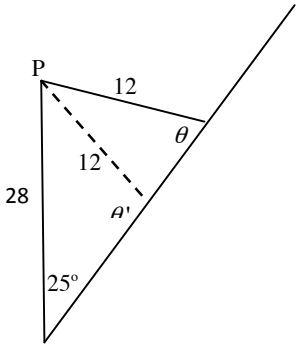
$\theta = 80^\circ$ *or* $\theta' = 100^\circ$

Next we must find the two possible angles at P:

$\angle P = 180^\circ - (25^\circ + 80^\circ) = 75^\circ$

$\angle P' = 180^\circ - (25^\circ + 100^\circ) = 55^\circ$

Therefore, they must head in the direction $S75^\circ E$ or $S55^\circ E$.



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**35.** $4\sin\theta - \csc\theta = 0$

$4\sin\theta - \dfrac{1}{\sin\theta} = 0$

$4\sin^2\theta - 1 = 0, \quad \sin\theta \neq 0$

$(2\sin\theta - 1)(2\sin\theta + 1) = 0$

$2\sin\theta - 1 = 0 \quad \text{or} \quad 2\sin\theta + 1 = 0$

$\sin\theta = \dfrac{1}{2} \qquad \sin\theta = -\dfrac{1}{2}$

$\theta = 30^\circ \text{ or } 150^\circ \qquad \theta = 210^\circ \text{ or } 330^\circ$

**37.** $2\cos\theta - \sin 2\theta = 0$

$2\cos\theta - 2\sin\theta\cos\theta = 0$

$2\cos\theta(1 - \sin\theta) = 0$

$2\cos\theta = 0 \quad \text{or} \quad 1 - \sin\theta = 0$

$\cos\theta = 0 \qquad \sin\theta = 1$

$\theta = 90^\circ \text{ or } 270^\circ \qquad \theta = 90^\circ$

**39.** $18\sec^2\theta - 17\tan\theta\sec\theta - 12 = 0$

$18\left(\dfrac{1}{\cos^2\theta}\right) - 17\left(\dfrac{\sin\theta}{\cos\theta}\right)\left(\dfrac{1}{\cos\theta}\right) - 12 = 0$

$18 - 17\sin\theta - 12\cos^2\theta = 0, \quad \cos\theta \neq 0$

$18 - 17\sin\theta - 12\left(1 - \sin^2\theta\right) = 0$

$18 - 17\sin\theta - 12 + 12\sin^2\theta = 0$

$12\sin^2\theta - 17\sin\theta + 6 = 0$

$12\sin^2\theta - 17\sin\theta + 6 = 0$

$\left(4\sin\theta - 3\right)\left(3\sin\theta - 2\right) = 0$

$4\sin\theta - 3 = 0 \quad \text{or} \quad 3\sin\theta - 2 = 0$

$\sin\theta = \dfrac{3}{4} \qquad \sin\theta = \dfrac{2}{3}$

$\theta = 48.6^\circ \text{ or } 131.4^\circ \qquad \theta = 41.8^\circ \text{ or } 138.2^\circ$

**41.** $2\cos x - \sec x + \tan x = 0$

$2\cos x - \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x} = 0$

$2\cos^2 x - 1 + \sin x = 0, \quad \cos x \neq 0$

$2(1 - \sin^2 x) - 1 + \sin x = 0$

$2 - 2\sin^2 x - 1 + \sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x - 1 = 0 \quad \text{or} \quad 2\sin x + 1 = 0$

$\sin x = 1 \qquad \sin x = -\dfrac{1}{2}$

$x = \dfrac{\pi}{2} + 2k\pi \qquad x = \dfrac{7\pi}{6} + 2k\pi \text{ or } \dfrac{11\pi}{6} + 2k\pi$

This answer doesn't check.

**43.** $\sin x + \cos x = 0$

$\sin x = -\cos x$

$\dfrac{\sin x}{\cos x} = -1, \quad \cos x \neq 0$

$\tan x = -1$

$\hat{x} = \dfrac{\pi}{4}$ and $x$ is in QII or QIV

$x = \dfrac{3\pi}{4} + 2k\pi \text{ or } x = \dfrac{7\pi}{4} + 2k\pi$ (This can also be written as $x = \dfrac{3\pi}{4} + k\pi$ )



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# 7.4 The Area of a Triangle

**EVEN SOLUTIONS**

**2.** To find the area of a triangle given two angles and one side, first find the missing angle and then use the appropriate formula.

**4.** To find the area of a triangle given all three sides, we can use Heron's formula. The first step is to find the semiperimeter.

**6.** Heron's formula is: $S = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$

**8.** The semiperimeter is: $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(153+174+232) = \frac{1}{2}(559) = 279.5$ cm

**10.** The semiperimeter is: $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(33+45+6) = \frac{1}{2}(84) = 42$ yd

**12.** Using the area formula: $S = \frac{1}{2}ab\sin C = \frac{1}{2}(10)(12)\sin 150° = 30$ cm$^2$

**14.** Using the area formula: $S = \frac{1}{2}ac\sin B = \frac{1}{2}(76.3)(42.8)\sin 13.6° \approx 384$ m$^2$

**16.** Using the area formula: $S = \frac{1}{2}bc\sin A = \frac{1}{2}(43.6)(75.2)\sin 124°40' \approx 1{,}350$ km$^2$

**18.** Note that $A = 180° - (57° + 31°) = 92°$. Using the area formula: $S = \frac{a^2 \sin B \sin C}{2\sin A} = \frac{3.7^2 \sin 57° \sin 31°}{2\sin 92°} \approx 2.96$ m$^2$

**20.** Note that $B = 180° - (110.4° + 31.8°) = 37.8°$. Using the area formula:

$$S = \frac{c^2 \sin A \sin B}{2\sin C} = \frac{240^2 \sin 110.4° \sin 37.8°}{2\sin 31.8°} \approx 31{,}400 \text{ in.}^2$$

**22.** Note that $A = 180° - (14°20' + 75°40') = 90°$. Using the area formula:

$$S = \frac{b^2 \sin A \sin C}{2\sin B} = \frac{7.22^2 \sin 90° \sin 75°40'}{2\sin 14°20'} \approx 102 \text{ ft}^2$$

**24.** Compute $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(12+23+34) = 34.5$. Using Heron's formula:

$$S = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{34.5(34.5-12)(34.5-23)(34.5-34)} \approx 66.8 \text{ in.}^2$$

**26.** Compute $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(48+57+63) = 84$. Using Heron's formula:

$$S = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{84(84-48)(84-57)(84-63)} \approx 1{,}310 \text{ yd}^2$$

**28.** Compute $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(8.32+12.36+5.34) = 13.01$. Using Heron's formula:

$$S = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{13.01(13.01-8.32)(13.01-12.36)(13.01-5.34)} \approx 17.4 \text{ ft}^2$$

**30.** Use $a = 24.1$, $b = 31.4$, and $c = 32.4$. Compute $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(24.1+31.4+32.4) = 43.95$. Using Heron's formula: $S = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{43.95(43.95-24.1)(43.95-31.4)(43.95-32.4)} \approx 355.609$ in.$^2$

Since the parallelogram consists of two of these triangles, its area is approximately $2(355.609) \approx 711$ in.$^2$.



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**32**. Note that $B = 180° - (25° + 110°) = 45°$. Using the area formula:

$$S = \frac{b^2 \sin A \sin C}{2 \sin B}$$

$$80 = \frac{b^2 \sin 25° \sin 110°}{2 \sin 45°}$$

$$160 \sin 45° = b^2 \sin 25° \sin 110°$$

$$b^2 = \frac{160 \sin 45°}{\sin 25° \sin 110°} \approx 284.89$$

$$b \approx \sqrt{284.89} \approx 16.9 \text{ in.}$$

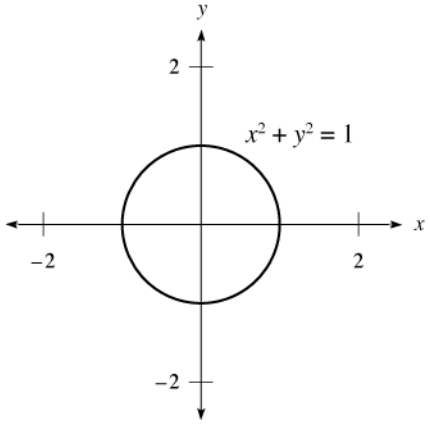
**34**. Since $\sin t = y$ and $\cos t = x$:

$$\sin^2 t + \cos^2 t = 1$$

$$y^2 + x^2 = 1$$

$$x^2 + y^2 = 1$$

The graph is a circle with center = (0,0) and radius = 1:



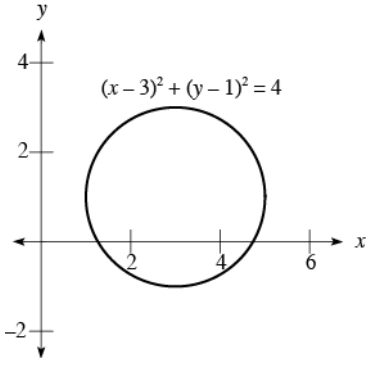

**36**. Since $\cos t = \frac{y-1}{2}$ and $\sin t = \frac{x-3}{2}$:

$$\sin^2 t + \cos^2 t = 1$$

$$\left(\frac{x-3}{2}\right)^2 + \left(\frac{y-1}{2}\right)^2 = 1$$

$$(x-3)^2 + (y-1)^2 = 4$$

The graph is a circle with center = (3,1) and radius = 2:



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**38**. Since $x = 2\tan t$, $\tan t = \frac{x}{2}$. Since $y = 3\sec t$, $\sec t = \frac{y}{3}$. Therefore:

$$\tan^2 t + 1 = \sec^2 t$$
$$\left(\frac{x}{2}\right)^2 + 1 = \left(\frac{y}{3}\right)^2$$
$$\frac{x^2}{4} + 1 = \frac{y^2}{9}$$
$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

The graph would be a hyperbola.

**40**. Using the double-angle formula for cosine: $y = \cos 2t = 1 - 2\sin^2 t$. Since $x = \sin t$, the equation is $y = 1 - 2x^2$.
The graph would be a portion of a parabola.

**42**. Using the area formula: $S = \frac{1}{2}ab\sin C = \frac{1}{2}(73.6)(41.5)\sin 22.3° = 580 \text{ mm}^2$

The correct answer is d.

**44**. The semiperimeter is: $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(17+41+28) = \frac{1}{2}(86) = 43$

The correct answer is c.

**ODD SOLUTIONS**

**1.** half, sides, sine **3.** sum, two **5.** $\frac{1}{2}(a+b+c)$

**7.** $s = \frac{1}{2}(a+b+c)$

$= \frac{1}{2}(3+4+5) = \frac{1}{2}(12) = 6$ ft

**9.** $s = \frac{1}{2}(a+b+c)$

$= \frac{1}{2}(2.1+2.3+3.9) = \frac{1}{2}(8.3) = 4.15$ m

**11.**

| | |
|---|---|
| $S = \frac{1}{2}ab\sin C$ | Formula for area of a triangle |
| $= \frac{1}{2}(50)(70)\sin 60^\circ$ | Substitute known values |
| $= 1{,}520 \text{ cm}^2$ | Round to 3 significant digits |

**13.**

| | |
|---|---|
| $S = \frac{1}{2}ac\sin B$ | Formula for area of a triangle |
| $= \frac{1}{2}(41.5)(34.5)\sin 151.5^\circ$ | Substitute known values |
| $= 342 \text{ m}^2$ | Round to 3 significant digits |

**15.**

| | |
|---|---|
| $S = \frac{1}{2}bc\sin A$ | Formula for area of a triangle |
| $= \frac{1}{2}(0.923)(0.387)\sin 43^\circ 20'$ | Substitute known values |
| $= 0.123 \text{ km}^2$ | Round to 3 significant digits |



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**17.** $C = 180^\circ - (46^\circ + 95^\circ) = 39^\circ$

$S = \dfrac{c^2 \sin A \sin B}{2 \sin C}$ Formula for area of a triangle

$= \dfrac{(6.8)^2 \sin 46^\circ \sin 95^\circ}{2 \sin 39^\circ}$ Substitute known values

$= 26.3 \text{ m}^2$ Round to 3 significant digits

**19.** $C = 180^\circ - (42.5^\circ + 71.4^\circ) = 66.1^\circ$

$S = \dfrac{a^2 \sin B \sin C}{2 \sin A}$ Formula for area of a triangle

$= \dfrac{(210)^2 \sin 71.4^\circ \sin 66.1^\circ}{2 \sin 42.5^\circ}$ Substitute known values

$= 28{,}300 \text{ in}^2$ Round to 3 significant digits

**21.** $B = 180^\circ - (43^\circ 30' + 120^\circ 30') = 16^\circ$

$S = \dfrac{a^2 \sin B \sin C}{2 \sin A}$ Formula for area of a triangle

$= \dfrac{(3.48)^2 \sin 16^\circ \sin 120.5^\circ}{2 \sin 43.5^\circ}$ Substitute known values

$= 2.09 \text{ ft}^2$ Round to 3 significant digits

**23.** $s = \dfrac{1}{2}(a + b + c)$ Formula for half the perimeter

$= \dfrac{1}{2}(44 + 66 + 88) = 99$ Substitute known values and simplify

$S = \sqrt{s(s-a)(s-b)(s-c)}$ Formula for area of a triangle

$= \sqrt{99(99-44)(99-66)(99-88)}$ Substitute known values

$= \sqrt{1{,}976{,}535}$ Simplify

$= 1{,}410 \text{ in}^2$ Round to 3 significant digits

**25.** $s = \dfrac{1}{2}(a + b + c)$ Formula for half the perimeter

$= \dfrac{1}{2}(4.8 + 6.3 + 7.5) = 9.3$ Substitute known values and simplify

$S = \sqrt{s(s-a)(s-b)(s-c)}$ Formula for area of a triangle

$= \sqrt{9.3(9.3-4.8)(9.3-6.3)(9.3-7.5)}$ Substitute known values

$= \sqrt{225.99}$ Simplify

$= 15.0 \text{ yd}^2$ Round to 3 significant digits

**27.** $s = \dfrac{1}{2}(a + b + c)$ Formula for half the perimeter

$= \dfrac{1}{2}(4.38 + 3.79 + 5.22) = 6.695$ Substitute known values and simplify

$S = \sqrt{s(s-a)(s-b)(s-c)}$ Formula for area of a triangle

$= \sqrt{6.695(6.695-4.38)(6.695-3.79)(6.695-5.22)}$ Substitute known values

$= \sqrt{66.41}$ Simplify

$= 8.15 \text{ ft}^2$ Round to 3 significant digits



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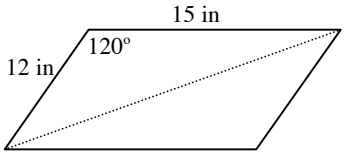
**29.** Area of the parallelogram is twice the area of the triangle:

$$\text{Area of parallelogram} = 2\left[\frac{1}{2}(12)(15)\sin 120^\circ\right]$$
$$= 156 \text{ in}^2$$




**31.** $C = 180^\circ - (30^\circ + 50^\circ) = 100^\circ$

$$S = \frac{c^2 \sin A \sin B}{2 \sin C}$$ Formula for area of a triangle

$$c^2 = \frac{2S \sin C}{\sin A \sin B}$$ Solve for $c^2$

$$= \frac{2(40)\sin 100^\circ}{\sin 30^\circ \sin 50^\circ}$$ Substitute known values

$$= 205.69$$ Simplify

$$c = 14.3 \text{ cm}$$ Round to 3 significant digits

**35.** $\cos^2 t + \sin^2 t = 1$

$$x^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

The graph is a circle with center at $(0,0)$ and radius of 1.

**37.** $\cos t - 3 = x$ $\qquad$ $\sin t + 2 = y$

$\cos t = x + 3$ $\qquad$ $\sin t = y - 2$

$\cos^2 t + \sin^2 t = 1$

$(x+3)^2 + (y-2)^2 = 1$

The graph is a circle with center at $(-3,2)$ and radius of 1.

**39.** $4\cot t = x$ $\qquad$ $4\csc t = y$ $\qquad$ $\csc^2 t = \cot^2 t + 1$

$\cot t = \frac{x}{4}$ $\qquad$ $\csc t = \frac{y}{4}$ $\qquad$ $\left(\frac{y}{4}\right)^2 = \left(\frac{x}{4}\right)^2 + 1$

$$\frac{y^2}{16} - \frac{x^2}{16} = 1$$

**41.** $\cos 2t = x$ and $\cos t = y$

$\cos 2t = 2\cos^2 t - 1$

$x = 2y^2 - 1$

## 7.5 Vectors: An Algebraic Approach

**EVEN SOLUTIONS**

**2**. If a vector **V** is in standard position and the tip of the vector corresponds to the point $(a, b)$, then we can write the vector in component form as $\langle a,b \rangle$. The $x$-coordinate, $a$, is called the horizontal component of **V**, and the $y$-coordinate, $b$, is called the vertical component of **V**.

**4**. A scalar is a real number. To multiply a vector in component form by a scalar, simply multiply each component by the scalar. This is called scalar multiplication.

**6**. The opposite of a vector is a vector with the same magnitude and opposite direction. To obtain the opposite of a vector, multiply the vector by –1.

**8**. The unit vector **i** points in the direction of the positive $x$-axis and is called the unit horizontal vector. The unit vector **j** points in the direction of the positive $y$-axis and is called the unit vertical vector.

**10**. The magnitude of $\mathbf{V} = \langle a,b \rangle = a\mathbf{i} + b\mathbf{j}$ is given by $\sqrt{a^2 + b^2}$.



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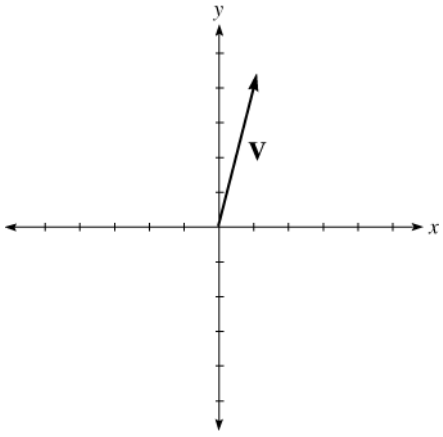
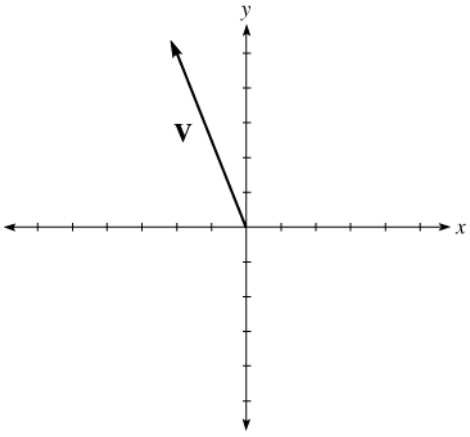
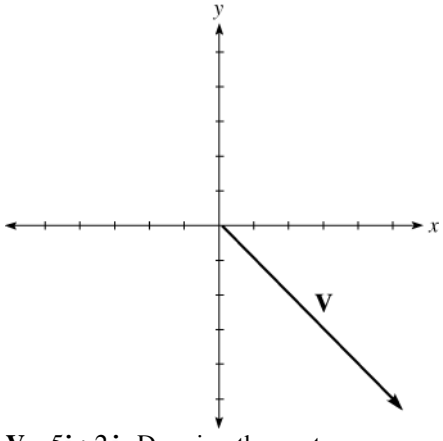
**12**. Write $\mathbf{V} = \langle 1,4 \rangle$. Drawing the vector:



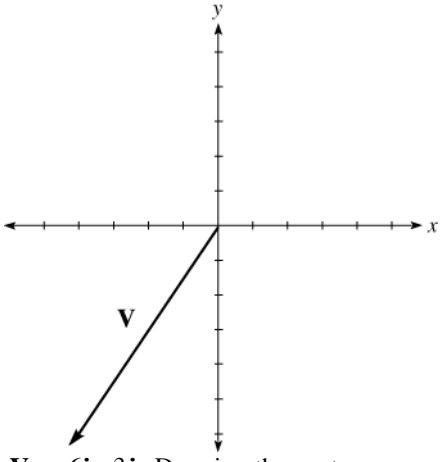

**14**. Write $\mathbf{V} = \langle -2,5 \rangle$. Drawing the vector:



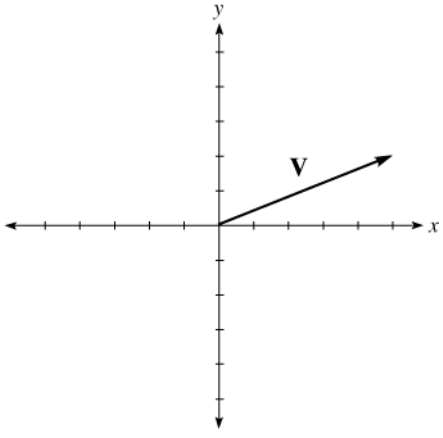

**16**. Write $\mathbf{V} = \langle 5,-5 \rangle$. Drawing the vector:



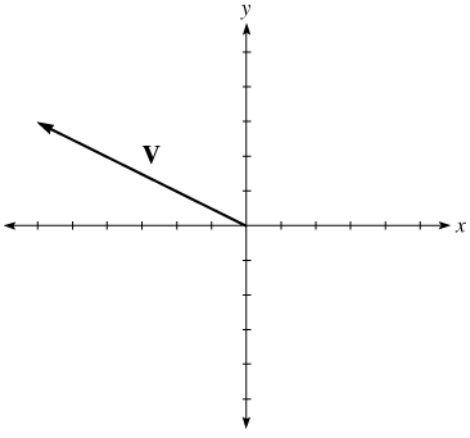

**18**. Write $\mathbf{V} = \langle -4,-6 \rangle$. Drawing the vector:




**20**. Write $\mathbf{V} = 5\mathbf{i} + 2\mathbf{j}$. Drawing the vector:




**22**. Write $\mathbf{V} = -6\mathbf{i} + 3\mathbf{j}$. Drawing the vector:



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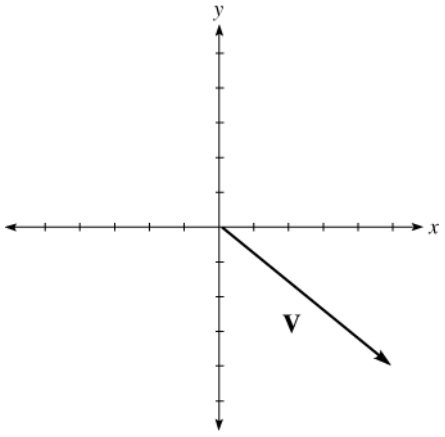
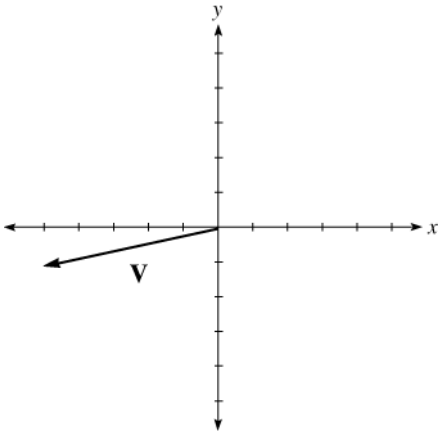
**24**. Write $\mathbf{V} = 5\mathbf{i} - 4\mathbf{j}$. Drawing the vector:




**26**. Write $\mathbf{V} = -5\mathbf{i} - \mathbf{j}$. Drawing the vector:




**28**. Let $\mathbf{V} = \langle -9,-2 \rangle$. The magnitude is given by: $|\mathbf{V}| = \sqrt{(-9)^2 + (-2)^2} = \sqrt{81+4} = \sqrt{85}$

**30**. Let $\mathbf{V} = \langle -7,0 \rangle$. The magnitude is given by: $|\mathbf{V}| = \sqrt{(-7)^2 + (0)^2} = \sqrt{49+0} = \sqrt{49} = 7$

**32**. The magnitude is given by: $|\mathbf{U}| = \sqrt{(15)^2 + (-8)^2} = \sqrt{225+64} = \sqrt{289} = 17$

**34**. The magnitude is given by: $|\mathbf{W}| = \sqrt{(-3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

**36**. Finding the indicated quantities:

$$\frac{1}{2}\mathbf{V} = \frac{1}{2}\langle -2,5 \rangle = \left\langle -1, \frac{5}{2} \right\rangle$$

$$-\mathbf{V} = -\langle -2,5 \rangle = \langle 2,-5 \rangle$$

$$4\mathbf{V} = 4\langle -2,5 \rangle = \langle -8,20 \rangle$$

**38**. Finding the indicated quantities:

$$\frac{1}{2}\mathbf{V} = \frac{1}{2}\left(-\sqrt{3}\mathbf{i} - \mathbf{j}\right) = -\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$$

$$-\mathbf{V} = -\left(-\sqrt{3}\mathbf{i} - \mathbf{j}\right) = \sqrt{3}\mathbf{i} + \mathbf{j}$$

$$4\mathbf{V} = 4\left(-\sqrt{3}\mathbf{i} - \mathbf{j}\right) = -4\sqrt{3}\mathbf{i} - 4\mathbf{j}$$

**40**. Finding the indicated quantities:

$$\mathbf{U} + \mathbf{V} = \langle -3,5 \rangle + \langle 3,-1 \rangle = \langle 0,4 \rangle$$

$$\mathbf{U} - \mathbf{V} = \langle -3,5 \rangle - \langle 3,-1 \rangle = \langle -6,6 \rangle$$

$$2\mathbf{U} - 3\mathbf{V} = 2\langle -3,5 \rangle - 3\langle 3,-1 \rangle = \langle -6,10 \rangle - \langle 9,-3 \rangle = \langle -15,13 \rangle$$

**42**. Finding the indicated quantities:

$$\mathbf{U} + \mathbf{V} = \langle 2,0 \rangle + \langle 0,-7 \rangle = \langle 2,-7 \rangle$$

$$\mathbf{U} - \mathbf{V} = \langle 2,0 \rangle - \langle 0,-7 \rangle = \langle 2,7 \rangle$$

$$2\mathbf{U} - 3\mathbf{V} = 2\langle 2,0 \rangle - 3\langle 0,-7 \rangle = \langle 4,0 \rangle - \langle 0,-21 \rangle = \langle 4,21 \rangle$$

**44**. Finding the indicated quantities:

$$\mathbf{U} + \mathbf{V} = \langle -6,-3 \rangle + \langle -2,5 \rangle = \langle -8,2 \rangle$$

$$\mathbf{U} - \mathbf{V} = \langle -6,-3 \rangle - \langle -2,5 \rangle = \langle -4,-8 \rangle$$

$$2\mathbf{U} - 3\mathbf{V} = 2\langle -6,-3 \rangle - 3\langle -2,5 \rangle = \langle -12,-6 \rangle - \langle -6,15 \rangle = \langle -6,-21 \rangle$$

**46**. Finding the indicated quantities:

$$\mathbf{U} + \mathbf{V} = (\mathbf{i} + 4\mathbf{j}) + (7\mathbf{i} - \mathbf{j}) = 8\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{U} - \mathbf{V} = (\mathbf{i} + 4\mathbf{j}) - (7\mathbf{i} - \mathbf{j}) = \mathbf{i} + 4\mathbf{j} - 7\mathbf{i} + \mathbf{j} = -6\mathbf{i} + 5\mathbf{j}$$

$$3\mathbf{U} + 2\mathbf{V} = 3(\mathbf{i} + 4\mathbf{j}) + 2(7\mathbf{i} - \mathbf{j}) = 3\mathbf{i} + 12\mathbf{j} + 14\mathbf{i} - 2\mathbf{j} = 17\mathbf{i} + 10\mathbf{j}$$

**48**. Finding the indicated quantities:

$$\mathbf{U} + \mathbf{V} = -3\mathbf{i} + 5\mathbf{j}$$

$$\mathbf{U} - \mathbf{V} = -3\mathbf{i} - 5\mathbf{j}$$

$$3\mathbf{U} + 2\mathbf{V} = 3(-3\mathbf{i}) + 2(5\mathbf{j}) = -9\mathbf{i} + 10\mathbf{j}$$



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**50**. Finding the indicated quantities:

$$\mathbf{U}+\mathbf{V}=(5\mathbf{i}+3\mathbf{j})+(-3\mathbf{i}-5\mathbf{j})=2\mathbf{i}-2\mathbf{j}$$

$$\mathbf{U}-\mathbf{V}=(5\mathbf{i}+3\mathbf{j})-(-3\mathbf{i}-5\mathbf{j})=5\mathbf{i}+3\mathbf{j}+3\mathbf{i}+5\mathbf{j}=8\mathbf{i}+8\mathbf{j}$$

$$3\mathbf{U}+2\mathbf{V}=3(5\mathbf{i}+3\mathbf{j})+2(-3\mathbf{i}-5\mathbf{j})=15\mathbf{i}+9\mathbf{j}-6\mathbf{i}-10\mathbf{j}=9\mathbf{i}-\mathbf{j}$$

**52**. The vector is: $\mathbf{U}=\langle 24\cos 120°, 24\sin 120°\rangle=\left\langle 24\left(-\frac{1}{2}\right), 24\left(\frac{\sqrt{3}}{2}\right)\right\rangle=\langle -12, 12\sqrt{3}\rangle=-12\mathbf{i}+12\sqrt{3}\mathbf{j}$

**54**. The vector is: $\mathbf{F}=\langle 30\cos 315°, 30\sin 315°\rangle=\left\langle 30\left(\frac{\sqrt{2}}{2}\right), 30\left(-\frac{\sqrt{2}}{2}\right)\right\rangle=\langle 15\sqrt{2}, -15\sqrt{2}\rangle=15\sqrt{2}\mathbf{i}-15\sqrt{2}\mathbf{j}$

**56**. The vector is: $\mathbf{V}=\langle 8.5\cos 97°, 8.5\sin 97°\rangle\approx\langle -1.0, 8.4\rangle=-1.0\mathbf{i}+8.4\mathbf{j}$

**58**. The vector is: $\mathbf{V}=\langle 330\cos 340°, 330\sin 340°\rangle\approx\langle 310, -110\rangle=310\mathbf{i}-110\mathbf{j}$

**60**. The magnitude is given by: $|\mathbf{V}|=\sqrt{(5)^2+(-5)^2}=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$

Since $\tan\theta=\frac{-5}{5}=-1$ and $\theta$ lies in the fourth quadrant, $\theta=315°$.

**62**. The magnitude is given by: $|\mathbf{F}|=\sqrt{\left(-2\sqrt{3}\right)^2+(2)^2}=\sqrt{12+4}=\sqrt{16}=4$

Since $\tan\theta=\frac{2}{-2\sqrt{3}}=-\frac{1}{\sqrt{3}}$ and $\theta$ lies in the second quadrant, $\theta=150°$.

**64**. The magnitude is given by: $|\mathbf{V}|=\sqrt{(-1)^2+(4)^2}=\sqrt{1+16}=\sqrt{17}\approx 4.1$

Since $\tan\theta=\frac{4}{-1}=-4$ and $\theta$ lies in the second quadrant, $\theta=180°-\tan^{-1}(4)\approx 104.0°$.

**66**. The magnitude is given by: $|\mathbf{F}|=\sqrt{(-6)^2+\left(-\sqrt{3}\right)^2}=\sqrt{36+3}=\sqrt{39}\approx 6.2$

Since $\tan\theta=\frac{-\sqrt{3}}{-6}=\frac{\sqrt{3}}{6}$ and $\theta$ lies in the third quadrant, $\theta=180°+\tan^{-1}\left(\frac{\sqrt{3}}{6}\right)\approx 196.1°$.

**68**. Write **W** as $\langle 0,-25\rangle$, **F** as $\langle |\mathbf{F}|\cos 10°, |\mathbf{F}|\sin 10°\rangle$, and **N** as $\langle -|\mathbf{N}|\cos 80°, |\mathbf{N}|\sin 80°\rangle$. Since **W** + **F** + **N** = **0**, we have:

$$0+|\mathbf{F}|\cos 10°-|\mathbf{N}|\cos 80°=0$$

$$-25+|\mathbf{F}|\sin 10°+|\mathbf{N}|\sin 80°=0$$

These simplify to the equations:

$$|\mathbf{F}|\cos 10°-|\mathbf{N}|\cos 80°=0$$

$$|\mathbf{F}|\sin 10°+|\mathbf{N}|\sin 80°=25$$

Solving the first equation for $|\mathbf{F}|$ results in $|\mathbf{F}|=\frac{|\mathbf{N}|\cos 80°}{\cos 10°}$. Substituting into the second equation:

$$\frac{|\mathbf{N}|\cos 80°}{\cos 10°}\cdot\sin 10°+|\mathbf{N}|\sin 80°=25$$

$$|\mathbf{N}|\cos 80°\sin 10°+|\mathbf{N}|\sin 80°\cos 10°=25\cos 10°$$

$$|\mathbf{N}|(\sin 80°\cos 10°+\cos 80°\sin 10°)=25\cos 10°$$

$$|\mathbf{N}|\sin(80°+10°)=25\cos 10°$$

$$|\mathbf{N}|=\frac{25\cos 10°}{\sin 90°}=25\cos 10°\approx 24.6 \text{ lb}$$

Therefore: $|\mathbf{F}|=\frac{|\mathbf{N}|\cos 80°}{\cos 10°}=\frac{25\cos 10°\cos 80°}{\cos 10°}=25\cos 80°\approx 4.34$ lb



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**70.** Write $\mathbf{W}$ as $\langle 0,-58\rangle$, $\mathbf{F}$ as $\langle |\mathbf{F}|\cos 8.5°, |\mathbf{F}|\sin 8.5°\rangle$, and $\mathbf{N}$ as $\langle -|\mathbf{N}|\cos 81.5°, |\mathbf{N}|\sin 81.5°\rangle$. Since $\mathbf{W} + \mathbf{F} + \mathbf{N} = \mathbf{0}$, we have:

$$0 + |\mathbf{F}|\cos 8.5° - |\mathbf{N}|\cos 81.5° = 0$$
$$-58 + |\mathbf{F}|\sin 8.5° + |\mathbf{N}|\sin 81.5° = 0$$

These simplify to the equations:

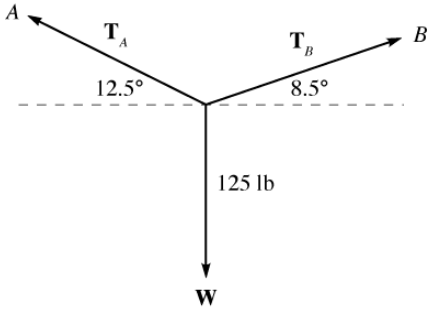
$$|\mathbf{F}|\cos 8.5° - |\mathbf{N}|\cos 81.5° = 0$$
$$|\mathbf{F}|\sin 8.5° + |\mathbf{N}|\sin 81.5° = 58$$

Solving the first equation for $|\mathbf{N}|$ results in $|\mathbf{N}| = \dfrac{|\mathbf{F}|\cos 8.5°}{\cos 81.5°}$. Substituting into the second equation:

$$|\mathbf{F}|\sin 8.5° + \frac{|\mathbf{F}|\cos 8.5°}{\cos 81.5°}\sin 81.5° = 58$$
$$|\mathbf{F}|\sin 8.5°\cos 81.5° + |\mathbf{F}|\cos 8.5°\sin 81.5°° = 58\cos 81.5°$$
$$|\mathbf{F}|(\sin 8.5°\cos 81.5° + \cos 81.5°\sin 8.5°) = 58\cos 81.5°$$
$$|\mathbf{F}|\sin(8.5° + 81.5°) = 58\cos 81.5°$$
$$|\mathbf{F}| = \frac{58\cos 81.5°}{\sin 90°} = 58\cos 81.5° \approx 8.57 \text{ lb}$$

Tyler must push with a force of approximately 8.57 lb.

**72.** Re-draw the figure as follows:




Write $\mathbf{W}$ as $\langle 0,-125\rangle$, $\mathbf{T}_A$ as $\langle -|\mathbf{T}_A|\cos 12.5°, |\mathbf{T}_A|\sin 12.5°\rangle$, and $\mathbf{T}_B$ as $\langle |\mathbf{T}_B|\cos 8.5°, |\mathbf{T}_B|\sin 8.5°\rangle$. Since $\mathbf{W} + \mathbf{T}_A + \mathbf{T}_B = \mathbf{0}$, we have:

$$0 - |\mathbf{T}_A|\cos 12.5° + |\mathbf{T}_B|\cos 8.5° = 0$$
$$-125 + |\mathbf{T}_A|\sin 12.5° + |\mathbf{T}_B|\sin 8.5° = 0$$

These simplify to the equations:

$$|\mathbf{T}_B|\cos 8.5° - |\mathbf{T}_A|\cos 12.5° = 0$$
$$|\mathbf{T}_B|\sin 8.5° + |\mathbf{T}_A|\sin 12.5° = 125$$

Solving the first equation for $|\mathbf{T}_B|$ results in $|\mathbf{T}_B| = \dfrac{|\mathbf{T}_A|\cos 12.5°}{\cos 8.5°}$. Substituting into the second equation:

$$\frac{|\mathbf{T}_A|\cos 12.5°}{\cos 8.5°} \bullet \sin 8.5° + |\mathbf{T}_A|\sin 12.5° = 125$$
$$|\mathbf{T}_A|\cos 12.5°\sin 8.5° + |\mathbf{T}_A|\sin 12.5°\cos 8.5° = 125\cos 8.5°$$
$$|\mathbf{T}_A|(\cos 12.5°\sin 8.5° + \sin 12.5°\cos 8.5°) = 125\cos 8.5°$$
$$|\mathbf{T}_A|\sin(12.5° + 8.5°) = 125\cos 8.5°$$
$$|\mathbf{T}_A|\sin 21° = 125\cos 8.5°$$
$$|\mathbf{T}_A| = \frac{125\cos 8.5°}{\sin 21°} \approx 345 \text{ lb}$$

Therefore: $|\mathbf{T}_B| = \dfrac{125\cos 8.5°\cos 12.5°}{\sin 21°\cos 8.5°} = \dfrac{125\cos 12.5°}{\sin 21°} \approx 341$ lb

The tensions at *A* and *B* are 345 lb and 341 lb, respectively.



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**74**. The vector is: $\mathbf{V} = \langle 12\cos 120°, 12\sin 120° \rangle = 12\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle = \langle -6, 6\sqrt{3} \rangle = -6\mathbf{i} + 6\sqrt{3}\mathbf{j}$

The correct answer is b.

**76**. Finding the required quantity: $\mathbf{U} + 2\mathbf{V} = (3\mathbf{i} + 7\mathbf{j}) + 2(\mathbf{i} - 4\mathbf{j}) = 3\mathbf{i} + 7\mathbf{j} + 2\mathbf{i} - 8\mathbf{j} = 5\mathbf{i} - \mathbf{j}$

The correct answer is d.

**ODD SOLUTIONS**

**1.** tail, origin

**3.** components

**5.** length, direction

**7.** unit

**9.** vector component, horizontal vector component, vertical vector component

For problems 11 through 25, see textbook answer section for graphs.

**27.** Let $\mathbf{V} = \langle -5, 6 \rangle$

$$|\mathbf{V}| = \sqrt{(-5)^2 + 6^2}$$
$$= \sqrt{25 + 36}$$
$$= \sqrt{61}$$

**29.** Let $\mathbf{V} = \langle 0, 5 \rangle$

$$|\mathbf{V}| = \sqrt{0^2 + 5^2}$$
$$= \sqrt{25}$$
$$= 5$$

**31.** Let $\mathbf{V} = \langle 5, 12 \rangle$

$$|\mathbf{V}| = \sqrt{(5)^2 + (12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169} = 13$$

**33.** $|\mathbf{V}| = \sqrt{a^2 + b^2}$

$$= \sqrt{1^2 + 2^2}$$
$$= \sqrt{1 + 4}$$
$$= \sqrt{5}$$

**35.** $\frac{1}{2}\mathbf{V} = \frac{1}{2}\langle -3, 7 \rangle = \left\langle -\frac{3}{2}, \frac{7}{2} \right\rangle$

$-\mathbf{V} = -\langle -3, 7 \rangle = \langle 3, -7 \rangle$

$4\mathbf{V} = 4\langle -3, 7 \rangle = \langle -12, 28 \rangle$

**37.** $\frac{1}{2}\mathbf{V} = \frac{1}{2}(2\mathbf{i} + 4\mathbf{j}) = \mathbf{i} + 2\mathbf{j}$

$-\mathbf{V} = -(2\mathbf{i} + 4\mathbf{j}) = -2\mathbf{i} - 4\mathbf{j}$

$4\mathbf{V} = 4(2\mathbf{i} + 4\mathbf{j}) = 8\mathbf{i} + 16\mathbf{j}$

**39.** $\mathbf{U} + \mathbf{V} = \langle 4, 4 \rangle + \langle 4, -4 \rangle = \langle 4 + 4, 4 + (-4) \rangle = \langle 8, 0 \rangle$

$\mathbf{U} - \mathbf{V} = \langle 4, 4 \rangle - \langle 4, -4 \rangle = \langle 4 - 4, 4 - (-4) \rangle = \langle 0, 8 \rangle$

$2\mathbf{U} - 3\mathbf{V} = 2\langle 4, 4 \rangle - 3\langle 4, -4 \rangle = \langle 8, 8 \rangle - \langle 12, -12 \rangle = \langle -4, 20 \rangle$

**41.** $\mathbf{U} + \mathbf{V} = \langle -5, 0 \rangle + \langle 0, 1 \rangle = \langle -5 + 0, 0 + 1) \rangle = \langle -5, 1 \rangle$

$\mathbf{U} - \mathbf{V} = \langle -5, 0 \rangle - \langle 0, 1 \rangle = \langle -5 - 0, 0 - 1 \rangle = \langle -5, -1 \rangle$

$2\mathbf{U} - 3\mathbf{V} = 2\langle -5, 0 \rangle - 3\langle 0, 1 \rangle = \langle -10, 0 \rangle - \langle 0, 3 \rangle = \langle -10, -3 \rangle$

**43.** $\mathbf{U} + \mathbf{V} = \langle 4, 1 \rangle + \langle -5, 2 \rangle = \langle 4 + (-5), 1 + 2 \rangle = \langle -1, 3 \rangle$

$\mathbf{U} - \mathbf{V} = \langle 4, 1 \rangle - \langle -5, 2 \rangle = \langle 4 - (-5), 1 - 2 \rangle = \langle 9, -1 \rangle$

$2\mathbf{U} - 3\mathbf{V} = 2\langle 4, 1 \rangle - 3\langle -5, 2 \rangle = \langle 8, 2 \rangle - \langle -15, 6 \rangle = \langle 23, -4 \rangle$

**45.** $\mathbf{U} + \mathbf{V} = (-\mathbf{i} + \mathbf{j}) + (\mathbf{i} + \mathbf{j}) = 2\mathbf{j}$

$\mathbf{U} - \mathbf{V} = (-\mathbf{i} + \mathbf{j}) - (\mathbf{i} + \mathbf{j}) = -\mathbf{i} + \mathbf{j} - \mathbf{i} - \mathbf{j} = -2\mathbf{i}$

$3\mathbf{U} + 2\mathbf{V} = 3(-\mathbf{i} + \mathbf{j}) + 2(\mathbf{i} + \mathbf{j}) = -3\mathbf{i} + 3\mathbf{j} + 2\mathbf{i} + 2\mathbf{j} = -\mathbf{i} + 5\mathbf{j}$

**47.** $\mathbf{U} + \mathbf{V} = (6\mathbf{i}) + (-8\mathbf{j}) = 6\mathbf{i} - 8\mathbf{j}$

$\mathbf{U} - \mathbf{V} = (6\mathbf{i}) - (-8\mathbf{j}) = 6\mathbf{i} + 8\mathbf{j}$

$3\mathbf{U} + 2\mathbf{V} = 3(6\mathbf{i}) + 2(-8\mathbf{j}) = 18\mathbf{i} - 16\mathbf{j}$

**49.** $\mathbf{U} + \mathbf{V} = (2\mathbf{i} + 5\mathbf{j}) + (5\mathbf{i} + 2\mathbf{j}) = 7\mathbf{i} + 7\mathbf{j}$

$\mathbf{U} - \mathbf{V} = (2\mathbf{i} + 5\mathbf{j}) - (5\mathbf{i} + 2\mathbf{j}) = 2\mathbf{i} + 5\mathbf{j} - 5\mathbf{i} - 2\mathbf{j} = -3\mathbf{i} + 3\mathbf{j}$

$\mathbf{3U + 2V} = 3(2\mathbf{i} + 5\mathbf{j}) + 2(5\mathbf{i} + 2\mathbf{j}) = 6\mathbf{i} + 15\mathbf{j} + 10\mathbf{i} + 4\mathbf{j} = 16\mathbf{i} + 19\mathbf{j}$



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**51.** $\mathbf{V} = \langle 18\cos 30^\circ, 18\sin 30^\circ \rangle$

$= \langle 9\sqrt{3}, 9 \rangle$

$= 9\sqrt{3}\mathbf{i} + 9\mathbf{j}$

**53.** $\mathbf{W} = \langle 8\cos 270^\circ, 8\sin 270^\circ \rangle$

$= \langle 0, -8 \rangle$

$= -8\mathbf{j}$

**55.** $\mathbf{V} = (5.8\cos 71^\circ)\mathbf{i} + (5.8\sin 71^\circ)\mathbf{j}$

$= 1.9\mathbf{i} + 5.5\mathbf{j}$

**57.** $\mathbf{V} = (0.55\cos 195^\circ)\mathbf{i} + (0.55\sin 195^\circ)\mathbf{j}$

$= -0.53\mathbf{i} - 0.14\mathbf{j}$

**59.** $\tan\theta = \dfrac{3}{3} = 1$ $\qquad |\mathbf{U}| = \sqrt{3^2 + 3^2}$

$\theta = 45^\circ$ $\qquad = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$

**61.** $\tan\theta = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$ $\qquad |\mathbf{W}| = \sqrt{(-1)^2 + (-\sqrt{3})^2}$

$\hat{\theta} = 60^\circ$ (in QIII) $\qquad = \sqrt{1+3}$

$\theta = 240^\circ$ $\qquad = \sqrt{4} = 2$

**63.** $||\mathbf{U}|| = \sqrt{3^2 + 5^2} = \sqrt{34} = 5.8$

$\tan\theta = \dfrac{5}{3}$ and is in QI

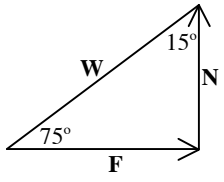
$\theta = 59.0^\circ$

**65.** $||W|| = \sqrt{(\sqrt{5})^2 + (-2)^2} = \sqrt{9} = 3$

$\tan\theta = \dfrac{-2}{\sqrt{5}}$ and is in QIV

$\theta = 318.2^\circ$

**67.** We are given that $|\mathbf{W}| = 8.0$ pounds.




$\mathbf{F}$ is a horizontal vector. $\qquad \mathbf{N}$ is a vertical vector.

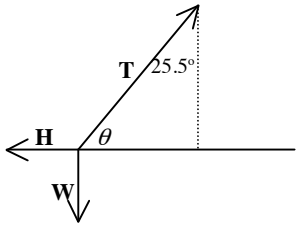
$|\mathbf{F}| = |\mathbf{W}|\cos 75^\circ$ $\qquad |\mathbf{N}| = |\mathbf{W}|\sin 75^\circ$

$= 8.0\cos 75^\circ$ $\qquad = 8.0\sin 75^\circ$

$= 2.1$ pounds $\qquad = 7.7$ pounds

**69.** We redraw the vectors and find the missing angles:

$\theta = 90^\circ - 25.5^\circ = 64.5^\circ$




$\mathbf{H} = -|\mathbf{H}|\mathbf{i}$, $\mathbf{W} = -95.5\mathbf{j}$, and $\mathbf{T} = |\mathbf{T}|\cos 64.5^\circ\mathbf{i} + |\mathbf{T}|\sin 64.5^\circ\mathbf{j}$

Also, $\mathbf{H} + \mathbf{W} + \mathbf{T} = 0$

Collecting all the **i** components and all the **j** components together, we have: $(-|\mathbf{H}| + |\mathbf{T}|\cos 64.5^\circ)\mathbf{i} + (-95.5 + |\mathbf{T}|\sin 64.5^\circ)\mathbf{j} = 0$

Therefore, $-|\mathbf{H}| + |\mathbf{T}|\cos 64.5^\circ = 0$ and $-95.5 + |\mathbf{T}|\sin 64.5^\circ = 0$

Solving the second equation for $|\mathbf{T}|$, we get: $|\mathbf{T}|\sin 64.5^\circ = 95.5$

$$|\mathbf{T}| = \frac{95.5}{\sin 64.5^\circ}$$

$$= 105.807$$

We substitute this into the first equation and solve for $|\mathbf{H}|$:

$$-|\mathbf{H}| + 105.807\cos 64.5^\circ = 0$$

$$|\mathbf{H}| = 105.807\cos 64.5^\circ$$

$= 45.6$ pounds (to 3 significant digits)

**71.** We are given that $|\mathbf{W}| = 22$ pounds.

$\mathbf{T_1} = |\mathbf{T_1}|\cos 30^\circ\mathbf{i} + |\mathbf{T_1}|\sin 30^\circ\mathbf{j}$

$= |\mathbf{T_1}|\left(\dfrac{\sqrt{3}}{2}\right)\mathbf{i} + |\mathbf{T_1}|\left(\dfrac{1}{2}\right)\mathbf{j}$

$\mathbf{T_2} = |\mathbf{T_2}|\cos 135^\circ\mathbf{i} + |\mathbf{T_2}|\sin 135^\circ\mathbf{j}$

$= |\mathbf{T_2}|\left(-\dfrac{\sqrt{2}}{2}\right)\mathbf{i} + |\mathbf{T_2}|\left(\dfrac{\sqrt{2}}{2}\right)\mathbf{j}$

$\mathbf{W} = 22\cos 270^\circ\mathbf{i} + 22\sin 270^\circ\mathbf{j} = 0\mathbf{i} - 22\mathbf{j}$

Since $\mathbf{T_1} + \mathbf{T_2} = \mathbf{W}$, we add the corresponding **i** and **j** components:

(1) $|\mathbf{T_1}|\left(\dfrac{\sqrt{3}}{2}\right) + |\mathbf{T_2}|\left(-\dfrac{\sqrt{2}}{2}\right) = 0$ and (2) $|\mathbf{T_1}|\left(\dfrac{1}{2}\right) + |\mathbf{T_2}|\left(\dfrac{\sqrt{2}}{2}\right) = -22$



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Solving this system using the elimination method, we get:

$$\frac{\sqrt{3}+1}{2}|\mathbf{T}_1| = |-22| \quad \text{or} \quad |\mathbf{T}_1| = \left|\frac{-22}{\frac{\sqrt{3}+1}{2}}\right| = 16 \text{ pounds}$$

Substituting the value for $|\mathbf{T}_1|$ in equation (1) we get:

$$16\left(\frac{\sqrt{3}}{2}\right)+|\mathbf{T}_2|\left(-\frac{\sqrt{2}}{2}\right)=0 \quad \text{or} \quad |\mathbf{T}_2| = \frac{8\sqrt{3}}{\sqrt{2}/2} = 20 \text{ pounds}$$

## 7.6 Vectors: The Dot Product

**EVEN SOLUTIONS**

**2**. The dot product of two vectors is a scalar quantity. For this reason, it is sometimes called the scalar product.

**4**. To find the angle between two nonzero vectors, find the inverse cosine of the quotient of the dot product of the vectors divided by the product of their magnitudes.

**6**. Two nonzero vectors are perpendicular if and only if their dot product is equal to zero.

**8**. If a constant force **F** is applied to an object, and the resulting movement of the object is represented by the displacement vector **d**, then the work performed by the force is given by the dot product of **F** and **d**.

**10**. Finding the dot product: $\langle 6,6\rangle \bullet \langle -3,5\rangle = (6)(-3)+(6)(5) = -18+30 = 12$

**12**. Finding the dot product: $\langle 13,-8\rangle \bullet \langle -4,-7\rangle = (13)(-4)+(-8)(-7) = -52+56 = 4$

**14**. Finding the dot product: $(-\mathbf{i}+\mathbf{j})\bullet(-\mathbf{i}-\mathbf{j}) = (-1)(-1)+(1)(-1) = 1-1 = 0$

**16**. Finding the dot product: $(6\mathbf{i})\bullet(-8\mathbf{j}) = (6)(0)+(0)(-8) = 0+0 = 0$

**18**. Finding the dot product: $(5\mathbf{i}+3\mathbf{j})\bullet(-5\mathbf{i}+3\mathbf{j}) = (5)(-5)+(3)(3) = -25+9 = -16$

**20**. Finding the dot product: $(2\mathbf{i}+9\mathbf{j})\bullet(-3\mathbf{i}-\mathbf{j}) = (2)(-3)+(9)(-1) = -6-9 = -15$

**22**. Finding the dot product: $(5\mathbf{i}-11\mathbf{j})\bullet(-20\mathbf{i}+9\mathbf{j}) = (5)(-20)+(-11)(9) = -100-99 = -199$

**24**. Using $\cos\theta = \dfrac{\mathbf{U}\bullet\mathbf{V}}{|\mathbf{U}||\mathbf{V}|}$ to find the angle:

$$\cos\theta = \frac{(-4)(0)+(0)(17)}{\sqrt{(-4)^2+0^2}\sqrt{0^2+17^2}} = \frac{0+0}{\sqrt{16}\sqrt{289}} = 0$$

$$\theta = 90°$$

**26**. Using $\cos\theta = \dfrac{\mathbf{U}\bullet\mathbf{V}}{|\mathbf{U}||\mathbf{V}|}$ to find the angle:

$$\cos\theta = \frac{(4)(7)+(5)(-4)}{\sqrt{4^2+5^2}\sqrt{7^2+(-4)^2}} = \frac{28-20}{\sqrt{41}\sqrt{65}} = \frac{8}{\sqrt{2665}} \approx 0.1550$$

$$\theta = 81.1°$$

**28**. Using $\cos\theta = \dfrac{\mathbf{U}\bullet\mathbf{V}}{|\mathbf{U}||\mathbf{V}|}$ to find the angle:

$$\cos\theta = \frac{(11)(-14)+(7)(6)}{\sqrt{11^2+7^2}\sqrt{(-14)^2+6^2}} = \frac{-154+42}{\sqrt{170}\sqrt{232}} = \frac{-112}{\sqrt{39{,}440}} \approx -0.5640$$

$$\theta = 124.3°$$

**30**. Compute their dot product: $\langle 1,0\rangle \bullet \langle 0,1\rangle = (1)(0)+(0)(1) = 0+0 = 0$

Thus $\cos\theta = 0$, so $\theta = 90°$. So $\mathbf{i}$ and $\mathbf{j}$ are perpendicular.



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**32.** Compute their dot product: $\langle 2,1\rangle \bullet \langle 1,-2\rangle = (2)(1)+(1)(-2) = 2-2 = 0$

Thus $\cos\theta = 0$, so $\theta = 90°$. So $2\mathbf{i}+\mathbf{j}$ and $\mathbf{i}-2\mathbf{j}$ are perpendicular.

**34.** Compute their dot product: $\langle -6,-5\rangle \bullet \langle 10,-12\rangle = (-6)(10)+(-5)(-12) = -60+60 = 0$

Thus $\cos\theta = 0$, so $\theta = 90°$. So $-6\mathbf{i}-5\mathbf{j}$ and $10\mathbf{i}-12\mathbf{j}$ are perpendicular.

**36.** Since the dot product must equal 0: $(a\mathbf{i}+b\mathbf{j})\bullet(-b\mathbf{i}+a\mathbf{j}) = (a)(-b)+(b)(a) = -ab+ab = 0$

**38.** The work is: $\mathbf{F}\bullet\mathbf{d} = (45\mathbf{i}-12\mathbf{j})\bullet(170\mathbf{i}+15\mathbf{j}) = (45)(170)+(-12)(15) = 7{,}470$ ft-lb

**40.** The work is: $\mathbf{F}\bullet\mathbf{d} = (-67\mathbf{i}+59\mathbf{j})\bullet(-96\mathbf{i}-28\mathbf{j}) = (-67)(-96)+(59)(-28) = 4{,}780$ ft-lb

**42.** The work is: $\mathbf{F}\bullet\mathbf{d} = (-54\mathbf{i}+0\mathbf{j})\bullet(20\mathbf{i}+0\mathbf{j}) = (-54)(20)+(0)(0) = -1{,}080$ ft-lb

**44.** The work is: $\mathbf{F}\bullet\mathbf{d} = (0\mathbf{i}+39\mathbf{j})\bullet(72\mathbf{i}+0\mathbf{j}) = (0)(72)+(39)(0) = 0$ ft-lb

**46.** Assume $\mathbf{U}\bullet\mathbf{V} = 0$. Since $\mathbf{U}\bullet\mathbf{V} = |\mathbf{U}||\mathbf{V}|\cos\theta$, $|\mathbf{U}||\mathbf{V}|\cos\theta = 0$. Since $\mathbf{U}$ and $\mathbf{V}$ are nonzero vectors, $\cos\theta = 0$ and thus $\theta = 90°$. Thus $\mathbf{U}$ is perpendicular to $\mathbf{V}$. Assume $\mathbf{U}$ is perpendicular to $\mathbf{V}$. Then $\theta = 90°$, so $\cos\theta = 0$. So $|\mathbf{U}||\mathbf{V}|\cos\theta = |\mathbf{U}||\mathbf{V}|\bullet 0 = 0$, so $\mathbf{U}\bullet\mathbf{V} = 0$. This proves both directions of the theorem.

**48.** The force vector is $\mathbf{F} = \langle 15\cos 25°, -15\sin 25°\rangle$ and the direction vector is $\mathbf{d} = \langle 52,0\rangle$.

The work is: $\mathbf{F}\bullet\mathbf{d} = \langle 15\cos 25°, -15\sin 25°\rangle \bullet \langle 52,0\rangle = 780\cos 25° \approx 710$ ft-lb

**50.** The force vector is $\mathbf{F} = \langle 25\cos 30°, 25\sin 30°\rangle$ and the direction vector is $\mathbf{d} = \langle 350,0\rangle$.

The work is: $\mathbf{F}\bullet\mathbf{d} = \langle 25\cos 30°, 25\sin 30°\rangle \bullet \langle 350,0\rangle = 8750\cos 30° \approx 7{,}600$ ft-lb

**52.** Using $\cos\theta = \dfrac{\mathbf{U}\bullet\mathbf{V}}{|\mathbf{U}||\mathbf{V}|}$ to find the angle:

$$\cos\theta = \frac{(3)(1)+(7)(-4)}{\sqrt{3^2+7^2}\sqrt{1^2+(-4)^2}} = \frac{3-28}{\sqrt{58}\sqrt{17}} = \frac{-25}{\sqrt{986}} \approx -0.7962$$

$$\theta = 142.8°$$

The correct answer is b.

**54.** The work is: $\mathbf{F}\bullet\mathbf{d} = (15\mathbf{i}-9\mathbf{j})\bullet(80\mathbf{i}+12\mathbf{j}) = (15)(80)+(-9)(12) = 1{,}092$ ft-lb

The correct answer is c.

**ODD SOLUTIONS**

**1.** multiply, add

**3.** magnitudes, cosine

**5.** orthogonal

**7.** projection, onto

**9.** $\langle 3,4\rangle \bullet \langle 5,5\rangle = 3(5)+4(5)$
$= 15+20 = 35$

**11.** $\langle -23,4\rangle \bullet \langle 15,-6\rangle = -23(15)+4(-6)$
$= -345-24 = -369$

**13.** $\mathbf{U}\bullet\mathbf{V} = 1(1)+1(-1)$
$= 1-1 = 0$

**15.** $\mathbf{U}\bullet\mathbf{V} = -3(0)+0(5)$
$= 0+0 = 0$

**17.** $\mathbf{U}\bullet\mathbf{V} = 2(5)+5(2)$
$= 10+10 = 20$

**19.** $\mathbf{U}\bullet\mathbf{V} = -4(-1)+(-3)(-2)$
$= 4+6 = 10$

**21.** $\mathbf{U}\bullet\mathbf{V} = -11(9)+7(-5)$
$= -99-35 = -134$

**23.** First, we find $\mathbf{U}\bullet\mathbf{V}$, $|\mathbf{U}|$, and $|\mathbf{V}|$:

$\mathbf{U}\bullet\mathbf{V} = \langle 13,0\rangle \bullet \langle 0,-6\rangle$
$= 13(0)+0(-6) = 0$

$|\mathbf{U}| = 13$ and $|\mathbf{V}| = 6$

$$\cos\theta = \frac{\mathbf{U}\bullet\mathbf{V}}{|\mathbf{U}||\mathbf{V}|} = \frac{0}{13(6)} = 0$$

$$\theta = 90.0°$$



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**25.** First, we find $\mathbf{U}\cdot\mathbf{V}, |\mathbf{U}|$, and $|\mathbf{V}|$:

$\mathbf{U}\cdot\mathbf{V} = \langle -3,5\rangle \cdot \langle 6,3\rangle$ $\quad |\mathbf{U}| = \sqrt{(-3)^2+5^2}$ $\quad$ and $\quad |\mathbf{V}| = \sqrt{6^2+3^2}$

$= -3(6)+5(3) = -3$ $\quad = \sqrt{9+25} = \sqrt{34}$ $\quad = \sqrt{36+9} = \sqrt{45}$

$$\cos\theta = \frac{\mathbf{U}\cdot\mathbf{V}}{|\mathbf{U}||\mathbf{V}|} = \frac{-3}{\sqrt{34}\sqrt{45}} = -0.0767$$

$\theta = 94.4^\circ$

**27.** First, we find $\mathbf{U}\cdot\mathbf{V}, |\mathbf{U}|$, and $|\mathbf{V}|$:

$\mathbf{U}\cdot\mathbf{V} = \langle 13,-8\rangle \cdot \langle 2,11\rangle$ $\quad |\mathbf{U}| = \sqrt{13^2+(-8)^2}$ $\quad$ and $\quad |\mathbf{V}| = \sqrt{2^2+11^2}$

$= 13(2)+(-8)(11)$ $\quad = \sqrt{169+64} = \sqrt{233}$ $\quad = \sqrt{4+121} = \sqrt{125}$

$= 26-88 = -62$

$$\cos\theta = \frac{\mathbf{U}\cdot\mathbf{V}}{|\mathbf{U}||\mathbf{V}|} = \frac{-62}{\sqrt{233}\sqrt{125}} = -0.3633$$

$\theta = 111.3^\circ$

**29.** To show that **i** and **j** are perpendicular, we must show that their dot product equals zero.

$\mathbf{i} = \langle 1,1\rangle$ and $\mathbf{j} = \langle 1,-1\rangle$

$\mathbf{i}\cdot\mathbf{j} = 1(1)+1(-1) = 1-1 = 0$

Therefore, they are perpendicular.

**31.** To show that **– i** and **j** are perpendicular, we must show that their dot product equals zero.

$-\mathbf{i} = \langle -1,0\rangle$ and $\mathbf{j} = \langle 0,1\rangle$

$-\mathbf{i}\cdot\mathbf{j} = -1(0)+0(1) = 0$

Therefore, they are perpendicular.

**33.** To show that **i** and **j** are perpendicular, we must show that their dot product equals zero.

$\mathbf{i} = \langle -4,-3\rangle$ and $\mathbf{j} = \langle 6,-8\rangle$

$\mathbf{i}\cdot\mathbf{j} = -4(6)+(-3)(-8) = -24+24 = 0$

Therefore, they are perpendicular.

**35.** $\langle a,6\rangle \cdot \langle 9,12\rangle = 0$

$9a+72 = 0$

$9a = -72$

$a = -8$

**37.** Work = $\mathbf{F}\cdot\mathbf{d}$

$= \langle 22,9\rangle \cdot \langle 30,4\rangle$

$= 22(30)+9(4) = 696$ ft-lb

**39.** Work = $\mathbf{F}\cdot\mathbf{d}$

$= \langle -6,19\rangle \cdot \langle 8,55\rangle$

$= -6(8)+19(55) = 997$ ft-lb

**41.** Work = $\mathbf{F}\cdot\mathbf{d}$

$= \langle 85,0\rangle \cdot \langle 6,0\rangle$

$= 85(6)+0(0) = 510$ ft-lb

**43.** Work = $\mathbf{F}\cdot\mathbf{d}$

$= \langle 0,39\rangle \cdot \langle 72,0\rangle$

$= 0(72)+39(0) = 0$ ft-lb

**45.** Let $\mathbf{U} = a\mathbf{i}+b\mathbf{j}$ and $\mathbf{V} = c\mathbf{i}+d\mathbf{j}$

Then $\mathbf{U}-\mathbf{V} = (a-c)\mathbf{i}+(b-d)\mathbf{j}$

$|\mathbf{U}-\mathbf{V}|^2 = (a-c)^2+(b-d)^2$

$= a^2-2ac+c^2+b^2-2bd+d^2$

(1) $= a^2+b^2+c^2+d^2-2(ac+bd)$

Using the Law of Cosines, we also have:

(2) $|\mathbf{U}-\mathbf{V}|^2 = |\mathbf{U}|^2+|\mathbf{V}|^2-2|\mathbf{U}||\mathbf{V}|\cos\theta$

We know that $|\mathbf{U}|^2 = a^2+b^2$, $|\mathbf{V}|^2 = c^2+d^2$, and $\mathbf{U}\cdot\mathbf{V} = ac+bd$



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Substituting these values into (2), we get:

(3) $|\mathbf{U}-\mathbf{V}|^2 = a^2+b^2+c^2+d^2-2|\mathbf{U}||\mathbf{V}|\cos\theta$

Setting (1) equal to (3) and simplifying, we get:

$$a^2+b^2+c^2+d^2-2(ac+bd) = a^2+b^2+c^2+d^2-2|\mathbf{U}||\mathbf{V}|\cos\theta$$
$$-2(ac+bd) = -2|\mathbf{U}||\mathbf{V}|\cos\theta$$
$$ac+bd = |\mathbf{U}||\mathbf{V}|\cos\theta$$
$$\mathbf{U}\cdot\mathbf{V} = |\mathbf{U}||\mathbf{V}|\cos\theta$$

**47.** $\mathbf{d} = \langle 75,0\rangle$

$\mathbf{F} = \langle 41\cos 20^\circ, 41\sin 20^\circ\rangle = \langle 38.53, 14.02\rangle$

Work = $\mathbf{F}\cdot\mathbf{d}$

$= \langle 75,0\rangle \cdot \langle 38.53, 14.03\rangle$

$= 75(38.53)+0(14.02) = 2{,}900$ ft-lb (rounded to 2 significant digits)

**49.** $\mathbf{d} = \langle 110,0\rangle$

$\mathbf{F} = \langle 85\cos 15^\circ, 85\sin 15^\circ\rangle = \langle 82.10, 22.00\rangle$

Work = $\mathbf{F}\cdot\mathbf{d}$

$= \langle 110,0\rangle \cdot \langle 82.10, 22.00\rangle$

$= 110(82.10)+0(22.00) = 9{,}000$ ft-lb (rounded to 2 significant digits)

## Chapter 7 Test

**1.** Using the law of sines to find $b$:

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 70^\circ} = \frac{3.8}{\sin 32^\circ}$$
$$b = \frac{3.8\sin 70^\circ}{\sin 32^\circ} \approx 6.7 \text{ in.}$$

**2.** First note that $C = 180^\circ - (38.2^\circ + 63.4^\circ) = 78.4^\circ$. Using the law of sines to find $a$:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 38.2^\circ} = \frac{42.0}{\sin 78.4^\circ}$$
$$a = \frac{42\sin 38.2^\circ}{\sin 78.4^\circ} \approx 26.5 \text{ cm}$$

Using the law of sines to find $b$:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 63.4^\circ} = \frac{42.0}{\sin 78.4^\circ}$$
$$b = \frac{42\sin 63.4^\circ}{\sin 78.4^\circ} \approx 38.3 \text{ cm}$$



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**3**. First note that $B = 180° - (24.7° + 106.1°) = 49.2°$ . Using the law of sines to find $a$:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 24.7°} = \frac{34.0}{\sin 49.2°}$$
$$a = \frac{34 \sin 24.7°}{\sin 49.2°} \approx 18.8 \text{ cm}$$

Using the law of sines to find $c$:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 106.1°} = \frac{34.0}{\sin 49.2°}$$
$$c = \frac{34 \sin 106.1°}{\sin 49.2°} \approx 43.2 \text{ cm}$$

**4**. Finding $c$ using the law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C = 10^2 + 12^2 - 2(10)(12)\cos 165° \approx 475.8$$
$$c \approx \sqrt{475.8} \approx 22 \text{ cm}$$

**5**. Finding $C$ using the law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$9^2 = 5^2 + 7^2 - 2(5)(7)\cos C$$
$$81 = 74 - 70\cos C$$
$$7 = -70\cos C$$
$$\cos C = -0.1$$
$$C \approx 95.7°$$

**6**. Finding $c$ using the law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C = 6.4^2 + 2.8^2 - 2(6.4)(2.8)\cos 119° \approx 66.176$$
$$c = \sqrt{66.176} \approx 8.1 \text{ m}$$

Finding $A$ using the law of sines:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{6.4}{\sin A} = \frac{8.1}{\sin 119°}$$
$$\sin A = \frac{6.4 \sin 119°}{8.1} \approx 0.6911$$
$$A \approx 44°$$

Finally $B = 180° - (44° + 119°) = 17°$ .

**7**. Finding $a$ using the law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A = 3.7^2 + 6.2^2 - 2(3.7)(6.2)\cos 35° \approx 14.547$$
$$a = \sqrt{14.547} \approx 3.8 \text{ m}$$

Finding $B$ using the law of sines:

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{3.7}{\sin B} = \frac{3.8}{\sin 35°}$$
$$\sin B = \frac{3.7 \sin 35°}{3.8} \approx 0.5585$$
$$B \approx 34°$$

Finally $C = 180° - (35° + 34°) = 111°$ .



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**8**. Finding $B$ using the law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12}{\sin 60^\circ} = \frac{42}{\sin B}$$
$$\sin B = \frac{42 \sin 60^\circ}{12} \approx 3.031$$

Since $-1 \le \sin B \le 1$, this is impossible. No such triangle exists.

**9**. Findng $B$ using the law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{29}{\sin 42^\circ} = \frac{21}{\sin B}$$
$$\sin B = \frac{21 \sin 42^\circ}{29} \approx 0.4845$$
$$B \approx 29^\circ, 151^\circ$$

But $B = 151^\circ$ is impossible, since 151° + 42° = 193° > 180°. So $B = 29^\circ$ is the only possible value of $B$, thus exactly one triangle exists meeting the specified conditions.

**10**. Findng $B$ using the law of sines:

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{7.9}{\sin B} = \frac{6.5}{\sin 51^\circ}$$
$$\sin B = \frac{7.9 \sin 51^\circ}{6.5} \approx 0.9445$$
$$B = 71^\circ, 109^\circ$$

If $B = 71^\circ$, $C = 180^\circ - (51^\circ + 71^\circ) = 58^\circ$. Finding $c$ using the law of sines:

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 58^\circ} = \frac{6.5}{\sin 51^\circ}$$
$$c = \frac{6.5 \sin 58^\circ}{\sin 51^\circ} \approx 7.1 \text{ ft}$$

If $B' = 109^\circ$, $C' = 180^\circ - (51^\circ + 109^\circ) = 20^\circ$. Finding $c'$ using the law of sines:

$$\frac{c'}{\sin C'} = \frac{a'}{\sin A'}$$
$$\frac{c'}{\sin 20^\circ} = \frac{6.5}{\sin 51^\circ}$$
$$c' = \frac{6.5 \sin 20^\circ}{\sin 51^\circ} \approx 2.9 \text{ ft}$$

**11**. First note that $C = 180^\circ - 38.2^\circ - 63.4^\circ = 78.4^\circ$.

Using the area formula: $S = \dfrac{c^2 \sin A \sin B}{2 \sin C} = \dfrac{(42.0)^2 \sin 38.2^\circ \sin 63.4^\circ}{2 \sin 78.4^\circ} \approx 498 \text{ cm}^2$

**12**. Using the area formula: $S = \dfrac{1}{2}ab \sin C = \dfrac{1}{2}(10)(12)\sin 165^\circ \approx 16 \text{ cm}^2$

**13**. Compute $s = \dfrac{1}{2}(a+b+c) = \dfrac{1}{2}(5+7+9) = 10.5$. Using Heron's formula:

$$S = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10.5(10.5-5)(10.5-7)(10.5-9)} \approx 17 \text{ km}^2$$



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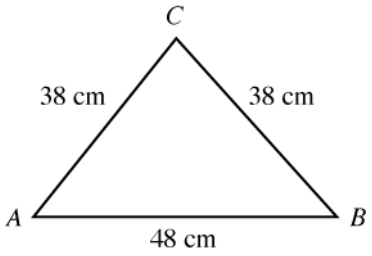
**14**. Drawing the figure:

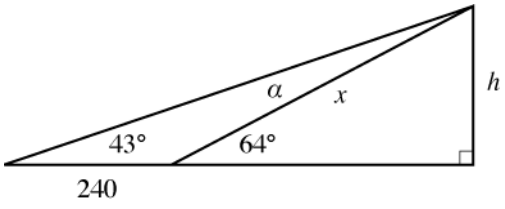



Finding $A$ using the law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$38^2 = 38^2 + 48^2 - 2(38)(48)\cos A$$
$$1444 = 3748 - 3648\cos A$$
$$-2304 = -3648\cos A$$
$$\cos A \approx 0.6316$$
$$A \approx 51°$$

The two equal angles are approximately 51°.

**15**. Drawing the figure:




Note that $\alpha = 180° - (43° + 116°) = 21°$. Find $x$ using the law of sines:

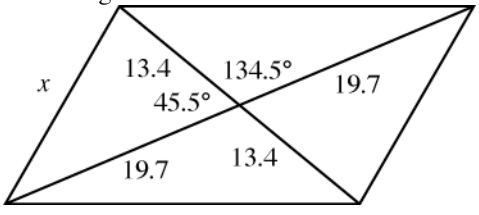
$$\frac{x}{\sin 43°} = \frac{240}{\sin 21°}$$
$$x = \frac{240\sin 43°}{\sin 21°} \approx 456.74 \text{ ft}$$

Therefore:

$$\sin 64° = \frac{h}{456.74}$$
$$h = 456.74\sin 64° \approx 410 \text{ ft}$$

The building is approximately 410 feet tall.

**16**. Draw the figure, where $x$ represents the length of the shorter side:




Find $x$ using the law of cosines:

$$x^2 = 13.4^2 + 19.7^2 - 2(13.4)(19.7)\cos 45.5° \approx 197.60$$
$$x = \sqrt{197.60} \approx 14.1 \text{ km}$$

The length of the shorter side is approximately 14.1 km.



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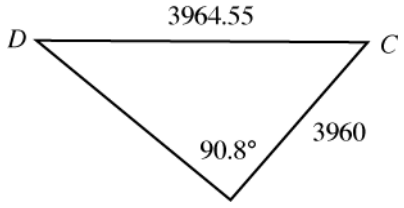
**17**. Re-draw the triangle to find angle $C$:




Finding $D$ using the law of sines:

$$\frac{3{,}960}{\sin D} = \frac{3{,}964.55}{\sin 90.8°}$$

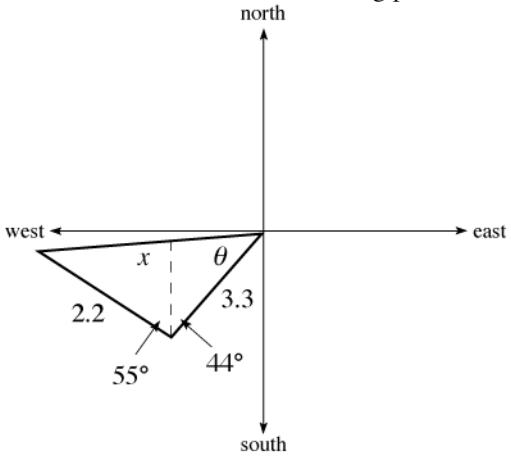
$$\sin D = \frac{3{,}960 \sin 90.8°}{3{,}964.55} \approx 0.9988$$

$$D \approx 87.14°$$

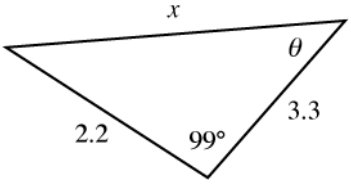
So $C = 180° - (90.8° + 87.14°) \approx 2.06°$. Converting to radians: $C = 2.06° \cdot \frac{\pi}{180°} = \frac{2.06\pi}{180}$ radians

The arc length is given by: $s = rC = 3960 \cdot \frac{2.06\pi}{180} \approx 142$ miles

**18**. Drawing the figure, where $x$ represents his distance from the starting point:




Re-draw the triangle:




Use the law of cosines to find $x$:

$$x^2 = 2.2^2 + 3.3^2 - 2(2.2)(3.3)\cos 99° \approx 18.00$$

$$x = \sqrt{18.00} \approx 4.24 \text{ mi}$$

Use the law of sines to find $\theta$:

$$\frac{2.2}{\sin\theta} = \frac{4.24}{\sin 99°}$$

$$\sin\theta = \frac{2.2 \sin 99°}{4.24} \approx 0.5125$$

$$\theta \approx 31°$$

He is approximately 4.2 miles from his starting point, with bearing S 75° W.



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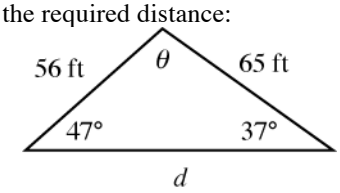
**19**. Drawing the figure, where $d$ represents the required distance:



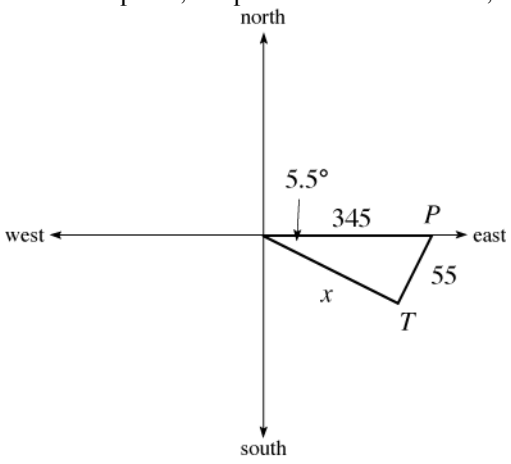

Note that $\theta = 180° - (37° + 47°) = 96°$. Finding $d$ using the law of cosines:

$$d^2 = 56^2 + 65^2 - 2(56)(65)\cos 96° \approx 8{,}121.97$$

$$d = \sqrt{8{,}121.97} \approx 90 \text{ ft}$$

The stakes are approximately 90 feet apart.

**20**. Drawing the figure, where $P$ represents the plane, $T$ represents the true course, and $x$ represents the ground speed:




Using the law of sines to find the angle at $T$:

$$\frac{345}{\sin T} = \frac{55}{\sin 5.5°}$$

$$\sin T = \frac{345 \sin 5.5°}{55} \approx 0.6012$$

$$T \approx 36.96°, 143.04°$$

If $T \approx 36.96°$, $P = 180° - (36.96° + 5.5°) \approx 137.54°$. Finding $x$ using the law of cosines:

$$x^2 = 345^2 + 55^2 - 2(345)(55)\cos 137.54° \approx 150{,}047$$

$$x = \sqrt{150{,}047} \approx 387 \text{ mph}$$

If $T \approx 143.04°$, $P = 180° - (143.04° + 5.5°) \approx 31.46°$. Finding $x$ using the law of cosines:

$$x^2 = 345^2 + 55^2 - 2(345)(55)\cos 31.46° \approx 89{,}678$$

$$x = \sqrt{89{,}678} \approx 299 \text{ mph}$$

The possibilities for the ground speed of the plane are 299 mph or 387 mph.



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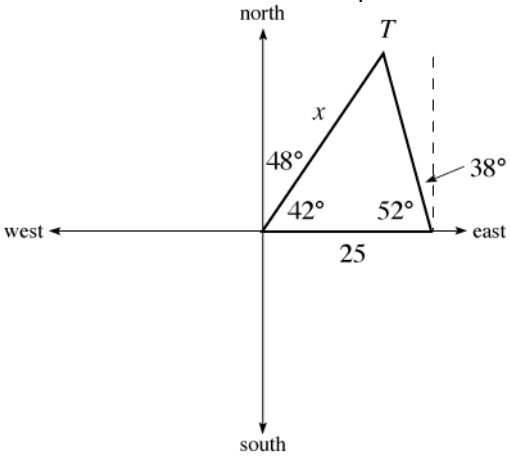
**21**. Drawing the figure, where $x$ represents the distance from the first person to the tree:

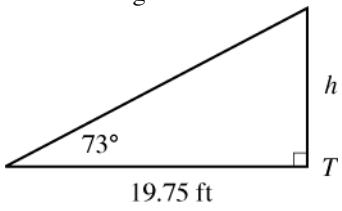



Note that $T = 180° - (42° + 52°) \approx 86°$ . Finding $x$ using the law of sines:

$$\frac{x}{\sin 52°} = \frac{25}{\sin 86°}$$

$$x = \frac{25 \sin 52°}{\sin 86°} \approx 19.75 \text{ ft}$$

Now consider the triangle where $h$ represents the height of the tree:



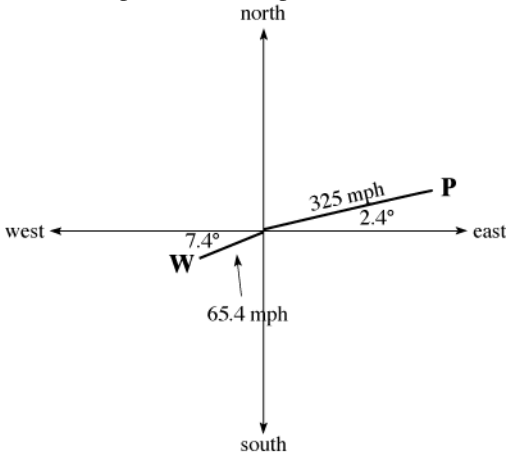

Since this is a right triangle (we are assuming the tree is vertical):

$$\tan 73° = \frac{h}{19.75}$$

$$h = 19.75 \tan 73° \approx 65 \text{ ft}$$

The tree is approximately 65 feet tall.

**22**. Drawing the figure, where $P$ represents the plane and $W$ represents the wind:



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The plane and wind vectors are given by:

$$\mathbf{P} = \langle 325\cos 2.4°, 325\sin 2.4° \rangle$$

$$\mathbf{W} = \langle -65.4\cos 7.4°, -65.4\sin 7.4° \rangle$$

Adding results in: $\mathbf{P} + \mathbf{W} = \langle 259.86, 5.19 \rangle$

The magnitude is given by: $|\mathbf{P} + \mathbf{W}| = \sqrt{259.86^2 + 5.19^2} \approx 260$ mph

The direction is given by: $\theta = \tan^{-1}\left(\dfrac{5.19}{259.86}\right) \approx 1.1°$

The ground speed is approximately 260 mph and the true course is 88.9° from due north.

**23**. Finding the magnitude: $|\mathbf{U}| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

**24**. Finding the vector: $3\mathbf{U} - 5\mathbf{V} = 3(5\mathbf{i} + 12\mathbf{j}) - 5(-4\mathbf{i} + \mathbf{j}) = 15\mathbf{i} + 36\mathbf{j} + 20\mathbf{i} - 5\mathbf{j} = 35\mathbf{i} + 31\mathbf{j}$

**25**. First find the vector: $2\mathbf{V} - \mathbf{W} = 2(-4\mathbf{i} + \mathbf{j}) - (\mathbf{i} - 4\mathbf{j}) = -8\mathbf{i} + 2\mathbf{j} - \mathbf{i} + 4\mathbf{j} = -9\mathbf{i} + 6\mathbf{j}$

Therefore: $|2\mathbf{V} - \mathbf{W}| = \sqrt{(-9)^2 + 6^2} = \sqrt{81 + 36} = \sqrt{117}$

**26**. Finding the dot product: $\mathbf{U} \cdot \mathbf{W} = (5)(1) + (12)(-4) = 5 - 48 = -43$

**27**. Since $\mathbf{U} \cdot \mathbf{V} = |\mathbf{U}||\mathbf{V}|\cos\theta$, we have:

$$\begin{aligned}
\mathbf{U} \cdot \mathbf{V} &= |\mathbf{U}||\mathbf{V}|\cos\theta \\
(5)(-4) + (12)(1) &= \sqrt{5^2 + 12^2}\sqrt{(-4)^2 + 1^2}\cos\theta \\
-20 + 12 &= \sqrt{169}\sqrt{17}\cos\theta \\
-8 &= \sqrt{2873}\cos\theta \\
\cos\theta &= \frac{-8}{\sqrt{2873}} \approx -0.1493 \\
\theta &\approx 98.6°
\end{aligned}$$

**28**. The vector is: $\mathbf{V} = \langle 38\cos 290°, 38\sin 290° \rangle \approx \langle 13, -36 \rangle = 13\mathbf{i} - 36\mathbf{j}$

**29**. Since the dot product must be 0:

$$\begin{aligned}
(5\mathbf{i} + 12\mathbf{j}) \cdot (4\mathbf{i} + b\mathbf{j}) &= 0 \\
20 + 12b &= 0 \\
12b &= -20 \\
b &= -\frac{5}{3}
\end{aligned}$$

**30**. The work is: $\mathbf{F} \cdot \mathbf{d} = (33\mathbf{i} - 4\mathbf{j}) \cdot (56\mathbf{i} + 10\mathbf{j}) = (33)(56) + (-4)(10) = 1{,}808$



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# Chapter 8
# Complex Numbers and Polar Coordinates

## 8.1 Complex Numbers

**EVEN SOLUTIONS**

**2**. When writing the square root of a negative number in terms of $i$, we usually place $i$ after the coefficient but in front of the radical.

**4**. The product of any nonzero real number and $i$ is called an imaginary number.

**6**. For any complex number $a+bi$, $a$ is called the real part and $b$ is called the imaginary part.

**8**. To add or subtract two complex numbers, simply combine like terms.

**10**. To divide complex numbers, multiply the numerator and denominator by the complex conjugate of the denominator.

**12**. This statement is true.

**14**. This statement is false.

**16**. Writing in terms of $i$: $\sqrt{-64} = i\sqrt{64} = 8i$

**18**. Writing in terms of $i$: $\sqrt{-225} = i\sqrt{225} = 15i$

**20**. Writing in terms of $i$: $\sqrt{-48} = i\sqrt{48} = 4i\sqrt{3}$

**22**. Writing in terms of $i$: $\sqrt{-21} = i\sqrt{21}$

**24**. Simplifying: $\sqrt{-25} \cdot \sqrt{-36} = i\sqrt{25} \cdot i\sqrt{36} = 5i \cdot 6i = 30i^2 = -30$

**26**. Simplifying: $\sqrt{-16} \cdot \sqrt{-49} = i\sqrt{16} \cdot i\sqrt{49} = 4i \cdot 7i = 28i^2 = -28$

**28**. Setting the real parts equal:

$$-x = 2$$
$$x = -2$$

Setting the imaginary parts equal:

$$10y = -5$$
$$y = -\frac{1}{2}$$

**30**. Setting the real parts equal:

$$x^2 - 6 = x$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = -2, 3$$

Setting the imaginary parts equal:

$$y^2 = 9$$
$$y = \pm\sqrt{9}$$
$$y = \pm 3$$

**32**. Setting the real parts equal:

$$\sin x = -\cos x$$
$$\tan x = -1$$
$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Setting the imaginary parts equal:

$$\cos y = -1$$
$$y = \pi$$

**34**. Setting the real parts equal:

$$\cos^2 x + 1 = 2\cos x$$
$$\cos^2 x - 2\cos x + 1 = 0$$
$$(\cos x - 1)^2 = 0$$
$$\cos x = 1$$
$$x = 0$$

Setting the imaginary parts equal:

$$\tan y = -1$$
$$y = \frac{3\pi}{4}, \frac{7\pi}{4}$$



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**36**. Combining the numbers: $(3-5i)+(-2-4i)=1-9i$

**38**. Combining the numbers: $(6+7i)-(4-i)=6+7i-4+i=2+8i$

**40**. Combining the numbers: $(7-3i)-(-4+10i)=7-3i+4-10i=11-13i$

**42**. Combining the numbers: $(2\cos x-3i\sin y)+(3\cos x-2i\sin y)=5\cos x-5i\sin y$

**44**. Combining the numbers: $[(4-5i)+(2+i)]-(2+5i)=4-5i+2+i-2-5i=4-9i$

**46**. Combining the numbers:

$$(10-2i)-[(2+6i)-(3-i)]=(10-2i)-(2+6i-3+i)=(10-2i)-(-1+7i)=10-2i+1-7i=11-9i$$

**48**. Computing the power: $i^{17}=i^{16}\cdot i=\left(i^4\right)^4\cdot i=1\cdot i=i$

**50**. Computing the power: $i^{22}=i^{20}\cdot i^2=\left(i^4\right)^5\cdot i^2=1\cdot(-1)=-1$

**52**. Computing the power: $i^{28}=\left(i^4\right)^7=1$

**54**. Computing the power: $i^{43}=i^{40}\cdot i^3=\left(i^4\right)^{10}\cdot i^2\cdot i=1\cdot(-1)\cdot i=-i$

**56**. Computing the product: $5i(3+4i)=15i+20i^2=15i-20=-20+15i$

**58**. Computing the product: $(-3+3i)(-2+9i)=6-27i-6i+27i^2=6-33i-27=-21-33i$

**60**. Computing the product: $(4+7i)^2=(4+7i)(4+7i)=16+28i+28i+49i^2=16+56i-49=-33+56i$

**62**. Computing the product: $(2+6i)(2-6i)=4-12i+12i-36i^2=4+36=40$

**64**. Computing the product: $(3+7i)(3-7i)=9-21i+21i-49i^2=9+49=58$

**66**. Computing the product: $4i(1-i)^2=4i(1-i)(1-i)=4i\left(1-i-i+i^2\right)=4i(1-2i-1)=4i(-2i)=-8i^2=8$

**68**. Finding the quotient: $\dfrac{3i}{2-i}\cdot\dfrac{2+i}{2+i}=\dfrac{6i+3i^2}{4-i^2}=\dfrac{6i-3}{4+1}=\dfrac{-3+6i}{5}=-\dfrac{3}{5}+\dfrac{6}{5}i$

**70**. Finding the quotient: $\dfrac{3+2i}{3-2i}\cdot\dfrac{3+2i}{3+2i}=\dfrac{9+12i+4i^2}{9-4i^2}=\dfrac{9+12i-4}{9+4}=\dfrac{5+12i}{13}=\dfrac{5}{13}+\dfrac{12}{13}i$

**72**. Finding the quotient: $\dfrac{5-2i}{-i}\cdot\dfrac{i}{i}=\dfrac{5i-2i^2}{-i^2}=\dfrac{5i+2}{1}=2+5i$

**74**. Finding the quotient: $\dfrac{2+i}{5-6i}\cdot\dfrac{5+6i}{5+6i}=\dfrac{10+17i+6i^2}{25-36i^2}=\dfrac{10+17i-6}{25+36}=\dfrac{4+17i}{61}=\dfrac{4}{61}+\dfrac{17}{61}i$

**76**. Computing the value: $z_2z_1=(2-3i)(2+3i)=4-9i^2=4+9=13$

**78**. Computing the value: $z_3z_1=(4+5i)(2+3i)=8+22i+15i^2=8+22i-15=-7+22i$

**80**. Computing the value: $3z_1+2z_2=3(2+3i)+2(2-3i)=6+9i+4-6i=10+3i$

**82**. Computing the value:

$$z_3(z_1-z_2)=(4+5i)[(2+3i)-(2-3i)]=(4+5i)(2+3i-2+3i)=(4+5i)(6i)=24i+30i^2=-30+24i$$

**84**. Computing the product: $(x-4i)(x+4i)=x^2-16i^2=x^2+16$



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**86**. Substituting $x = 3+2i$ into the equation:

$$\begin{aligned} x^2-6x+13 &= (3+2i)^2-6(3+2i)+13 \\ &= 9+12i+4i^2-18-12i+13 \\ &= 9+12i-4-18-12i+13 \\ &= (9-4-18+13)+(12-12)i \\ &= 0 \end{aligned}$$

**88**. Substituting $x = a-bi$ into the equation:

$$\begin{aligned} x^2-2ax+\left(a^2+b^2\right) &= (a-bi)^2-2a(a-bi)+\left(a^2+b^2\right) \\ &= a^2-2abi+b^2i^2-2a^2+2abi+a^2+b^2 \\ &= a^2-2abi-b^2-2a^2+2abi+a^2+b^2 \\ &= \left(a^2-b^2-2a^2+a^2+b^2\right)+(-2ab+2ab)i \\ &= 0 \end{aligned}$$

**90**. Solving $x-y=10$ for $x$ yields $x = y+10$. Substituting into the second equation:

$$\begin{aligned} xy &= -40 \\ (y+10)y &= -40 \\ y^2+10y &= -40 \\ y^2+10y+40 &= 0 \end{aligned}$$

Using $a = 1$, $b = 10$, and $c = 40$ in the quadratic formula:

$$y = \frac{-10\pm\sqrt{10^2-4(1)(40)}}{2(1)} = \frac{-10\pm\sqrt{100-160}}{2} = \frac{-10\pm\sqrt{-60}}{2} = \frac{-10\pm 2i\sqrt{15}}{2} = -5\pm i\sqrt{15}$$

If $y = -5+i\sqrt{15}$, $x = \left(-5+i\sqrt{15}\right)+10 = 5+i\sqrt{15}$. If $y = -5-i\sqrt{15}$, $x = \left(-5-i\sqrt{15}\right)+10 = 5-i\sqrt{15}$.

The solutions are $\left(5+i\sqrt{15},-5+i\sqrt{15}\right)$ and $\left(5-i\sqrt{15},-5-i\sqrt{15}\right)$.

**92**. Solving $3x+y=6$ for $y$ yields $y = 6-3x$. Substituting into the second equation:

$$\begin{aligned} xy &= 9 \\ x(6-3x) &= 9 \\ 6x-3x^2 &= 9 \\ 3x^2-6x+9 &= 0 \\ x^2-2x+3 &= 0 \end{aligned}$$

Using $a = 1$, $b = -2$, and $c = 3$ in the quadratic formula:

$$y = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(3)}}{2(1)} = \frac{2\pm\sqrt{4-12}}{2} = \frac{2\pm\sqrt{-8}}{2} = \frac{2\pm 2i\sqrt{2}}{2} = 1\pm i\sqrt{2}$$

Substituting each value to find $y$:

$x = 1+i\sqrt{2}$: $\quad y = 6-3\left(1+i\sqrt{2}\right) = 6-3-3i\sqrt{2} = 3-3i\sqrt{2}$

$x = 1-i\sqrt{2}$: $\quad y = 6-3\left(1-i\sqrt{2}\right) = 6-3+3i\sqrt{2} = 3+3i\sqrt{2}$

The solutions are $\left(1+i\sqrt{2},3-3i\sqrt{2}\right)$ and $\left(1-i\sqrt{2},3+3i\sqrt{2}\right)$.

**94**. Let $z = a+bi$, so the conjugate is $\overline{z} = a-bi$. The sum is given by: $z+\overline{z} = (a+bi)+(a-bi) = 2a$

Thus the sum $z+\overline{z}$ is a real number.

**96**. If $x = -8$ and $y = -15$, $r = \sqrt{(-8)^2+(-15)^2} = \sqrt{64+225} = \sqrt{289} = 17$. Therefore $\sin\theta = \frac{y}{r} = -\frac{15}{17}$ and $\cos\theta = \frac{x}{r} = -\frac{8}{17}$.



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**98**. If $\sin\theta = -\frac{1}{2}$ and $\theta$ terminates in quadrant III, $\theta = 210°$.

**100**. First note $A = 180° - (24.2° + 63.8°) = 92.0°$. Find $a$ using the law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 92°} = \frac{5.92}{\sin 24.2°}$$
$$a = \frac{5.92 \sin 92°}{\sin 24.2°} \approx 14.4 \text{ inches}$$

Find $c$ using the law of sines:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 63.8°} = \frac{5.92}{\sin 24.2°}$$
$$c = \frac{5.92 \sin 63.8°}{\sin 24.2°} \approx 13.0 \text{ inches}$$

**102**. Find $A$ using the law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{625}{\sin A} = \frac{521}{\sin 32.8°}$$
$$\sin A = \frac{625 \sin 32.8°}{521} \approx 0.6498$$
$$A \approx 40.5°, 139.5°$$

If $A = 40.5°$, $C = 180° - (40.5° + 32.8°) = 106.7°$. Find $c$ using the law of sines:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 106.7°} = \frac{521}{\sin 32.8°}$$
$$c = \frac{521 \sin 106.7°}{\sin 32.8°} \approx 921 \text{ ft}$$

If $A' = 139.5°$, $C' = 180° - (139.5° + 32.8°) = 7.7°$. Find $c'$ using the law of sines:

$$\frac{c'}{\sin C'} = \frac{b}{\sin B}$$
$$\frac{c'}{\sin 7.7°} = \frac{521}{\sin 32.8°}$$
$$c' = \frac{521 \sin 7.7°}{\sin 32.8°} \approx 129 \text{ ft}$$

**104**. Simplifying: $[(8 + 5i) - (4 - 2i)] + (-6 + 3i) = (8 + 5i - 4 + 2i) + (-6 + 3i) = (4 + 7i) + (-6 + 3i) = -2 + 10i$

The correct answer is d.

**106**. Finding the quotient: $\frac{5 + 2i}{4 - 3i} \cdot \frac{4 + 3i}{4 + 3i} = \frac{20 + 23i + 6i^2}{16 - 9i^2} = \frac{20 + 23i - 6}{16 + 9} = \frac{14 + 23i}{25} = \frac{14}{25} + \frac{23}{25}i$

The correct answer is c.



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**ODD SOLUTIONS**

**1.** imaginary unit

**3.** negative, before

**5.** sum

**7.** real, imaginary

**9.** conjugate, real

**11.** **a.** iii **b.** i **c.** iv **d.** ii

**13.** true

**15.** $\sqrt{-16} = i\sqrt{16}$
$= 4i$

**17.** $\sqrt{-121} = i\sqrt{121}$
$= 11i$

**19.** $\sqrt{-18} = i\sqrt{18}$
$= 3i\sqrt{2}$

**21.** $\sqrt{-8} = i\sqrt{8}$
$= 2i\sqrt{2}$

**23.** $\sqrt{-4}\sqrt{-9} = i\sqrt{4}\,i\sqrt{9}$
$= 2 \cdot 3 \cdot i^2$
$= 6(-1) = -6$

**25.** $\sqrt{-1}\sqrt{-9} = i \cdot i\sqrt{9}$
$= 3i^2$
$= 3(-1) = -3$

**27.** $4 = 6x$ and $7 = -14y$

$x = \frac{4}{6} = \frac{2}{3}$ $\quad y = -\frac{7}{14} = -\frac{1}{2}$

**29.** $x^2 - 2x = 8$ and $y^2 = 2y - 1$

$x^2 - 2x - 8 = 0$ $\quad 0 = y^2 - 2y + 1$

$(x-4)(x+2) = 0$ $\quad 0 = (y-1)(y-1)$

$x - 4 = 0$ or $x + 2 = 0$ $\quad 0 = y - 1$

$x = 4$ or $x = -2$ $\quad y = 1$

**31.** $\cos x = \sin x$ and $\sin y = 1$

$1 = \frac{\sin x}{\cos x}$ $\quad y = \frac{\pi}{2}$

$\tan x = 1$

$x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

**33.** $\sin^2 x - 2\sin x + 1 = 0$ and $\tan y = 1$

$(\sin x - 1)(\sin x - 1) = 0$ $\quad y = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

$\sin x - 1 = 0$

$\sin x = 1$

$x = \frac{\pi}{2}$

**35.** $7 + 2i + 3 - 4i = 10 - 2i$

**37.** $(5 + 2i) - (3 + 6i) = 5 + 2i - 3 - 6i$
$= (5 - 3) + (2 - 6)i$
$= 2 - 4i$

**39.** $(11 - 6i) - (2 - 4i) = 11 - 6i - 2 + 4i$
$= (11 - 2) + (-6 + 4)i$
$= 9 - 2i$

**41.** $(3\cos x + 4i\sin y) + (2\cos x - 7i\sin y)$
$= (3\cos x + 2\cos x) + (4\sin y - 7\sin y)i$
$= 5\cos x - 3i\sin y$

**43.** $[(3 + 2i) - (6 + i)] + (5 + i) = 3 + 2i - 6 - i + 5 + i$
$= (3 - 6 + 5) + (2i - i + i)$
$= 2 + 2i$

**45.** $(7 - 4i) - [(-2 + i) - (3 + 7i)] = 7 - 4i - (-2 + i - 3 - 7i)$
$= 7 - 4i + 2 - i + 3 + 7i$
$= 12 + 2i$

**47.** $i^{12} = (i^4)^3 = 1^3 = 1$

**49.** $i^{15} = (i^4)^3 \cdot i^3$
$= 1(-i) = -i$

**51.** $i^{34} = (i^4)^8 \cdot i^2$
$= 1^8 \cdot (-1) = -1$

**53.** $i^{33} = (i^4)^8 \cdot i$
$= 1^8 \cdot i = i$

**55.** $-6i(3 - 8i) = -18i + 48i^2$
$= -18i + 48(-1) = -48 - 18i$

**57.** $(2 - 4i)(3 + i) = 6 + 2i - 12i - 4i^2$
$= 6 - 10i - 4(-1)$
$= 6 - 10i + 4 = 10 - 10i$

**59.** $(3 - 2i)^2 = 9 - 12i + 4i^2$
$= 9 - 12i + 4(-1)$
$= 9 - 12i - 4 = 5 - 12i$



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**61.** $(4+5i)(4-5i) = 16-25i^2$
$= 16-25(-1)$
$= 16+25 = 41$

**63.** $(7+2i)(7-2i) = 49-4i^2$
$= 49-4(-1)$
$= 49+4 = 53$

**65.** $3i(1+i)^2 = 3i(1+2i+i^2)$
$= 3i+6i^2-3i$
$= 6(-1) = -6$

**67.** $\dfrac{2i}{3+i}\cdot\dfrac{3-i}{3-i} = \dfrac{6-2i^2}{9-i^2}$
$= \dfrac{6i-2(-1)}{9-(-1)}$
$= \dfrac{2+6i}{10} = \dfrac{1}{5}+\dfrac{3}{5}i$

**69.** $\dfrac{2+3i}{2-3i}\cdot\dfrac{2+3i}{2+3i} = \dfrac{4+12i+9i^2}{4-9i^2}$
$= \dfrac{4+12i-9}{4-(-9)}$
$= \dfrac{-5+12i}{13} = -\dfrac{5}{13}+\dfrac{12}{13}i$

**71.** $\dfrac{5-2i}{i}\cdot\dfrac{i}{i} = \dfrac{5i-2i^2}{i^2}$
$= \dfrac{5i-2(-1)}{-1}$
$= \dfrac{2+5i}{-1} = -2-5i$

**73.** $\dfrac{5+4i}{3+6i}\cdot\dfrac{3-6i}{3-6i} = \dfrac{15-18i-24i^2}{9-36i^2}$
$= \dfrac{15-18i+24}{9-(-36)}$
$= \dfrac{39-18i}{45} = \dfrac{39}{45}-\dfrac{18}{45}i$
$= \dfrac{13}{15}-\dfrac{2}{5}i$

**75.** $z_1z_2 = (2+3i)(2-3i)$
$= 4-9i^2$
$= 4-(-9) = 13$

**77.** $z_1z_3 = (2+3i)(4+5i)$
$= 8+22i+15i^2$
$= 8+22i-15 = -7+22i$

**79.** $2z_1+3z_2 = 2(2+3i)+3(2-3i)$
$= 4+6i+6-9i$
$= 10-3i$

**81.** $z_3(z_1+z_2) = (4+5i)\big[(2+3i)+(2-3i)\big]$
$= (4+5i)(4)$
$= 16+20i$

**83.** $(x+3i)(x-3i) = x^2-9i^2$
$= x^2+9$

**85.** $x^2-4x+13 = (2+3i)^2-4(2+3i)+13$
$= 4+12i+9i^2-8-12i+13$
$= 4+12i-9-8-12i+13$
$= 0$

**87.** $x^2-2ax+(a^2+b^2)$
$= (a+bi)^2-2a(a+bi)+a^2+b^2$
$= a^2+2abi+b^2i^2-2a^2-2abi+a^2+b^2$
$= b^2(-1)+b^2 = 0$

**89.** If $x+y=8$, then $y=8-x$. Substituting this into the second equation, we get:

$x(8-x) = 20$
$8x-x^2 = 20$
$x^2-8x+20 = 0$ where $a = 1$, $b = -8$, and $c = 20$

$$x = \frac{-(-8)\pm\sqrt{(-8)^2-4(1)(20)}}{2(1)}$$
$$= \frac{8\pm\sqrt{-16}}{2}$$
$$= \frac{8\pm 4i}{2} = 4\pm 2i$$

Now, we find the corresponding $y$-values:

If $x = 4+2i$, then $y = 8-(4+2i) = 4-2i$

If $x = 4-2i$, then $y = 8-(4-2i) = 4+2i$



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**91.** If $2x + y = 4$, then $y = 4 - 2x$. Substituting this into the second equation, we get:

$$x(4 - 2x) = 8$$
$$4x - 2x^2 = 8$$
$$2x^2 - 4x + 8 = 0 \text{ or } x^2 - 2x + 4 = 0 \quad \text{where } a = 1, b = -2, \text{ and } c = 4$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{-12}}{2}$$
$$= \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$$

Now, we find the corresponding $y$-values:

If $x = 1 + i\sqrt{3}$, then $y = 4 - 2(1 + i\sqrt{3}) = 2 - 2i\sqrt{3}$.

If $x = 1 - i\sqrt{3}$, then $y = 4 - 2(1 - i\sqrt{3}) = 2 + 2i\sqrt{3}$.

**93.**
$$(a + bi)(a - bi) = a^2 - b^2 i^2$$
$$= a^2 - b^2(-1)$$
$$= a^2 + b^2$$

This is a real number. Therefore, the product of a complex number and its conjugate is a real number.

**95.**
$$r = \sqrt{3^2 + (-4)^2}$$
$$= \sqrt{25} = 5$$
$$\sin\theta = -\frac{4}{5}$$
$$\cos\theta = \frac{3}{5}$$

**97.** $\hat{\theta} = 45^\circ$ and is in QII

$\theta = 180^\circ - 45^\circ = 135^\circ$

**99.**
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 243^2 + 157^2 - 2(243)(157)\cos 73.1^\circ$$
$$= 61{,}517$$
$$a = 248 \text{ cm}$$

$$\sin C = \frac{c\sin A}{a}$$
$$= \frac{157\sin 73.1^\circ}{248} = 0.6057$$
$$C = 37.3^\circ$$
$$B = 180^\circ - (73.1^\circ + 37.3^\circ) = 69.6^\circ$$

**101.**
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$= \frac{42.1^2 + 56.8^2 - 63.4^2}{2(42.1)(56.8)} = 0.2047$$
$$C = 78.2^\circ$$

$$\sin B = \frac{b\sin C}{c}$$
$$= \frac{56.8\sin 78.2^\circ}{63.4} = 0.8770$$
$$B = 61.3^\circ$$
$$A = 180^\circ - (78.2^\circ + 61.3^\circ) = 40.5^\circ$$



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## 8.2 Trigonometric Form for Complex Numbers

**EVEN SOLUTIONS**

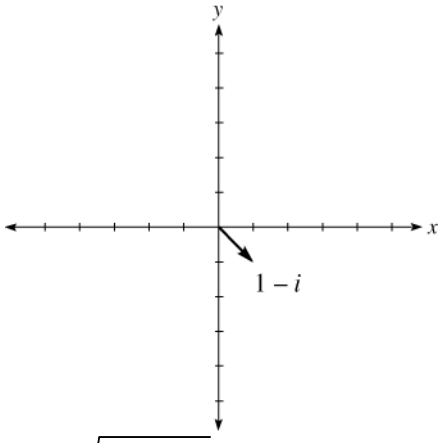
**2**. The graph of the complex number $a + bi$ is a vector that extends from the origin to the point $(a, b)$ in the complex plane.

**4**. The argument of a complex number $z$ is the smallest positive angle from the positive real axis to the graph of $z$.

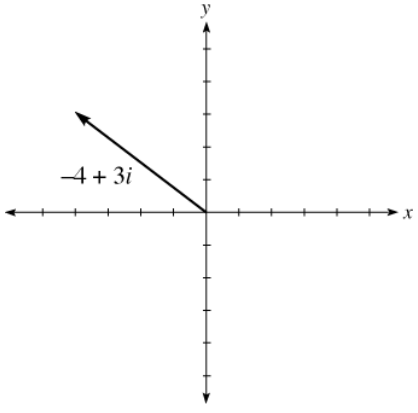
**6**. When we write $z = r\text{cis}\theta$, $r$ is the modulus of $z$ and $\theta$ is the argument.

**8**. To convert a complex number $z = x + yi$ from standard form into trigonometric form, use the relationships $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$, keeping in mind the quadrant that the graph of $z$ lies in.

**10**. The absolute value is given by: $|1 - i| = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$. Graphing the complex number:




**12**. The absolute value is given by: $|-4 + 3i| = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$. Graphing the complex number:



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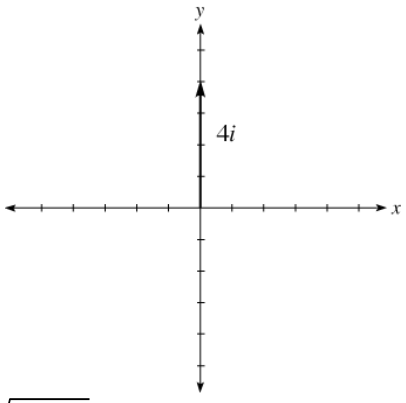
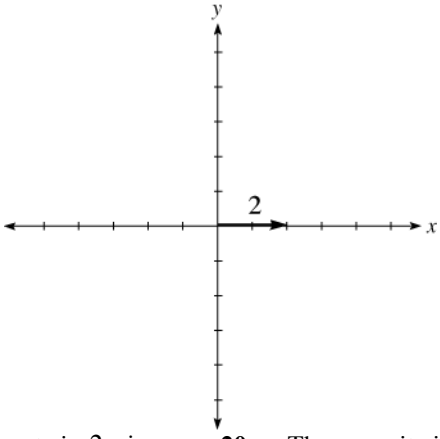
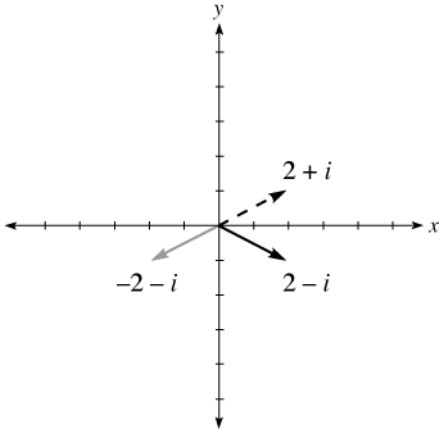
**14**. The absolute value is given by: $|4i| = \sqrt{0^2 + 4^2} = \sqrt{0+16} = \sqrt{16} = 4$ . Graphing the complex number:



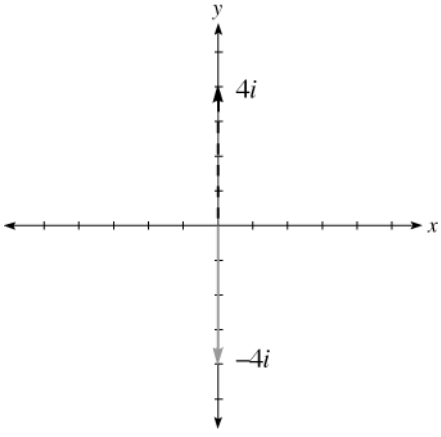

**16**. The absolute value is given by: $|2| = \sqrt{2^2 + 0^2} = \sqrt{4+0} = \sqrt{4} = 2$ . Graphing the complex number:




**18**. The opposite is $-2 - i$ and the conjugate is $2 - i$ :




**20**. The opposite is $-4i$ and the conjugate is $-4i$ :



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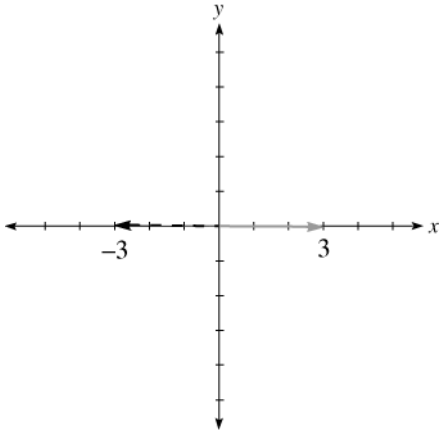
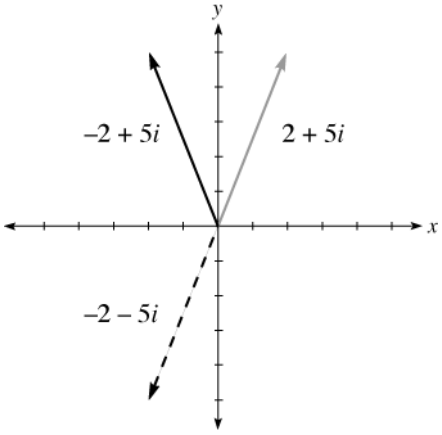
**22**. The opposite is 3 and the conjugate is –3:




**24**. The opposite is $2+5i$ and the conjugate is $-2+5i$ :




**26**. Writing in standard form: $4\left(\cos 30^\circ + i\sin 30^\circ\right) = 4\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 2\sqrt{3} + 2i$

**28**. Writing in standard form: $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 8\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -4 + 4i\sqrt{3}$

**30**. Writing in standard form: $1\text{cis}240^\circ = \cos 240^\circ + i\sin 240^\circ = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

**32**. Writing in standard form: $\sqrt{2}\ \text{cis}\ \frac{7\pi}{4} = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = 1 - i$

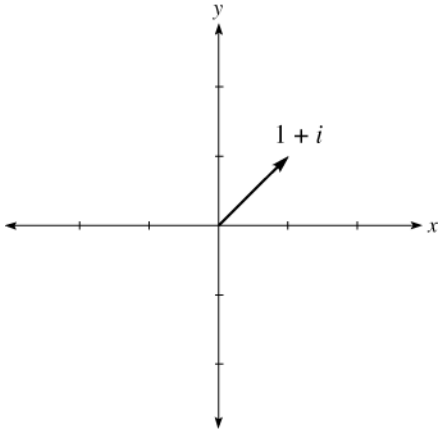
**34**. Writing in standard form: $5\left(\cos 70^\circ + i\sin 70^\circ\right) = \left(5\cos 70^\circ\right) + \left(5\sin 70^\circ\right)i \approx 1.71 + 4.70i$

**36**. Note these are radian angles. Writing in standard form: $50\left(\cos 3 + i\sin 3\right) = \left(50\cos 3\right) + \left(50\sin 3\right)i \approx -49.50 + 7.06i$

**38**. Writing in standard form: $2\text{cis}261^\circ = 2\left(\cos 261^\circ + i\sin 261^\circ\right) = \left(2\cos 261^\circ\right) + \left(2\sin 261^\circ\right)i \approx -0.31 - 1.98i$

**40**. Note these are radian angles. Writing in standard form:
$10\text{cis}5.5 = 10\left(\cos 5.5 + i\sin 5.5\right) = \left(10\cos 5.5\right) + \left(10\sin 5.5\right)i \approx 7.09 - 7.06i$

**42**. Sketching the graph:




Here $x = 1$, $y = 1$, and $r = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$ . Since $\tan\theta = \frac{y}{x} = 1$ , $\theta = 45^\circ$ .

So the trigonometric form is $1 + i = \sqrt{2}\left(\cos 45^\circ + i\sin 45^\circ\right) = \sqrt{2}\ \text{cis}\ 45^\circ = \sqrt{2}\ \text{cis}\ \frac{\pi}{4}$ .



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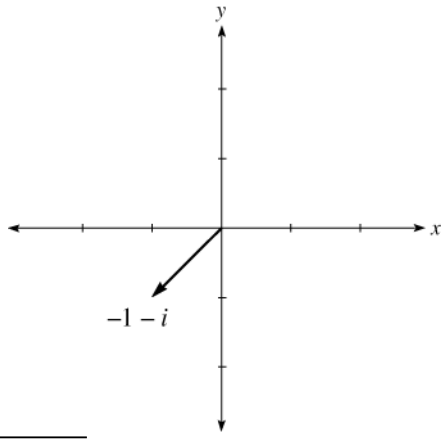
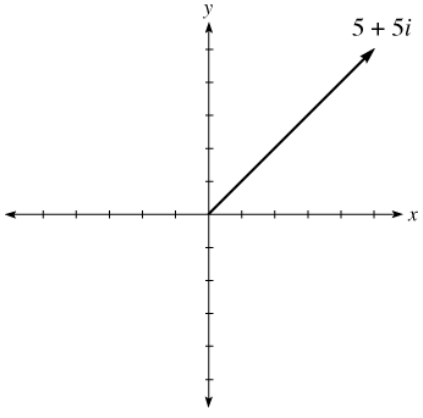
**44**. Sketching the graph:




Here $x = -1$, $y = -1$, and $r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$. Since $\tan\theta = \frac{y}{x} = \frac{-1}{-1} = 1$, $\theta = 225°$.

So the trigonometric form is $-1 - i = \sqrt{2}(\cos 225° + i \sin 225°) = \sqrt{2} \text{ cis } 225° = \sqrt{2} \text{ cis } \frac{5\pi}{4}$.

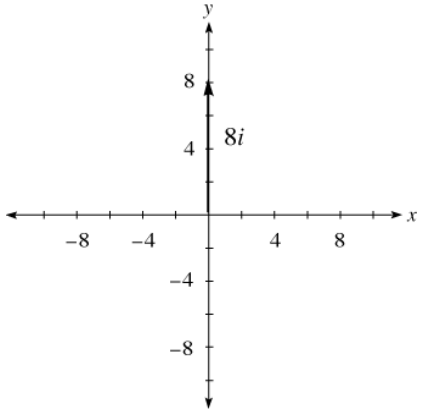
**46**. Sketching the graph:




Here $x = 5$, $y = 5$, and $r = \sqrt{5^2 + 5^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$. Since $\tan\theta = \frac{y}{x} = \frac{5}{5} = 1$, $\theta = 45°$.

So the trigonometric form is $5 + 5i = 5\sqrt{2}(\cos 45° + i \sin 45°) = 5\sqrt{2} \text{ cis } 45° = 5\sqrt{2} \text{ cis } \frac{\pi}{4}$.

**48**. Sketching the graph:




Here $x = 0$, $y = 8$, and $r = \sqrt{0^2 + 8^2} = \sqrt{0+64} = \sqrt{64} = 8$. Since $\tan\theta$ is undefined, $\theta = 90°$.

So the trigonometric form is $8i = 8(\cos 90° + i \sin 90°) = 8 \text{ cis } 90° = 8 \text{ cis } \frac{\pi}{2}$.



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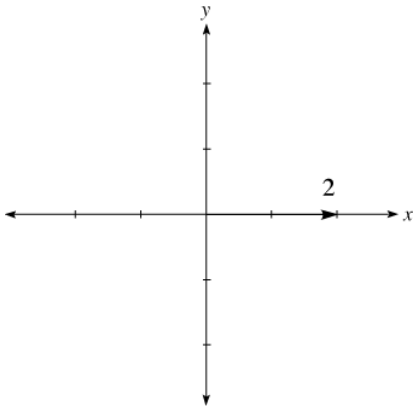
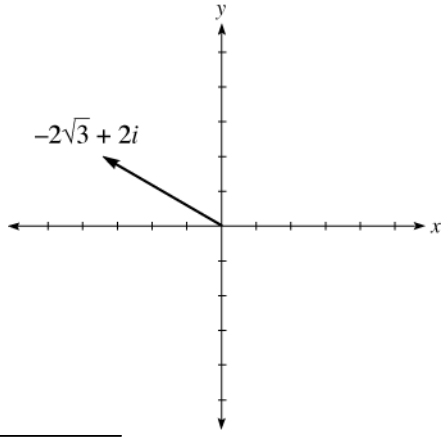
**50**. Sketching the graph:




Here $x = 2, y = 0$, and $r = \sqrt{2^2 + 0^2} = \sqrt{4+0} = \sqrt{4} = 2$. Since $\tan\theta = \frac{0}{2} = 0$, $\theta = 0°$.

So the trigonometric form is $2 = 2(\cos 0° + i \sin 0°) = 2 \text{ cis } 0° = 2 \text{ cis } 0$.

**52**. Sketching the graph:




Here $x = -2\sqrt{3}$, $y = 2$, and $r = \sqrt{\left(-2\sqrt{3}\right)^2 + 2^2} = \sqrt{12+4} = \sqrt{16} = 4$. Since $\tan\theta = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}}$, $\theta = 150°$.

So the trigonometric form is $-2\sqrt{3} + 2i = 4(\cos 150° + i \sin 150°) = 4 \text{ cis } 150° = 4 \text{ cis } \frac{5\pi}{6}$.

**54**. Here $x = 5, y = 12$, and $r = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169} = 13$. Since $\tan\theta = \frac{12}{5}$ in quadrant I, $\theta = 67.38°$. So the trigonometric form is $5 + 12i = 13(\cos 67.38° + i \sin 67.38°) = 13 \text{ cis } 67.38°$.

**56**. Here $x = -8, y = -15$, and $r = \sqrt{(-8)^2 + (-15)^2} = \sqrt{64+225} = \sqrt{289} = 17$. Since $\tan\theta = \frac{15}{8}$ in quadrant III, $\theta = 180° + 61.93° = 241.93°$. So the trigonometric form is $-8 - 15i = 17(\cos 241.93° + i \sin 241.93°) = 17 \text{ cis } 241.93°$.

**58**. Here $x = 6, y = 2$, and $r = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$. Since $\tan\theta = \frac{2}{6} = \frac{1}{3}$ in quadrant I, $\theta = 18.43°$.

So the trigonometric form is $6 + 2i = 2\sqrt{10}(\cos 18.43° + i \sin 18.43°) = 2\sqrt{10} \text{ cis } 18.43°$.

**60**. Here $x = -2, y = -3$, and $r = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$. Since $\tan\theta = \frac{3}{2}$ in quadrant III, $\theta = 180° + 56.31° = 236.31°$. So the trigonometric form is $-2 - 3i = \sqrt{13}(\cos 236.31° + i \sin 236.31°) = \sqrt{13} \text{ cis } 236.31°$.

**62**. The trigonometric form is $5 + 12i = 13(\cos 67.38° + i \sin 67.38°) = 13 \text{ cis } 67.38°$.

**64**. The trigonometric form is $-8 - 15i = 17(\cos 241.93° + i \sin 241.93°) = 17 \text{ cis } 241.93°$.



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**66**. The trigonometric form is $6+2i=2\sqrt{10}\left(\cos 18.43°+i\sin 18.43°\right)=2\sqrt{10}\text{ cis }18.43°$.

**68**. The trigonometric form is $-2-3i=\sqrt{13}\left(\cos 236.31°+i\sin 236.31°\right)=\sqrt{13}\text{ cis }236.31°$.

**70**. Writing each complex number in standard form:

$$2\left(\cos 60°+i\sin 60°\right)=2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=1+i\sqrt{3}$$

$$2\left[\cos\left(-60°\right)+i\sin\left(-60°\right)\right]=2\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=1-i\sqrt{3}$$

Note that the two numbers are conjugates.

**72**. Finding the magnitude: $|z|=\sqrt{\cos^2\theta+\left(-\sin\theta\right)^2}=\sqrt{\cos^2\theta+\sin^2\theta}=\sqrt{1}=1$

**74**. Using the addition formula for sine:

$$\sin 75°=\sin\left(30°+45°\right)=\sin 30°\cos 45°+\cos 30°\sin 45°=\frac{1}{2}\bullet\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\bullet\frac{\sqrt{2}}{2}=\frac{\sqrt{2}+\sqrt{6}}{4}$$

**76**. Since $A$ terminates in quadrant I, $\cos A>0$ and thus: $\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

Since $B$ terminates in quadrant I, $\cos B>0$ and thus: $\cos B=\sqrt{1-\sin^2 B}=\sqrt{1-\left(\frac{5}{13}\right)^2}=\sqrt{1-\frac{25}{169}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$

Using the addition formula for cosine: $\cos\left(A+B\right)=\cos A\cos B-\sin A\sin B=\frac{4}{5}\bullet\frac{12}{13}-\frac{3}{5}\bullet\frac{5}{13}=\frac{48}{65}-\frac{15}{65}=\frac{33}{65}$

**78**. Using the addition formula for cosine: $\cos 30°\cos 90°-\sin 30°\sin 90°=\cos\left(30°+90°\right)=\cos 120°=-\frac{1}{2}$

**80**. Using the law of sines:

$$\frac{\sin B}{b}=\frac{\sin A}{a}$$

$$\frac{\sin B}{567}=\frac{\sin 45.6°}{678}$$

$$\sin B=\frac{567\sin 45.6°}{678}\approx 0.5975$$

$$B\approx 36.7°,143.3°$$

Since 143.3° + 45.6° = 188.9° > 180°, only $B\approx 36.7°$ is valid.

**82**. Using the law of sines:

$$\frac{\sin B}{b}=\frac{\sin A}{a}$$

$$\frac{\sin B}{567}=\frac{\sin 45.6°}{789}$$

$$\sin B=\frac{567\sin 45.6°}{789}\approx 0.5134$$

$$B\approx 30.9°,149.1°$$

Since 149.1° + 45.6° = 194.7° > 180°, only $B\approx 30.9°$ is valid.

**84**. Here $x=-4$, $y=4$, and $r=\sqrt{\left(-4\right)^2+4^2}=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$. Since $\tan\theta=-1$ in quadrant II, $\theta=135°$.

So the trigonometric form is $-4+4i=4\sqrt{2}\left(\cos 135°+i\sin 135°\right)=4\sqrt{2}\text{ cis }135°$. The correct answer is a.

**86**. The correct answer is d.



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**ODD SOLUTIONS**

**1.** Argand , real, imaginary

**3.** modulus, origin, $(a, b)$

**5.** standard, trigonometric

**7.** evaluate, distribute

**9.** $|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

**11.** $|-3-4i| = \sqrt{(-3)^2+(-4)^2} = \sqrt{25} = 5$

**13.** $|0-5i| = \sqrt{0^2+(-5)^2} = \sqrt{25} = 5$

**15.** $|-4+0i| = \sqrt{(-4)^2+0^2} = \sqrt{16} = 4$

**17.** Opposite of $2 - i = -(2-i) = -2+i$
Conjugate of $2 - i = 2 + i$

**19.** Opposite of $-3i = 3i$
Conjugate of $-3i = 3i$

**21.** Opposite of $5 = -5$
Conjugate of $5 = 5$

**23.** Opposite of $-5-2i = -(-5-2i) = 5+2i$
Conjugate of $-5-2i = -5+2i$

**25.** $2(\cos 30^\circ + i\sin 30^\circ) = 2\left[\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right]$
$= \sqrt{3} + i$

**27.** $4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 4\left[-\frac{1}{2} + i\left(\frac{\sqrt{3}}{2}\right)\right]$
$= -2 + 2i\sqrt{3}$

**29.** $\cos 210^\circ + i\sin 210^\circ = -\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)$
$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

**31.** $\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4} = \frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

**33.** $10(\cos 12^\circ + i\sin 12^\circ) = 10\left[0.978 + i(0.208)\right]$
$= 9.78 + 2.08\,i$

**35.** $100(\cos 2.5 + i\sin 2.5) = 100\left[-0.8011 + i(0.5985)\right]$
$= -80.11 + 59.85\,i$

**37.** $\cos 205^\circ + i\sin 205^\circ = -0.91 + i(-0.42)$
$= -0.91 - 0.42\,i$

**39.** $4(\cos 6 + i\sin 6) = 4\left[0.960 + i(-0.279)\right]$
$= 3.84 - 1.12\,i$

**41.** We have $x = -1$ and $y = 1$; therefore, $r = \sqrt{(-1)^2+1^2} = \sqrt{2}$.
We also know that $\tan\theta = \frac{1}{-1}$ and $\theta$ is in QII. Therefore, $\theta = 135^\circ$ or $\frac{3\pi}{4}$.
In trigonometric form, $z = r(\cos\theta + i\sin\theta) = \sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$ or $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

**43.** We have $x = 1$ and $y = -1$; therefore, $r = \sqrt{1^2+(-1)^2} = \sqrt{2}$
We also know that $\tan\theta = \frac{-1}{1}$ and $\theta$ is in QIV. Therefore, $\theta = 315^\circ$ or $\frac{7\pi}{4}$
In trigonometric form, $z = r(\cos\theta + i\sin\theta) = \sqrt{2}(\cos 315^\circ + i\sin 315^\circ)$ or $\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

**45.** We have $x = 3$ and $y = 3$; therefore, $r = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$.
We also know that $\tan\theta = \frac{3}{3} = 1$ and $\theta$ is in QI. Therefore, $\theta = 45^\circ$ or $\frac{\pi}{4}$.
In trigonometric form, $z = r(\cos\theta + i\sin\theta) = 3\sqrt{2}\left(\cos 45^\circ + i\sin 45^\circ\right)$ or $3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ .



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**47.** We have $x = 0$ and $y = -8$; therefore, $r = \sqrt{0^2 + (-8)^2} = \sqrt{64} = 8$.

We also know that $\theta = 270^\circ$ or $\frac{3\pi}{2}$ from its graph.

In trigonometric form, $z = r(\cos\theta + i\sin\theta) = 8\left(\cos 270^\circ + i\sin 270^\circ\right)$ or $8\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$.

**49.** We have $x = -9$ and $y = 0$; therefore, $r = \sqrt{(-9)^2 + 0^2} = \sqrt{81} = 9$.

We also know that $\theta = 180^\circ$ or $\pi$ from its graph.

In trigonometric form, $z = r(\cos\theta + i\sin\theta) = 9(\cos 180^\circ + i\sin 180^\circ)$ or $9(\cos\pi + i\sin\pi)$.

**51.** We have $x = -2$ and $y = 2\sqrt{3}$; therefore $r = \sqrt{(-2)^2 + \left(2\sqrt{3}\right)^2} = \sqrt{16} = 4$.

We also know that $\tan\theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$ and $\theta$ is in QII. Therefore, $\theta = 120^\circ$ or $\frac{2\pi}{3}$.

In trigonometric form, $z = r(\cos\theta + i\sin\theta) = 4\left(\cos 120^\circ + i\sin 120^\circ\right)$ or $4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

**53.** We have $x = 3$ and $y = -4$; therefore, $r = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$.

We also know that $\tan\theta = \frac{-4}{3}$ and $\theta$ is in QIV. Therefore, $\hat{\theta} = 53.13^\circ$ and $\theta = 306.87^\circ$.

In trigonometric form, $z = r(\cos\theta + i\sin\theta) = 5(\cos 306.87^\circ + i\sin 306.87^\circ)$.

**55.** We have $x = -20$ and $y = 21$; therefore, $r = \sqrt{(-20)^2 + 21^2} = \sqrt{841} = 29$.

We also know that $\tan\theta = \frac{21}{-20}$ and $\theta$ is in QII. Therefore, $\hat{\theta} = 46.40^\circ$ and $\theta = 133.60^\circ$.

In trigonometric form, $z = r(\cos\theta + i\sin\theta) = 29(\cos 133.60^\circ + i\sin 133.60^\circ)$.

**57.** We have $x = 7$ and $y = -1$; therefore, $r = \sqrt{7^2 + (-1)^2} = \sqrt{50} = 5\sqrt{2}$.

We also know that $\tan\theta = -\frac{1}{7}$ and $\theta$ is in QIV. Therefore, $\hat{\theta} = 8.13^\circ$ and $\theta = 351.87^\circ$.

In trigonometric form, $z = r(\cos\theta + i\sin\theta) = 5\sqrt{2}(\cos 351.87^\circ + i\sin 351.87^\circ)$.

**59.** We have $x = -11$ and $y = 2$; therefore, $r = \sqrt{(-11)^2 + 2^2} = \sqrt{125} = 5\sqrt{5}$.

We also know that $\tan\theta = \frac{2}{-11}$ and $\theta$ is in QII. Therefore, $\hat{\theta} = 10.30^\circ$ and $\theta = 169.70^\circ$.

In trigonometric form, $z = r(\cos\theta + i\sin\theta) = 5\sqrt{5}(\cos 169.70^\circ + i\sin 169.70^\circ)$.

For problems 61 through 67, see textbook answer section for answers.

**69.** $2(\cos 30^\circ + i\sin 30^\circ) = 2\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \sqrt{3} + i$

$2\left[\cos(-30^\circ) + i\sin(-30^\circ)\right] = 2\left[\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right] = \sqrt{3} - i$

**71.** $|z| = \sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = 1$

**79.**
$$\begin{aligned}\cos 75^\circ &= \cos(30^\circ + 45^\circ)\\ &= \cos 30^\circ\cos 45^\circ - \sin 30^\circ\sin 45^\circ\\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$



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**81.** $\sin A = \frac{3}{5}$ $\qquad$ $\sin B = \frac{5}{13}$

$\cos A = \sqrt{1-\left(\frac{3}{5}\right)^2}$ $\qquad$ $\cos B = \sqrt{1-\left(\frac{5}{13}\right)^2}$

$= \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$ $\qquad$ $= \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$= \frac{3}{5}\cdot\frac{12}{13} + \frac{4}{5}\cdot\frac{5}{13} = \frac{36}{65} + \frac{20}{65} = \frac{56}{65}$

**83.** $\sin 30^\circ \cos 90^\circ + \cos 30^\circ \sin 90^\circ = \sin\left(30^\circ + 90^\circ\right)$

$= \sin 120^\circ = \frac{\sqrt{3}}{2}$

**85.** $\sin B = \frac{b\sin A}{a}$

$= \frac{567\sin 45.6^\circ}{234} = 1.7312$

No triangle exists because sin B is greater than 1 (which is impossible).

**87.** $\sin B = \frac{b\sin A}{a}$

$= \frac{567\sin 45.6^\circ}{456} = 0.8884$

$B = 62.7^\circ$ or $117.3^\circ$

Two triangles exist because $a < b$.

## 8.3 Products and Quotients in Trigonometric Form

**EVEN SOLUTIONS**

**2**. To divide two complex numbers in trigonometric form, divide their moduli and subtract their arguments.

**4**. To find the *n*th power of a complex number, find the *n*th power of the modulus and multiply the argument by *n*.

**6**. Multiplying in trigonometric form:

$$3\left(\cos 20^\circ + i\sin 20^\circ\right)\cdot 4\left(\cos 195^\circ + i\sin 195^\circ\right) = 12\left[\cos\left(20^\circ + 195^\circ\right) + i\sin\left(20^\circ + 195^\circ\right)\right] = 12\left(\cos 215^\circ + i\sin 215^\circ\right)$$

**8**. Multiplying in trigonometric form:

$$7\left(\cos 110^\circ + i\sin 110^\circ\right)\cdot 8\left(\cos 201^\circ + i\sin 201^\circ\right) = 56\left[\cos\left(110^\circ + 201^\circ\right) + i\sin\left(110^\circ + 201^\circ\right)\right]$$

$$= 56\left(\cos 311^\circ + i\sin 311^\circ\right)$$

**10**. Multiplying in trigonometric form:

$$2\text{cis}\frac{2\pi}{3}\cdot 4\text{cis}\frac{7\pi}{6} = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\cdot 4\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

$$= 8\left[\cos\left(\frac{2\pi}{3} + \frac{7\pi}{6}\right) + i\sin\left(\frac{2\pi}{3} + \frac{7\pi}{6}\right)\right]$$

$$= 8\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$$

$$= 8\text{ cis }\frac{11\pi}{6}$$

**12**. Finding the product in standard form: $z_1 z_2 = (1+i)(2+2i) = 2+4i+2i^2 = 2+4i-2 = 4i$

Writing each number in trigonometric form:

$z_1 = 1+i = \sqrt{2}\left(\cos 45^\circ + i\sin 45^\circ\right)$ $\qquad$ $z_2 = 2+2i = 2\sqrt{2}\left(\cos 45^\circ + i\sin 45^\circ\right)$

Multiplying in trigonometric form:

$$z_1 z_2 = \sqrt{2}\left(\cos 45^\circ + i\sin 45^\circ\right)\cdot 2\sqrt{2}\left(\cos 45^\circ + i\sin 45^\circ\right) = 4\left(\cos 90^\circ + i\sin 90^\circ\right) = 4\left(0+1i\right) = 4i$$

Note that the products are equal.



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**14**. Finding the product in standard form: $z_1 z_2 = \left(-1+i\sqrt{3}\right)\left(\sqrt{3}+i\right) = -\sqrt{3}+2i+i^2\sqrt{3} = -\sqrt{3}+2i-\sqrt{3} = -2\sqrt{3}+2i$

Writing each number in trigonometric form:

$z_1 = -1+i\sqrt{3} = 2\left(\cos 120° + i\sin 120°\right)$ $\qquad$ $z_2 = \sqrt{3}+i = 2\left(\cos 30° + i\sin 30°\right)$

Multiplying in trigonometric form:

$$z_1 z_2 = 2\left(\cos 120° + i\sin 120°\right)\cdot 2\left(\cos 30° + i\sin 30°\right) = 4\left(\cos 150° + i\sin 150°\right) = 4\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = -2\sqrt{3}+2i$$

Note that the products are equal.

**16**. Finding the product in standard form: $z_1 z_2 = (2i)(-5i) = -10i^2 = 10$

Writing each number in trigonometric form:

$z_1 = 2i = 2\left(\cos 90° + i\sin 90°\right)$ $\qquad$ $z_2 = -5i = 5\left(\cos 270° + i\sin 270°\right)$

Multiplying in trigonometric form:

$$z_1 z_2 = 2\left(\cos 90° + i\sin 90°\right)\cdot 5\left(\cos 270° + i\sin 270°\right) = 10\left(\cos 360° + i\sin 360°\right) = 10\left(1+0i\right) = 10$$

Note that the products are equal.

**18**. Finding the product in standard form: $z_1 z_2 = (1+i)(3i) = 3i+3i^2 = -3+3i$

Writing each number in trigonometric form:

$z_1 = 1+i = \sqrt{2}\left(\cos 45° + i\sin 45°\right)$ $\qquad$ $z_2 = 3i = 3\left(\cos 90° + i\sin 90°\right)$

Multiplying in trigonometric form:

$$z_1 z_2 = \sqrt{2}\left(\cos 45° + i\sin 45°\right)\cdot 3\left(\cos 90° + i\sin 90°\right) = 3\sqrt{2}\left(\cos 135° + i\sin 135°\right) = 3\sqrt{2}\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right) = -3+3i$$

Note that the products are equal.

**20**. Finding the product in standard form: $z_1 z_2 = -3\left(\sqrt{3}+i\right) = -3\sqrt{3}-3i$

Writing each number in trigonometric form:

$z_1 = -3 = 3\left(\cos 180° + i\sin 180°\right)$ $\qquad$ $z_2 = \sqrt{3}+i = 2\left(\cos 30° + i\sin 30°\right)$

Multiplying in trigonometric form:

$$z_1 z_2 = 3\left(\cos 180° + i\sin 180°\right)\cdot 2\left(\cos 30° + i\sin 30°\right) = 6\left(\cos 210° + i\sin 210°\right) = 6\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right) = -3\sqrt{3}-3i$$

Note that the products are equal.

**22**. The trigonometric forms are:

$4i = 4\left(\cos 90° + i\sin 90°\right)$ $\qquad$ $2 = 2\left(\cos 0° + i\sin 0°\right)$

Therefore:

$$\begin{aligned} 4i\cdot 2 &= 4\left(\cos 90° + i\sin 90°\right)\cdot 2\left(\cos 0° + i\sin 0°\right) \\ &= 8\left(\cos 90°\cos 0° + i\sin 90°\sin 0° + i\cos 90°\sin 0° + i^2\sin 90°\sin 0°\right) \\ &= 8\left(0\cdot 1 + i\cdot 1\cdot 1 + i\cdot 0\cdot 0 + (-1)\cdot 1\cdot 0\right) \\ &= 8\left(0+i+0+0\right) \\ &= 8i \end{aligned}$$

**24**. Using deMoivre's Theorem:

$$\begin{aligned} \left[4\left(\cos 15° + i\sin 15°\right)\right]^3 &= 4^3\left[\cos\left(3\cdot 15°\right) + i\sin\left(3\cdot 15°\right)\right] \\ &= 64\left(\cos 45° + i\sin 45°\right) \\ &= 64\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) \\ &= 32\sqrt{2}+32i\sqrt{2} \end{aligned}$$



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**26**. Using deMoivre's Theorem:

$$\left[3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]^5 = 3^5\left[\cos\left(5\cdot\frac{\pi}{6}\right)+i\sin\left(5\cdot\frac{\pi}{6}\right)\right] = 243\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right) = 243\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = -\frac{243\sqrt{3}}{2}+\frac{243}{2}i$$

**28**. Using deMoivre's Theorem:

$$(\text{cis } 18°)^{10} = (\cos 18° + i\sin 18°)^{10} = \cos(10\cdot 18°)+i\sin(10\cdot 18°) = \cos 180° + i\sin 180° = -1+0i = -1$$

**30**. Using deMoivre's Theorem:

$$\begin{aligned}\left(\sqrt{2}\ \text{cis}\frac{\pi}{8}\right)^{10} &= \left[\sqrt{2}\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)\right]^{10}\\ &= \left(\sqrt{2}\right)^{10}\left[\cos\left(10\cdot\frac{\pi}{8}\right)+i\sin\left(10\cdot\frac{\pi}{8}\right)\right]\\ &= 32\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)\\ &= 32\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\\ &= -16\sqrt{2}-16i\sqrt{2}\end{aligned}$$

**32**. First write $1+i$ in trigonometric form: $1+i=\sqrt{2}(\cos 45°+i\sin 45°)$. Using deMoivre's Theorem:

$$\begin{aligned}(1+i)^5 &= \left[\sqrt{2}(\cos 45°+i\sin 45°)\right]^5\\ &= \left(\sqrt{2}\right)^5\left[\cos(5\cdot 45°)+i\sin(5\cdot 45°)\right]\\ &= 4\sqrt{2}(\cos 225°+i\sin 225°)\\ &= 4\sqrt{2}\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)\\ &= -4-4i\end{aligned}$$

**34**. First write $\sqrt{3}+i$ in trigonometric form: $\sqrt{3}+i=2(\cos 30°+i\sin 30°)$. Using deMoivre's Theorem:

$$\begin{aligned}\left(\sqrt{3}+i\right)^4 &= \left[2(\cos 30°+i\sin 30°)\right]^4\\ &= 2^4\left[\cos(4\cdot 30°)+i\sin(4\cdot 30°)\right]\\ &= 16(\cos 120°+i\sin 120°)\\ &= 16\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\\ &= -8+8i\sqrt{3}\end{aligned}$$

**36**. First write $-1+i$ in trigonometric form: $-1+i=\sqrt{2}(\cos 135°+i\sin 135°)$. Using deMoivre's Theorem:

$$\begin{aligned}(-1+i)^8 &= \left[\sqrt{2}(\cos 135°+i\sin 135°)\right]^8\\ &= \left(\sqrt{2}\right)^8\left[\cos(8\cdot 135°)+i\sin(8\cdot 135°)\right]\\ &= 16(\cos 1080°+i\sin 1080°)\\ &= 16(1+0i)\\ &= 16\end{aligned}$$



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**38**. First write $-2-2i$ in trigonometric form: $-2-2i = 2\sqrt{2}\left(\cos 225° + i\sin 225°\right)$. Using deMoivre's Theorem:

$$\begin{aligned}\left(-2-2i\right)^3 &= \left[2\sqrt{2}\left(\cos 225° + i\sin 225°\right)\right]^3 \\ &= \left(2\sqrt{2}\right)^3\left[\cos\left(3\cdot 225°\right) + i\sin\left(3\cdot 225°\right)\right] \\ &= 16\sqrt{2}\left(\cos 675° + i\sin 675°\right) \\ &= 16\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) \\ &= 16 - 16i\end{aligned}$$

**40**. Write $1-i$ in trigonometric form: $1-i = \sqrt{2}\left(\cos 315° + i\sin 315°\right)$

Therefore: $\left(1-i\right)^{-1} = \left[\sqrt{2}\left(\cos 315° + i\sin 315°\right)\right]^{-1} = \frac{1}{\sqrt{2}}\left[\cos\left(-315°\right) + i\sin\left(-315°\right)\right] = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \frac{1}{2} + \frac{1}{2}i$

**42**. Write $\sqrt{3}+i$ in trigonometric form: $\sqrt{3}+i = 2\left(\cos 30° + i\sin 30°\right)$

Therefore: $\left(\sqrt{3}+i\right)^{-1} = \left[2\left(\cos 30° + i\sin 30°\right)\right]^{-1} = \frac{1}{2}\left[\cos\left(-30°\right) + i\sin\left(-30°\right)\right] = \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{\sqrt{3}}{4} - \frac{1}{4}i$

**44**. Dividing in trigonometric form: $\frac{30\left(\cos 80° + i\sin 80°\right)}{10\left(\cos 15° + i\sin 15°\right)} = \frac{30}{10}\left[\cos\left(80° - 15°\right) + i\sin\left(80° - 15°\right)\right] = 3\left(\cos 65° + i\sin 65°\right)$

**46**. Dividing in trigonometric form:

$$\frac{21\left(\cos 163° + i\sin 163°\right)}{14\left(\cos 44° + i\sin 44°\right)} = \frac{21}{14}\left[\cos\left(163° - 44°\right) + i\sin\left(163° - 44°\right)\right] = 1.5\left(\cos 119° + i\sin 119°\right)$$

**48**. Dividing in trigonometric form:

$$\frac{6\text{ cis}\frac{2\pi}{3}}{8\text{ cis}\frac{\pi}{2}} = \frac{6\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)}{8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)} = \frac{6}{8}\left[\cos\left(\frac{2\pi}{3} - \frac{\pi}{2}\right) + i\sin\left(\frac{2\pi}{3} - \frac{\pi}{2}\right)\right] = \frac{3}{4}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 0.75\text{ cis}\frac{\pi}{6}$$

**50**. Finding the quotient in standard form: $\frac{z_1}{z_2} = \frac{2-2i}{1-i} = \frac{2(1-i)}{1-i} = 2$

Writing each number in trigonometric form:

$z_1 = 2-2i = 2\sqrt{2}\left(\cos 315° + i\sin 315°\right)$ $\qquad$ $z_2 = 1-i = \sqrt{2}\left(\cos 315° + i\sin 315°\right)$

Dividing in trigonometric form: $\frac{z_1}{z_2} = \frac{2\sqrt{2}\left(\cos 315° + i\sin 315°\right)}{\sqrt{2}\left(\cos 315° + i\sin 315°\right)} = 2\left(\cos 0° + i\sin 0°\right) = 2\left(1+0i\right) = 2$

Note that the quotients are equal.

**52**. Finding the quotient in standard form: $\frac{z_1}{z_2} = \frac{1+i\sqrt{3}}{2i}\cdot\frac{i}{i} = \frac{i+i^2\sqrt{3}}{2i^2} = \frac{i-\sqrt{3}}{-2} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

Writing each number in trigonometric form:

$z_1 = 1+i\sqrt{3} = 2\left(\cos 60° + i\sin 60°\right)$ $\qquad$ $z_2 = 2i = 2\left(\cos 90° + i\sin 90°\right)$

Dividing in trigonometric form: $\frac{z_1}{z_2} = \frac{2\left(\cos 60° + i\sin 60°\right)}{2\left(\cos 90° + i\sin 90°\right)} = 1\left(\cos\left(-30°\right) + i\sin\left(-30°\right)\right) = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

Note that the quotients are equal.



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**54**. Finding the quotient in standard form: $\dfrac{z_1}{z_2}=\dfrac{6+6i}{-3-3i}=\dfrac{6(1+i)}{-3(1+i)}=\dfrac{6}{-3}=-2$

Writing each number in trigonometric form:

$z_1=6+6i=6\sqrt{2}\left(\cos 45°+i\sin 45°\right)$ $\qquad$ $z_2=-3-3i=3\sqrt{2}\left(\cos 225°+i\sin 225°\right)$

Dividing in trigonometric form: $\dfrac{z_1}{z_2}=\dfrac{6\sqrt{2}\left(\cos 45°+i\sin 45°\right)}{3\sqrt{2}\left(\cos 225°+i\sin 225°\right)}=2\left(\cos\left(-180°\right)+i\sin\left(-180°\right)\right)=2\left(-1+0i\right)=-2$

Note that the quotients are equal.

**56**. Finding the quotient in standard form: $\dfrac{z_1}{z_2}=\dfrac{-6}{3}=-2$

Writing each number in trigonometric form:

$z_1=-6=6\left(\cos 180°+i\sin 180°\right)$ $\qquad$ $z_2=3=3\left(\cos 0°+i\sin 0°\right)$

Dividing in trigonometric form: $\dfrac{z_1}{z_2}=\dfrac{6\left(\cos 180°+i\sin 180°\right)}{3\left(\cos 0°+i\sin 0°\right)}=2\left(\cos 180°+i\sin 180°\right)=2\left(-1+0i\right)=-2$

Note that the quotients are equal.

**58**. First write each complex number in trigonometric form:

$\sqrt{3}+i=2\left(\cos 30°+i\sin 30°\right)$

$2i=2\left(\cos 90°+i\sin 90°\right)$

$1+i=\sqrt{2}\left(\cos 45°+i\sin 45°\right)$

Now simplify the expression:

$$\frac{\left(\sqrt{3}+i\right)^4\left(2i\right)^5}{\left(1+i\right)^{10}}=\frac{\left[2\left(\cos 30°+i\sin 30°\right)\right]^4\left[2\left(\cos 90°+i\sin 90°\right)\right]^5}{\left[\sqrt{2}\left(\cos 45°+i\sin 45°\right)\right]^{10}}$$

$$=\frac{16\left(\cos 120°+i\sin 120°\right)\cdot 32\left(\cos 450°+i\sin 450°\right)}{32\left(\cos 450°+i\sin 450°\right)}$$

$$=\frac{512\left(\cos 570°+i\sin 570°\right)}{32\left(\cos 450°+i\sin 450°\right)}$$

$$=16\left(\cos 120°+i\sin 120°\right)$$

$$=16\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$=-8+8i\sqrt{3}$$



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**60**. First write each complex number in trigonometric form:

$$1+i\sqrt{3}=2\left(\cos 60°+i\sin 60°\right)$$

$$\sqrt{3}-i=2\left(\cos 330°+i\sin 330°\right)$$

$$1-i\sqrt{3}=2\left(\cos 300°+i\sin 300°\right)$$

Now simplify the expression:

$$\frac{\left(1+i\sqrt{3}\right)^4\left(\sqrt{3}-i\right)^2}{\left(1-i\sqrt{3}\right)^3}=\frac{\left[2\left(\cos 60°+i\sin 60°\right)\right]^4\left[2\left(\cos 330°+i\sin 330°\right)\right]^2}{\left[2\left(\cos 300°+i\sin 300°\right)\right]^3}$$

$$=\frac{16\left(\cos 240°+i\sin 240°\right)\cdot 4\left(\cos 660°+i\sin 660°\right)}{8\left(\cos 900°+i\sin 900°\right)}$$

$$=\frac{64\left(\cos 900°+i\sin 900°\right)}{8\left(\cos 900°+i\sin 900°\right)}$$

$$=8\left(\cos 0°+i\sin 0°\right)$$

$$=8\left(1+0i\right)$$

$$=8$$

**62**. Substituting into the equation:

$$x^2-2x+4=\left[2\left(\cos 300°+i\sin 300°\right)\right]^2-2\cdot 2\left(\cos 300°+i\sin 300°\right)+4$$

$$=4\left(\cos 600°+i\sin 600°\right)-4\left(\cos 300°+i\sin 300°\right)+4$$

$$=4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)-4\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)+4$$

$$=-2-2i\sqrt{3}-2+2i\sqrt{3}+4$$

$$=0$$

Thus $x=2\left(\cos 300°+i\sin 300°\right)$ is a solution to $x^2-2x+4=0$.

**64**. To show $x$ is a cube root of –1, we must show $x^3=-1$. First write the number in trigonometric form:

$$x=\frac{1}{2}+\frac{\sqrt{3}}{2}i=\cos 60°+i\sin 60°$$

Therefore: $x^3=\left(\cos 60°+i\sin 60°\right)^3=\cos 180°+i\sin 180°=-1+0i=-1$

So $x=\frac{1}{2}+\frac{\sqrt{3}}{2}i$ is a cube root of –1.

**66**. Since $90°<A<180°$, $\sin A>0$ and thus: $\sin A=\sqrt{1-\cos^2 A}=\sqrt{1-\left(-\frac{1}{3}\right)^2}=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$

Using the double-angle formula for sine: $\sin 2A=2\sin A\cos A=2\left(\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{3}\right)=-\frac{4\sqrt{2}}{9}$

**68**. Since $90°<A<180°$, $45°<\frac{A}{2}<90°$, thus $\cos\frac{A}{2}>0$. Using the half-angle formula for cosine:

$$\cos\frac{A}{2}=\sqrt{\frac{1+\cos A}{2}}=\sqrt{\frac{1-\frac{1}{3}}{2}}=\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$



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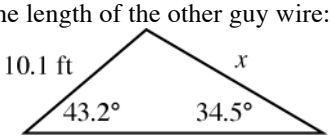
**70**. Drawing the figure, where $x$ represents the length of the other guy wire:



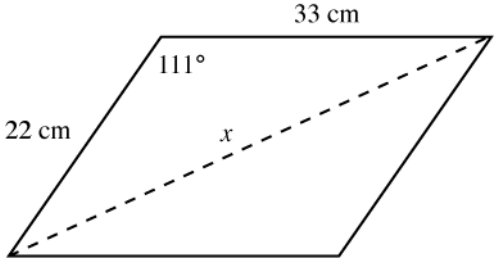

Using the law of sines:

$$\frac{x}{\sin 43.2^\circ} = \frac{10.1}{\sin 34.5^\circ}$$

$$x = \frac{10.1 \sin 43.2^\circ}{\sin 34.5^\circ} \approx 12.2 \text{ ft}$$

The other guy wire is approximately 12.2 ft long.

**72**. Drawing the figure, where $x$ represents the length of the longer diagonal:




Using the law of cosines:

$$x^2 = 22^2 + 33^2 - 2(22)(33)\cos 111^\circ \approx 2093$$

$$x = \sqrt{2093} \approx 46 \text{ cm}$$

The longer diagonal is approximately 46 cm long.

**74**. Dividing in trigonometric form:

$$\frac{8(\cos 120^\circ + i \sin 120^\circ)}{2(\cos 40^\circ + i \sin 40^\circ)} = \frac{8}{2}\left[\cos(120^\circ - 40^\circ) + i \sin(120^\circ - 40^\circ)\right] = 4(\cos 80^\circ + i \sin 80^\circ)$$

The correct answer is b.

**76**. First write each complex number in trigonometric form:

$$1 - i = \sqrt{2}(\cos 315^\circ + i \sin 315^\circ)$$

$$-\sqrt{3} - i = 2(\cos 210^\circ + i \sin 210^\circ)$$

$$1 + i\sqrt{3} = 2(\cos 60^\circ + i \sin 60^\circ)$$

Now simplify the expression:

$$\frac{(1-i)^4(-\sqrt{3}-i)^2}{(1+i\sqrt{3})^5} = \frac{\left[\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)\right]^4 \left[2(\cos 210^\circ + i \sin 210^\circ)\right]^2}{\left[2(\cos 60^\circ + i \sin 60^\circ)\right]^5}$$

$$= \frac{4(\cos 1260^\circ + i \sin 1260^\circ) \cdot 4(\cos 420^\circ + i \sin 420^\circ)}{32(\cos 300^\circ + i \sin 300^\circ)}$$

$$= \frac{16(\cos 1680^\circ + i \sin 1680^\circ)}{32(\cos 300^\circ + i \sin 300^\circ)}$$

$$= \frac{1}{2}(\cos 1380^\circ + i \sin 1380^\circ)$$

$$= \frac{1}{2}(\cos 300^\circ + i \sin 300^\circ)$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= \frac{1}{4} - \frac{\sqrt{3}}{4}i$$

The correct answer is d.



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**ODD SOLUTIONS**

**1.** multiply, add

**3.** powers

**5.** $5(\cos 15^\circ + i\sin 15^\circ)\cdot 2(\cos 25^\circ + i\sin 25^\circ) = 5\cdot 2\left[\cos(15^\circ + 25^\circ) + i\sin(15^\circ + 25^\circ)\right]$

$= 10(\cos 40^\circ + i\sin 40^\circ)$

**7.** $9(\cos 115^\circ + i\sin 115^\circ)\cdot 4(\cos 51^\circ + i\sin 51^\circ) = 9\cdot 4\left[\cos(115^\circ + 51^\circ) + i\sin(115^\circ + 51^\circ)\right]$

$= 36(\cos 166^\circ + i\sin 166^\circ)$

**9.** $2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\cdot 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 2\cdot 2\left[\cos\left(\frac{3\pi}{4} + \frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{4} + \frac{\pi}{4}\right)\right]$

$= 4(\cos\pi + i\sin\pi) = 4\text{cis}\pi$

**11.** $z_1 z_2 = (1+i)(-1+i)$

$= -1 + i - i + i^2$

$= -1 - 1 = -2$

$z_1 = 1 + i$ where $x = 1$, $y = 1$, and $r = \sqrt{1^2 + 1^2} = \sqrt{2}$

Also $\tan\theta = 1$ and $\theta$ is in QI. Therefore, $\theta = 45^\circ$.

$z_1 = \sqrt{2}\text{cis}\left(45^\circ\right)$ in trigonometric form

$z_2 = -1 + i$ where $x = -1$, $y = 1$, and $r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

Also $\tan\theta = -1$ and $\theta$ is in QII. Therefore, $\theta = 135^\circ$.

$z_2 = \sqrt{2}\text{cis}(135^\circ)$ in trigonometric form

$z_1 z_2 = \sqrt{2}\text{cis}(45^\circ)\cdot\sqrt{2}\text{cis}(135^\circ)$

$= \sqrt{2}\cdot\sqrt{2}\text{cis}(45^\circ + 135^\circ)$

$= 2\text{cis}(180^\circ) = 2(-1 + i\cdot 0) = -2$

**13.** $z_1 z_2 = (1 + i\sqrt{3})(-\sqrt{3} + i)$

$= -\sqrt{3} - 2i + i^2\sqrt{3}$

$= -\sqrt{3} - 2i - \sqrt{3} = -2\sqrt{3} - 2i$

$z_1 = 1 + i\sqrt{3}$ where $x = 1$, $y = \sqrt{3}$, and $r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$

Also $\tan\theta = \sqrt{3}$ and $\theta$ is in QI. Therefore, $\theta = 60^\circ$.

$z_1 = 2\text{cis}(60^\circ)$ in trigonometric form

$z_2 = -\sqrt{3} + i$ where $x = -\sqrt{3}$, $y = 1$, and

$r = \sqrt{\left(-\sqrt{3}\right)^2 + 1^2} = 2$

Also $\tan\theta = \frac{-1}{\sqrt{3}}$ and $\theta$ is in QII. Therefore, $\theta = 150^\circ$.

$z_2 = 2\text{cis}(150^\circ)$ in trigonometric form

$z_1 z_2 = 2\text{cis}(60^\circ)\cdot 2\text{cis}(150^\circ)$

$= 2\cdot 2\text{cis}(60^\circ + 150^\circ)$

$= 4\text{cis}(210^\circ) = 4\left(-\frac{\sqrt{3}}{2} - i\cdot\frac{1}{2}\right) = -2\sqrt{3} - 2i$

**15.** $z_1 z_2 = (3i)(-4i)$

$= -12i^2$

$= -12(-1) = 12$

$z_1 = 3i$ where $x = 0$, $y = 3$, and $r = 3$, and $\theta = 90^\circ$

$z_1 = 3\text{cis}\left(90^\circ\right)$ in trigonometric form

$z_2 = -4i$ where $x = 0$, $y = -4$, $r = 4$ and $\theta = 270^\circ$

$z_2 = 4\text{cis}(270^\circ)$ in trigonometric form

$z_1 z_2 = 3\text{cis}(90^\circ)\cdot 4\text{cis}(270^\circ)$

$= 3\cdot 4\text{cis}(90^\circ + 270^\circ)$

$= 12\text{cis}(360^\circ) = 12(1 + i\cdot 0) = 12$



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**17.** $z_1 z_2 = (1+i)(4i)$

$= 4i + 4i^2$

$= 4i + 4(-1) = -4 + 4i$

$z_1 = 1+i$ where $x = 1$, $y = 1$, and $r = \sqrt{1^2+1^2} = \sqrt{2}$.

Also $\tan\theta = 1$ and $\theta$ is in QI. Therefore, $\theta = 45^\circ$.

$z_1 = \sqrt{2}\text{cis}(45^\circ)$ in trigonometric form

$z_2 = 4i$ where $x = 0$, $y = 4$, $r = 4$ and $\theta = 90^\circ$

$z_2 = 4\text{cis}(90^\circ)$ in trigonometric form

$z_1 z_2 = \sqrt{2}\text{cis}(45^\circ)\cdot 4\text{cis}(90^\circ)$

$= \sqrt{2}\cdot 4\text{cis}(45^\circ + 90^\circ)$

$= 4\sqrt{2}\text{cis}(135^\circ) = 4\sqrt{2}\left(\frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -4 + 4i$

**19.** $z_1 z_2 = -5(1+i\sqrt{3})$

$= -5 - 5i\sqrt{3}$

$z_1 = -5$ where $x = -5$, $y = 0$, $\theta = 180^\circ$ and $r = 5$.

$z_1 = 5\text{cis}(180^\circ)$ in trigonometric form

$z_2 = 1 + i\sqrt{3}$ where $x = 1$, $y = \sqrt{3}$ and $r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = \sqrt{4} = 2$

Also $\tan\theta = \sqrt{3}$ and $\theta$ is in QI. Therefore, $\theta = 60^\circ$.

$z_2 = 2\text{cis}(60^\circ)$ in trigonometric form

$z_1 z_2 = 5\text{cis}(180^\circ)\cdot 2\text{cis}(60^\circ)$

$= 5\cdot 2\text{cis}(180^\circ + 60^\circ)$

$= 10\text{cis}(240^\circ) = 10\left(\frac{-1}{2} + i\frac{-\sqrt{3}}{2}\right) = -5 - 5i\sqrt{3}$

**21.** $2i = 2(\cos 90^\circ + i\sin 90^\circ)$ and $3i = 3(\cos 90^\circ + i\sin 90^\circ)$ (from 8.2)

$2i\cdot 3i = \left[2(\cos 90^\circ + i\sin 90^\circ)\right]\left[3(\cos 90^\circ + i\sin 90^\circ)\right]$

$= 2\cdot 3\left(\cos^2 90^\circ + 2i\cos 90^\circ \sin 90^\circ + i^2 \sin^2 90^\circ\right)$

$= 6\left[0 + 2i(0)(1) + (-1)(1)^2\right]$

$= 6(-1) = -6$

**23.** $\left[2(\cos 10^\circ + i\sin 10^\circ)\right]^6 = 2^6\left[\cos(6\cdot 10^\circ) + i\sin(6\cdot 10^\circ)\right]$

$= 64(\cos 60^\circ + i\sin 60^\circ)$

$= 64\left(\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right) = 32 + 32i\sqrt{3}$

**25.** $\left[3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^4 = 3^4\left[\cos\left(4\cdot\frac{\pi}{3}\right) + i\sin\left(4\cdot\frac{\pi}{3}\right)\right]$

$= 81\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$= 81\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -\frac{81}{2} - \frac{81\sqrt{3}}{2}i$

**27.** $\left(\text{cis}12^\circ\right)^{10} = 1^{10}\left[\text{cis}(10\cdot 12^\circ)\right]$

$= 1(\cos 120^\circ + i\sin 120^\circ) = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$



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**29.** $\left(\sqrt{2}\,\text{cis}\,\frac{7\pi}{18}\right)^6 = \left(\sqrt{2}\right)^6\left[\text{cis}\left(6\cdot\frac{7\pi}{18}\right)\right]$

$= 8\left(\cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3}\right) = 8\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \qquad \left(\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}\right)$

$= 8\left(\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right) = 4 + 4i\sqrt{3}$

**31.** In problem 11, we found that $1 + i$ was $\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$ in trigonometric form.

$z^4 = \left(\sqrt{2}\right)^4\left[\cos(4\cdot 45^\circ) + i\sin(4\cdot 45^\circ)\right]$

$= 4(\cos 180^\circ + i\sin 180^\circ) = 4\left[-1 + i(0)\right] = -4$

**33.** In problem 13, we found that $-\sqrt{3} + i$ was $2(\cos 150^\circ + i\sin 150^\circ)$ in trigonometric form.

$z^4 = (2)^4\left[\cos(4\cdot 150^\circ) + i\sin(4\cdot 150^\circ)\right]$

$= 16(\cos 600^\circ + i\sin 600^\circ)$

$= 16\left[-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right] = -8 - 8i\sqrt{3}$

**35.** In problem 11, we found that $1 + i$ was $\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$ in trigonometric form.

$z^6 = \left(\sqrt{2}\right)^6\left[\cos(6\cdot 45^\circ) + i\sin(6\cdot 45^\circ)\right]$

$= 8(\cos 270^\circ + i\sin 270^\circ)$

$= 8\left[0 + i(-1)\right] = -8i$

**37.** First we write $-2 + 2i$ in trigonometric form: $x = -2$, $y = 2$, and $r = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

Also $\tan\theta = \frac{-2}{2} = -1$ and $\theta$ is in QII. Therefore, $\theta = 135^\circ$ and $z = 2\sqrt{2}\,\text{cis}(135^\circ)$.

$z^3 = \left(2\sqrt{2}\right)^3\left[\text{cis}(3\cdot 135^\circ)\right]$

$= 16\sqrt{2}\left(\cos 405^\circ + i\sin 405^\circ\right)$

$= 16\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\cdot\frac{\sqrt{2}}{2}\right) = 16 + 16i$

**39.** $1 + i = \sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$ in trigonometric form

$(1+i)^{-1} = \left(\sqrt{2}\right)^{-1}\left[\cos(-1\cdot 45^\circ) + i\sin(-1\cdot 45^\circ)\right]$

$= \frac{1}{\sqrt{2}}\left[\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{1}{2} - \frac{1}{2}i$

**41.** $z = \sqrt{3} - i = 2(\cos 330^\circ + i\sin 330^\circ)$ in trigonometric form

$z^{-1} = 2^{-1}\left[\cos(-1\cdot 330^\circ) + i\sin(-1\cdot 330^\circ)\right]$

$= \frac{1}{2}\left[\cos(-330^\circ) + i\sin(-330^\circ)\right]$

$= \frac{1}{2}\left[\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right] = \frac{\sqrt{3}}{4} + \frac{1}{4}i$

**43.** $\frac{20\left(\cos 75^\circ + i\sin 75^\circ\right)}{5\left(\cos 40^\circ i\sin 40^\circ\right)} = \frac{20}{5}\left[\cos(75^\circ - 40^\circ) + i\sin(75^\circ - 40^\circ)\right]$

$= 4(\cos 35^\circ + i\sin 35^\circ)$



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**45.** $\dfrac{18\left(\cos 51^\circ + i\sin 51^\circ\right)}{12\left(\cos 32^\circ i\sin 32^\circ\right)} = \dfrac{18}{12}\left[\cos(51^\circ - 32^\circ) + i\sin(51^\circ - 32^\circ)\right]$

$= 1.5(\cos 19^\circ + i\sin 19^\circ)$

**47.** $\dfrac{4\text{cis}\dfrac{\pi}{2}}{8\text{cis}\dfrac{\pi}{6}} = \dfrac{4}{8}\text{cis}\left(\dfrac{\pi}{2} - \dfrac{\pi}{6}\right)$

$= 0.5\text{cis}\dfrac{\pi}{3}$

**49.** $\dfrac{z_1}{z_2} = \dfrac{2+2i}{1+i}\cdot\dfrac{1-i}{1-i}$

$= \dfrac{2-2i^2}{1-i^2} = \dfrac{2+2}{1+1} = \dfrac{4}{2} = 2$

$z_1 = 2+2i$ where $x = 2, y = 2,$ and $r = \sqrt{2^2+2^2} = 2\sqrt{2}.$

Also, $\tan\theta = \dfrac{2}{2} = 1$ and $\theta$ is in QI. Therefore, $\theta = 45^\circ$ and $z_1 = 2\sqrt{2}\text{cis}45^\circ$.

In problem 7, we found that $z_2 = \sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$ in trigonometric form.

$\dfrac{z_1}{z_2} = \dfrac{2\sqrt{2}}{\sqrt{2}}\text{cis}(45^\circ - 45^\circ)$

$= 2\text{cis}0^\circ = 2(1 + i\cdot 0) = 2$

**51.** $\dfrac{z_1}{z_2} = \dfrac{\sqrt{3}+i}{2i}\cdot\dfrac{i}{i}$

$= \dfrac{i\sqrt{3}+i^2}{2i^2} = \dfrac{-1+i\sqrt{3}}{-2} = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

$z_1 = \sqrt{3}+i$ where $x = \sqrt{3}, y = 1,$ and $r = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2.$

Also, $\tan\theta = \dfrac{1}{\sqrt{3}}$ and $\theta$ is in QI. Therefore, $\theta = 30^\circ$ and $z_1 = 2\text{cis}30^\circ$.

$z_2 = 2i$ where $x = 0, y = 2, r = 2,$ and $\theta = 90^\circ$. In trigonometric form, $z_2 = 2\text{cis}90^\circ$.

$\dfrac{z_1}{z_2} = \dfrac{2\,cis30^\circ}{2\,cis90^\circ} = \dfrac{2}{2}\text{cis}(30^\circ - 90^\circ)$

$= \text{cis}(-60^\circ) = \dfrac{1}{2} + i\left(-\dfrac{\sqrt{3}}{2}\right) = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

**53.** $\dfrac{z_1}{z_2} = \dfrac{4+4i}{2-2i}\cdot\dfrac{2+2i}{2+2i} = \dfrac{8+16i+8i^2}{4-4i^2} = \dfrac{8+16i-8}{4+4} = \dfrac{16i}{8} = 2i$

$z_1 = 4+4i$ where $x = 4, y = 4,$ and $r = \sqrt{4^2+4^2} = \sqrt{32} = 4\sqrt{2}.$

Also, $\tan\theta = \dfrac{4}{4} = 1$ and $\theta$ is in QI. Therefore, $\theta = 45^\circ$ and $z_1 = 4\sqrt{2}\text{cis}45^\circ$.

$z_2 = 2-2i$ where $x = 2, y = -2,$ and $r = \sqrt{2^2+(-2)^2} = \sqrt{8} = 2\sqrt{2}$ .

Also, $\tan\theta = \dfrac{2}{-2} = -1$ and $\theta$ is in QIV. Therefore, $\theta = 315^\circ$ and $z_2 = 2\sqrt{2}\text{cis}315^\circ$.

$\dfrac{z_1}{z_2} = \dfrac{4\sqrt{2}\,cis45^\circ}{2\sqrt{2}\,cis315^\circ} = \dfrac{4\sqrt{2}}{2\sqrt{2}}\text{cis}(45^\circ - 315^\circ)$

$= 2\text{cis}(-270^\circ) = 2(\cos 90^\circ + i\sin 90^\circ) = 2(0 + i\cdot 1) = 2i$



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**55.** $\dfrac{z_1}{z_2} = \dfrac{8}{-4} = -2$

$z_1 = 8$ where $x = 8, y = 0, r = 8,$ and $\theta = 0^\circ$. In trigonometric form, $z_1 = 8\text{cis}0^\circ$.

$z_2 = -4$ where $x = -4, y = 0, r = 4,$ and $\theta = 180^\circ$. In trigonometric form, $z_1 = 4\text{cis}180^\circ$.

$$\frac{z_1}{z_2}\ \frac{8\ cis\ 0^\circ}{4\ cis\ 180^\circ} = \frac{8}{4}\text{cis}(0^\circ - 180^\circ) = 2\text{cis}(-180^\circ) = 2(\cos 180^\circ + i\sin 180^\circ) = 2(-1 + i\cdot 0) = -2$$

**57.** In problem 11, we found that $z_1 = 1 + i$ was $\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$ in trigonometric form.

In problem 51, we found that $z_2 = 2i$ was $2(\cos 90^\circ + i\sin 90^\circ)$ in trigonometric form.

In problem 37, we found that $z_3 = -2 + 2i$ was $2\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$ in trigonometric form.

$$\frac{(z_1)^4(z_2)^2}{z_3} = \frac{\left[\left(\sqrt{2}\right)^4 \text{cis}(4\cdot 45^\circ)\right]\left[(2)^2\text{cis}(2\cdot 90^\circ)\right]}{2\sqrt{2}\,\text{cis}\left(135^\circ\right)}$$

$$= \frac{4\cdot 4\left(\text{cis}180^\circ\right)\left(\text{cis}180^\circ\right)}{2\sqrt{2}\text{cis}135^\circ}$$

$$= \frac{16\text{cis}360^\circ}{2\sqrt{2}\text{cis}135^\circ}$$

$$= \frac{8}{\sqrt{2}}\left[\text{cis}(360^\circ - 135^\circ)\right] = \frac{8}{\sqrt{2}}\text{cis}225^\circ$$

$$= \frac{8}{\sqrt{2}}\left(-\frac{\sqrt{2}}{2} + i\cdot -\frac{\sqrt{2}}{2}\right) = -4 - 4i$$

**59.** First, we find the trigonometric form of each of the complex numbers:

$z_1 = 2 + 2i = 2\sqrt{2}\left(\cos 45^\circ + i\sin 45^\circ\right)$, $z_2 = -3 + 3i = 3\sqrt{2}\left(\cos 135^\circ + i\sin 135^\circ\right)$, and $z_3 = \sqrt{3} + i = 2\left(\cos 30^\circ + i\sin 30^\circ\right)$

$$\frac{(z_1)^5(z_2)^3}{(z_3)^{10}} = \frac{\left[\left(2\sqrt{2}\right)^5 \text{cis}(5\cdot 45^\circ)\right]\left[\left(3\sqrt{2}\right)^3\text{cis}(3\cdot 135^\circ)\right]}{2^{10}\,\text{cis}\left(10\cdot 30^\circ\right)}$$

$$= \frac{128\sqrt{2}\cdot 54\sqrt{2}\left(\text{cis}225^\circ\right)\left(\text{cis}405^\circ\right)}{1024\text{cis}300^\circ}$$

$$= \frac{13{,}824\text{cis}(225^\circ + 405^\circ)}{1024\text{cis}300^\circ}$$

$$= \frac{27}{2}\text{cis}330^\circ$$

$$\frac{27}{2}\left(\cos 330^\circ + i\sin 330^\circ\right) = \frac{27}{2}\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = \frac{27\sqrt{3}}{4} - \frac{27}{4}i$$

**61.** $x^2 - 2x + 4 = (2\text{cis}60^\circ)^2 - 2(2\text{cis}60^\circ) + 4$

$$= 4\text{cis}(120^\circ) - 4\text{cis}(60^\circ) + 4$$

$$= 4\left(-\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right) - 4\left(\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right) + 4$$

$$= -2 + 2i\sqrt{3} - 2 - 2i\sqrt{3} + 4 = 0$$



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**63.** $w^4 = \left[2(\cos 15^\circ + i\sin 15^\circ)\right]^4$

$= 2^4\left[\cos(4\cdot 15^\circ) + i\sin(4\cdot 15^\circ)\right]$

$= 16\left[\cos(60^\circ) + i\sin(60^\circ)\right]$

$= 16\left(\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right) = 8 + 8i\sqrt{3}$

**69.** $\cos 2A = 2\cos^2 A - 1$

$= 2\left(-\frac{1}{3}\right)^2 - 1$

$= \frac{2}{9} - 1 = -\frac{7}{9}$

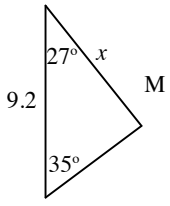
**71.** $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$

$= \sqrt{\frac{1-(-1/3)}{2}}$

$= \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$

**73.** $\angle M = 180^\circ - (27^\circ + 35^\circ) = 118^\circ$

$x = \frac{9.2\sin 35^\circ}{\sin 118^\circ} = 6.0$ miles



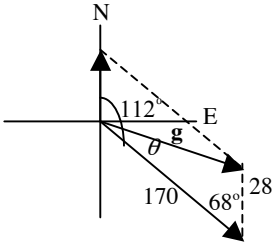

**75.** $|\mathbf{g}| = \sqrt{170^2 + 28^2 - 2(170)(28)\cos 68^\circ}$

$= \sqrt{26{,}118} = 160$ mph (to 2 significant digits)

$\sin\theta = \frac{28\sin 68^\circ}{160} = 0.1623$

$\theta = 9^\circ$ (to the nearest degree)

The true course is $112^\circ - \theta = 112^\circ - 9^\circ = 103^\circ$. The ground speed is 160 miles per hour.



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## 8.4 Roots of a Complex Number

**EVEN SOLUTIONS**

**2**. To find the principle *n*th root of a complex number, find the *n*th root of the modulus and divide the argument by *n*. To find the remaining *n*th roots, add $\frac{360°}{n}$ or $\frac{2\pi}{n}$ to the argument repeatedly until you have them all.

**4**. When graphed, all of the *n*th roots of a complex number will be evenly distributed around a circle of radius $r^{1/n}$.

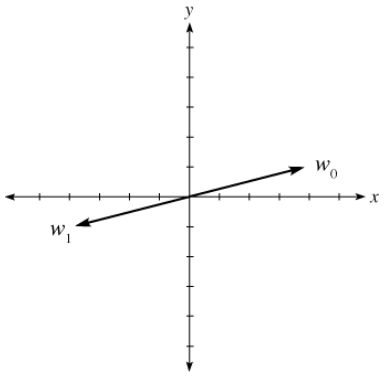
**6**. The square roots are given by:

$$w_k = 16^{1/2}\left[\cos\frac{30°+360°k}{2} + i\sin\frac{30°+360°k}{2}\right] = 4\left[\cos(15°+180°k) + i\sin(15°+180°k)\right]$$

Substituting $k = 0,1$:

$w_0 = 4(\cos 15° + i\sin 15°)$ $\qquad$ $w_1 = 4(\cos 195° + i\sin 195°)$

Graphing these solutions:



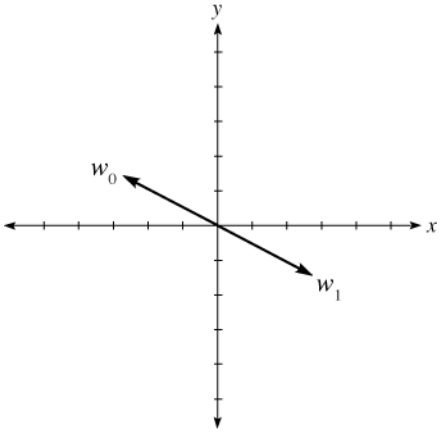

**8**. The square roots are given by:

$$w_k = 9^{1/2}\left[\cos\frac{310°+360°k}{2} + i\sin\frac{310°+360°k}{2}\right] = 3\left[\cos(155°+180°k) + i\sin(155°+180°k)\right]$$

Substituting $k = 0,1$:

$w_0 = 3(\cos 155° + i\sin 155°)$ $\qquad$ $w_1 = 3(\cos 335° + i\sin 335°)$

Graphing these solutions:



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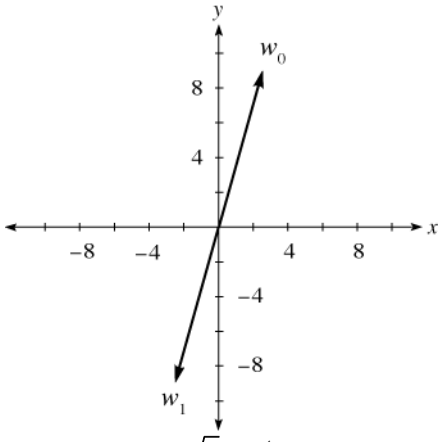
**10**. The square roots are given by:

$$w_k = 81^{1/2}\left[\cos\frac{\frac{5\pi}{6}+2\pi k}{2}+i\sin\frac{\frac{5\pi}{6}+2\pi k}{2}\right]=9\left[\cos\left(\frac{5\pi}{12}+\pi k\right)+i\sin\left(\frac{5\pi}{12}+\pi k\right)\right]$$

Substituting $k = 0,1$:

$$w_0 = 9\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)=9\ \text{cis}\frac{5\pi}{12} \qquad w_1 = 9\left(\cos\frac{17\pi}{12}+i\sin\frac{17\pi}{12}\right)=9\ \text{cis}\frac{17\pi}{12}$$

Graphing these solutions:




**12**. First write the number in trigonometric form: $-2+2i\sqrt{3}=4\left(\cos120°+i\sin120°\right)$

The square roots are given by:

$$w_k = 4^{1/2}\left[\cos\frac{120°+360°k}{2}+i\sin\frac{120°+360°k}{2}\right]=2\left[\cos\left(60°+180°k\right)+i\sin\left(60°+180°k\right)\right]$$

Substituting $k = 0,1$:

$$w_0 = 2\left(\cos60°+i\sin60°\right)=2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=1+i\sqrt{3}$$

$$w_1 = 2\left(\cos240°+i\sin240°\right)=2\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=-1-i\sqrt{3}$$

**14**. First write the number in trigonometric form: $-36i=36\left(\cos270°+i\sin270°\right)$

The square roots are given by:

$$w_k = 36^{1/2}\left[\cos\frac{270°+360°k}{2}+i\sin\frac{270°+360°k}{2}\right]=6\left[\cos\left(135°+180°k\right)+i\sin\left(135°+180°k\right)\right]$$

Substituting $k = 0,1$:

$$w_0 = 6\left(\cos135°+i\sin135°\right)=6\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)=-3\sqrt{2}+3i\sqrt{2}$$

$$w_1 = 6\left(\cos315°+i\sin315°\right)=6\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)=3\sqrt{2}-3i\sqrt{2}$$

**16**. First write the number in trigonometric form: $9=9\left(\cos0°+i\sin0°\right)$

The square roots are given by: $w_k = 9^{1/2}\left[\cos\frac{0°+360°k}{2}+i\sin\frac{0°+360°k}{2}\right]=3\left[\cos\left(180°k\right)+i\sin\left(180°k\right)\right]$

Substituting $k = 0,1$:

$$w_0 = 3\left(\cos0°+i\sin0°\right)=3\left(1+0i\right)=3 \qquad w_1 = 3\left(\cos180°+i\sin180°\right)=3\left(-1+0i\right)=-3$$



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**18**. First write the number in trigonometric form: $1-i\sqrt{3}=2\left(\cos 300°+i\sin 300°\right)$

The square roots are given by:

$$w_k=2^{1/2}\left[\cos\frac{300°+360°k}{2}+i\sin\frac{300°+360°k}{2}\right]=\sqrt{2}\left[\cos\left(150°+180°k\right)+i\sin\left(150°+180°k\right)\right]$$

Substituting $k=0,1$:

$$w_0=\sqrt{2}\left(\cos 150°+i\sin 150°\right)=\sqrt{2}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=-\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i$$

$$w_1=\sqrt{2}\left(\cos 330°+i\sin 330°\right)=\sqrt{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)=\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}i$$

**20**. The cube roots are given by:

$$w_k=27^{1/3}\left[\cos\frac{303°+360°k}{3}+i\sin\frac{303°+360°k}{3}\right]=3\left[\cos\left(101°+120°k\right)+i\sin\left(101°+120°k\right)\right]$$

Substituting $k=0,1,2$:

$$w_0=3\left(\cos 101°+i\sin 101°\right)$$

$$w_1=3\left(\cos 221°+i\sin 221°\right)$$

$$w_2=3\left(\cos 341°+i\sin 341°\right)$$

**22**. First write the number in trigonometric form: $-4\sqrt{3}+4i=8\left(\cos 150°+i\sin 150°\right)$

The cube roots are given by: $w_k=8^{1/3}\left[\cos\frac{150°+360°k}{3}+i\sin\frac{150°+360°k}{3}\right]=2\left[\cos\left(50°+120°k\right)+i\sin\left(50°+120°k\right)\right]$

Substituting $k=0,1,2$:

$$w_0=2\left(\cos 50°+i\sin 50°\right)$$

$$w_1=2\left(\cos 170°+i\sin 170°\right)$$

$$w_2=2\left(\cos 290°+i\sin 290°\right)$$

**24**. First write the number in trigonometric form: $8=8\left(\cos 0°+i\sin 0°\right)$

The cube roots are given by: $w_k=8^{1/3}\left[\cos\frac{0°+360°k}{3}+i\sin\frac{0°+360°k}{3}\right]=2\left[\cos\left(120°k\right)+i\sin\left(120°k\right)\right]$

Substituting $k=0,1,2$:

$$w_0=2\left(\cos 0°+i\sin 0°\right)$$

$$w_1=2\left(\cos 120°+i\sin 120°\right)$$

$$w_2=2\left(\cos 240°+i\sin 240°\right)$$

**26**. First write the number in trigonometric form: $-64i=64\left(\cos 270°+i\sin 270°\right)$

The cube roots are given by:

$$w_k=64^{1/3}\left[\cos\frac{270°+360°k}{3}+i\sin\frac{270°+360°k}{3}\right]=4\left[\cos\left(90°+120°k\right)+i\sin\left(90°+120°k\right)\right]$$

Substituting $k=0,1,2$:

$$w_0=4\left(\cos 90°+i\sin 90°\right)$$

$$w_1=4\left(\cos 210°+i\sin 210°\right)$$

$$w_2=4\left(\cos 330°+i\sin 330°\right)$$



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**28**. Since $x^3 - 27 = 0, x^3 = 27$ . We wish to find the three cube roots of 27. Writing 27 in trigonometric form:

$$27 = 27(\cos 0° + i \sin 0°)$$

The cube roots are given by: $w_k = 27^{1/3}\left[\cos\frac{0° + 360°k}{3} + i\sin\frac{0° + 360°k}{3}\right] = 3\left[\cos(120°k) + i\sin(120°k)\right]$

Substituting $k = 0,1,2$:

$$w_0 = 3(\cos 0° + i\sin 0°) = 3(1 + 0i) = 3$$

$$w_1 = 3(\cos 120° + i\sin 120°) = 3\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$$

$$w_2 = 3(\cos 240° + i\sin 240°) = 3\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$$

**30**. Since $x^4 - 16 = 0, x^4 = 16$ . We wish to find the four fourth roots of 16. Writing 16 in trigonometric form:

$$16 = 16(\cos 0° + i \sin 0°)$$

The fourth roots are given by: $w_k = 16^{1/4}\left[\cos\frac{0° + 360°k}{4} + i\sin\frac{0° + 360°k}{4}\right] = 2\left[\cos(90°k) + i\sin(90°k)\right]$

Substituting $k = 0,1,2,3$:

$$w_0 = 2(\cos 0° + i\sin 0°) = 2(1 + 0i) = 2$$

$$w_1 = 2(\cos 90° + i\sin 90°) = 2(0 + 1i) = 2i$$

$$w_2 = 2(\cos 180° + i\sin 180°) = 2(-1 + 0i) = -2$$

$$w_3 = 2(\cos 270° + i\sin 270°) = 2(0 - 1i) = -2i$$

**32**. The fourth roots are given by: $w_k = 1^{1/4}\left[\cos\frac{\frac{4\pi}{3} + 2\pi k}{4} + i\sin\frac{\frac{4\pi}{3} + 2\pi k}{4}\right] = \cos\left(\frac{\pi}{3} + \frac{\pi}{2}k\right) + i\sin\left(\frac{\pi}{3} + \frac{\pi}{2}k\right)$

Substituting $k = 0,1,2,3$:

$$w_0 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$$

$$w_1 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}$$

$$w_2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

$$w_3 = \cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}$$

**34**. The fifth roots are given by:

$$w_k = \left(10^{10}\right)^{1/5}\left[\cos\frac{75° + 360°k}{5} + i\sin\frac{75° + 360°k}{5}\right] = 100\left[\cos(15° + 72°k) + i\sin(15° + 72°k)\right]$$

Substituting $k = 0,1,2,3,4$:

$$w_0 = 100(\cos 15° + i\sin 15°) \approx 96.59 + 25.88i$$

$$w_1 = 100(\cos 87° + i\sin 87°) \approx 5.23 + 99.86i$$

$$w_2 = 100(\cos 159° + i\sin 159°) \approx -93.36 + 35.84i$$

$$w_3 = 100(\cos 231° + i\sin 231°) \approx -62.93 - 77.71i$$

$$w_4 = 100(\cos 303° + i\sin 303°) \approx 54.46 - 83.87i$$



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**36**. Write 1 in trigonometric form: $1 = \cos 0° + i \sin 0°$

The sixth roots are given by: $w_k = 1^{1/6}\left[\cos\dfrac{0° + 360°k}{6} + i\sin\dfrac{0° + 360°k}{6}\right] = \cos(60°k) + i\sin(60°k)$

Substituting $k = 0,1,2,3,4,5$:

$w_0 = \cos 0° + i \sin 0°$

$w_1 = \cos 60° + i \sin 60°$

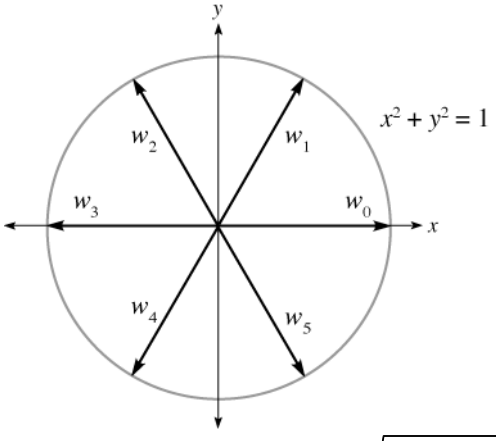
$w_2 = \cos 120° + i \sin 120°$

$w_3 = \cos 180° + i \sin 180°$

$w_4 = \cos 240° + i \sin 240°$

$w_5 = \cos 300° + i \sin 300°$

Graphing these roots:




**38**. Using the quadratic formula with $a = 1$, $b = 2$, and $c = 4$: $x^2 = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)} = \dfrac{-2 \pm \sqrt{-12}}{2} = \dfrac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$

Thus $x^2 = -1 + i\sqrt{3}$ or $x^2 = -1 - i\sqrt{3}$. Writing $-1 + i\sqrt{3}$ in trigonometric form: $-1 + i\sqrt{3} = 2(\cos 120° + i \sin 120°)$

Its square roots are given by:

$$w_k = 2^{1/2}\left[\cos\frac{120° + 360°k}{2} + i\sin\frac{120° + 360°k}{2}\right] = \sqrt{2}\left[\cos(60° + 180°k) + i\sin(60° + 180°k)\right]$$

Substituting $k = 0,1$:

$w_0 = \sqrt{2}(\cos 60° + i \sin 60°)$ $\qquad$ $w_1 = \sqrt{2}(\cos 240° + i \sin 240°)$

Writing $-1 - i\sqrt{3}$ in trigonometric form: $-1 - i\sqrt{3} = 2(\cos 240° + i \sin 240°)$

Its square roots are given by:

$$w_k = 2^{1/2}\left[\cos\frac{240° + 360°k}{2} + i\sin\frac{240° + 360°k}{2}\right] = \sqrt{2}\left[\cos(120° + 180°k) + i\sin(120° + 180°k)\right]$$

Substituting $k = 0,1$:

$w_0 = \sqrt{2}(\cos 120° + i \sin 120°)$ $\qquad$ $w_1 = \sqrt{2}(\cos 300° + i \sin 300°)$

Summarizing, the solutions are $\sqrt{2}(\cos\theta + i\sin\theta)$, where $\theta = 60°, 120°, 240°, 300°$.



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**40**. Using the quadratic formula with $a = 1$, $b = -2$, and $c = 2$: $x^2 = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \dfrac{2 \pm \sqrt{-4}}{2} = \dfrac{2 \pm 2i}{2} = 1 \pm i$

Thus $x^2 = 1 + i$ or $x^2 = 1 - i$. Writing $1 + i$ in trigonometric form: $1 + i = \sqrt{2}\left(\cos 45° + i \sin 45°\right)$

Its square roots are given by:

$$w_k = \left(\sqrt{2}\right)^{1/2}\left[\cos\frac{45° + 360°k}{2} + i\sin\frac{45° + 360°k}{2}\right] = \sqrt[4]{2}\left[\cos\left(22.5° + 180°k\right) + i\sin\left(22.5° + 180°k\right)\right]$$

Substituting $k = 0,1$:

$$w_0 = \sqrt[4]{2}\left(\cos 22.5° + i\sin 22.5°\right) \qquad w_1 = \sqrt[4]{2}\left(\cos 202.5° + i\sin 202.5°\right)$$

Writing $1 - i$ in trigonometric form: $1 - i = \sqrt{2}\left(\cos 315° + i\sin 315°\right)$

Its square roots are given by:

$$w_k = \left(\sqrt{2}\right)^{1/2}\left[\cos\frac{315° + 360°k}{2} + i\sin\frac{315° + 360°k}{2}\right] = \sqrt[4]{2}\left[\cos\left(157.5° + 180°k\right) + i\sin\left(157.5° + 180°k\right)\right]$$

Substituting $k = 0,1$:

$$w_0 = \sqrt[4]{2}\left(\cos 157.5° + i\sin 157.5°\right) \qquad w_1 = \sqrt[4]{2}\left(\cos 337.5° + i\sin 337.5°\right)$$

Summarizing, the solutions are $\sqrt[4]{2}\left(\cos\theta + i\sin\theta\right)$, where $\theta = 22.5°, 157.5°, 202.5°, 337.5°$.

**42**. Using Euler's Formula in the derivation:

$$\left(w_k\right)^n = r \text{ cis } \left(\theta + 2k\pi\right)$$

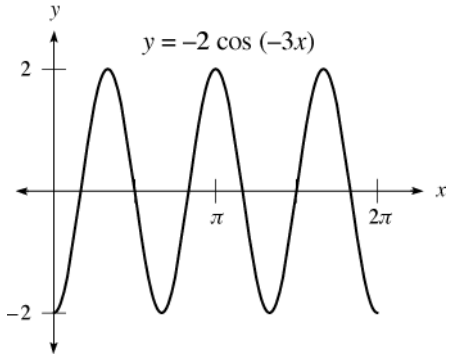
$$w_k = \left(r \text{ cis } \left(\theta + 2k\pi\right)\right)^{1/n}$$

$$w_k = r^{1/n}\left(e^{(\theta + 2k\pi)i}\right)^{1/n}$$

$$w_k = r^{1/n}\left(e^{(\theta/n + 2k\pi/n)i}\right)$$

$$w_k = r^{1/n}\text{cis}\left(\frac{\theta}{n} + \frac{2\pi}{n}k\right)$$

**44**. Note that $y = -2\cos\left(-3x\right) = -2\cos 3x$. The amplitude is 2, the period is $\dfrac{2\pi}{3}$, and the graph is a reflection of $y = 2\cos 3x$ across the $x$-axis:



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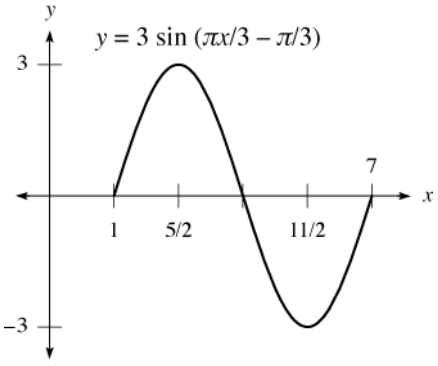
**46**. The amplitude is 3, the period is $\dfrac{2\pi}{\pi/3}=6$, and the horizontal shift is $\dfrac{\pi/3}{\pi/3}=1$:




**48**. Using the area formula: $S=\dfrac{1}{2}ac\sin B=\dfrac{1}{2}(44.4)(22.2)\sin 21.8°\approx 183 \text{ cm}^2$

**50**. First compute $s=\dfrac{1}{2}(a+b+c)=\dfrac{1}{2}(5.4+4.3+3.2)=6.45$. Using Heron's formula:

$$S=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{6.45(6.45-5.4)(6.45-4.3)(6.45-3.2)}\approx 6.9 \text{ ft}^2$$

**52**. The fifth roots are given by:

$$w_k=32^{1/5}\left[\cos\frac{40°+360°k}{5}+i\sin\frac{40°+360°k}{5}\right]=2\left[\cos(8°+72°k)+i\sin(8°+72°k)\right]$$

Substituting $k=0,1,2,3,4$:

$w_0=2(\cos 8°+i\sin 8°)=2 \text{ cis } 8°$

$w_1=2(\cos 80°+i\sin 80°)=2 \text{ cis } 80°$

$w_2=2(\cos 152°+i\sin 152°)=2 \text{ cis } 152°$

$w_3=2(\cos 224°+i\sin 224°)=2 \text{ cis } 224°$

$w_4=2(\cos 296°+i\sin 296°)=2 \text{ cis } 296°$

The correct answer is a

**54**. Since $x^3+64=0, x^3=-64$. We wish to find the three cube roots of –64. Writing –64 in trigonometric form:

$$-64=64(\cos 180°+i\sin 180°)$$

The cube roots are given by:

$$w_k=64^{1/3}\left[\cos\frac{180°+360°k}{3}+i\sin\frac{180°+360°k}{3}\right]=4\left[\cos(60°+120°k)+i\sin(60°+120°k)\right]$$

Substituting $k=0,1,2$:

$$w_0=4(\cos 60°+i\sin 60°)=4\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=2+2i\sqrt{3}$$

$$w_1=4(\cos 180°+i\sin 180°)=4(-1+0i)=-4$$

$$w_2=4(\cos 300°+i\sin 300°)=4\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=2-2i\sqrt{3}$$

The correct answer is a.



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**ODD SOLUTIONS**

**1.** $n$

**3.** 72

**5.** The 2 square roots will be

$$w_k = 4^{1/2}\left(\text{cis}\frac{30^\circ + 360^\circ k}{2}\right) \text{ for } k = 0,1$$

$$= 2\text{cis}\left(15^\circ + 180^\circ k\right)$$

We replace $k$ with 0 and 1:

when $k = 0,\ w_0 = 2(\cos 15^\circ + i\sin 15^\circ)$

when $k = 1,\ w_1 = 2(\cos 195^\circ + i\sin 195^\circ)$

**7.** The 2 square roots will be

$$w_k = 25^{1/2}\left(\text{cis}\frac{210^\circ + 360^\circ k}{2}\right) \text{ for } k = 0,1$$

$$= 5\text{cis}\left(105^\circ + 180^\circ k\right)$$

We replace $k$ with 0 and 1:

when $k = 0,\ w_0 = 5(\cos 105^\circ + i\sin 105^\circ)$

when $k = 1,\ w_1 = 5(\cos 285^\circ + i\sin 285^\circ)$

**9.** The 2 square roots will be

$$w_k = 49^{1/2}\left(\text{cis}\frac{\pi + 2k\pi}{2}\right) \text{ for } k = 0,1$$

$$= 7\text{cis}\left(\frac{\pi}{2} + \pi k\right)$$

We replace $k$ with 0 and 1:

when $k = 0,\ w_0 = 7\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

when $k = 1,\ w_1 = 7\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

**11.** First, we put $2 + 2i\sqrt{3}$ in trigonometric form: $x = 2, y = 2\sqrt{3}$, and $r = \sqrt{2^2 + (2\sqrt{3})^2} = 4$

Also, $\tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$ and $\theta$ is in QI. Therefore, $\theta = 60^\circ$.

In trigonometric form $2 + 2i\sqrt{3} = 4\left(\cos 60^\circ + i\sin 60^\circ\right)$

The 2 square roots will be $w_k = 4^{1/2}\left(\text{cis}\frac{60^\circ + 360^\circ k}{2}\right)$ for $k = 0,1$

$$= 2\text{cis}\left(30^\circ + 180^\circ k\right)$$

We replace $k$ with 0 and 1:

when $k = 0,\ w_0 = 2(\cos 30^\circ + i\sin 30^\circ) = 2\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \sqrt{3} + i$

when $k = 1,\ w_1 = 2(\cos 210^\circ + i\sin 210^\circ) = 2\left(-\frac{\sqrt{3}}{2} + i\cdot -\frac{1}{2}\right) = -\sqrt{3} - i$

**13.** In trigonometric form, $4i = 4(\cos 90^\circ + i\sin 90^\circ)$

The 2 square roots will be $w_k = 4^{1/2}\left(\text{cis}\frac{90^\circ + 360^\circ k}{2}\right)$ for $k = 0,1$

$$= 2\text{cis}\left(45^\circ + 180^\circ k\right)$$

We replace $k$ with 0 and 1:

when $k = 0,\ w_0 = 2(\cos 45^\circ + i\sin 45^\circ) = 2\left(\frac{\sqrt{2}}{2} + i\cdot\frac{\sqrt{2}}{2}\right) = \sqrt{2} + i\sqrt{2}$

when $k = 1,\ w_1 = 2(\cos 225^\circ + i\sin 225^\circ) = 2\left(-\frac{\sqrt{2}}{2} + i\cdot -\frac{\sqrt{2}}{2}\right) = -\sqrt{2} - i\sqrt{2}$



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**15.** In trigonometric form, $-25 = 25(\cos 180^\circ + i\sin 180^\circ)$

The 2 square roots will be $w_k = 25^{1/2}\left(\text{cis}\dfrac{180^\circ + 360^\circ k}{2}\right)$ for $k = 0,1$

$$= 5\text{cis}\left(90^\circ + 180^\circ k\right)$$

We replace $k$ with 0 and 1:

when $k = 0$, $w_0 = 5(\cos 90^\circ + i\sin 90^\circ) = 5\left(0 + i\cdot 1\right) = 5i$

when $k = 1$, $w_1 = 5(\cos 270^\circ + i\sin 270^\circ) = 5\left(0 + i\cdot -1\right) = -5i$

**17.** In trigonometric form, $1 + i\sqrt{3} = 2(\cos 60^\circ + i\sin 60^\circ)$

The 2 square roots will be $w_k = 2^{1/2}\left(\text{cis}\dfrac{60^\circ + 360^\circ k}{2}\right) = \sqrt{2}\text{cis}\left(30^\circ + 180^\circ k\right)$ for $k = 0,1$

We replace $k$ with 0 and 1:

when $k = 0$, $w_0 = \sqrt{2}(\cos 30^\circ + i\sin 30^\circ) = \sqrt{2}\left(\dfrac{\sqrt{3}}{2} + i\cdot\dfrac{1}{2}\right) = \dfrac{\sqrt{6}}{2} + \dfrac{\sqrt{2}}{2}i$

when $k = 1$, $w_1 = \sqrt{2}(\cos 210^\circ + i\sin 210^\circ) = \sqrt{2}\left(-\dfrac{\sqrt{3}}{2} + i\cdot -\dfrac{1}{2}\right) = -\dfrac{\sqrt{6}}{2} - \dfrac{\sqrt{2}}{2}i$

**19.** The 3 cube roots will be $w_k = 8^{1/3}\left(\text{cis}\dfrac{210^\circ + 360^\circ k}{3}\right)$ for $k = 0,1,2$

$$= 2\text{cis}\left(70^\circ + 120^\circ k\right)$$

We replace $k$ with 0, 1, and 2:

when $k = 0$, $w_0 = 2(\cos 70^\circ + i\sin 70^\circ)$

when $k = 1$, $w_1 = 2(\cos 190^\circ + i\sin 190^\circ)$

when $k = 2$, $w_2 = 2(\cos 310^\circ + i\sin 310^\circ)$

**21.** In trigonometric form, $4\sqrt{3} + 4i = 8(\cos 30^\circ + i\sin 30^\circ)$

The 3 cube roots will be $w_k = 8^{1/3}\left(\text{cis}\dfrac{30^\circ + 360^\circ k}{3}\right)$ for $k = 0,1,2$

$$= 2\text{cis}\left(10^\circ + 120^\circ k\right)$$

We replace $k$ with 0, 1, and 2:

when $k = 0$, $w_0 = 2(\cos 10^\circ + i\sin 10^\circ)$

when $k = 1$, $w_1 = 2(\cos 130^\circ + i\sin 130^\circ)$

when $k = 2$, $w_2 = 2(\cos 250^\circ + i\sin 250^\circ)$

**23.** In trigonometric form, $-27 = 27(\cos 180^\circ + i\sin 180^\circ)$

The 3 cube roots will be $w_k = 27^{1/3}\left(\text{cis}\dfrac{180^\circ + 360^\circ k}{3}\right)$ for $k = 0,1,2$

$$= 3\text{cis}\left(60^\circ + 120^\circ k\right)$$

We replace $k$ with 0, 1, and 2:

when $k = 0$, $w_0 = 3(\cos 60^\circ + i\sin 60^\circ)$

when $k = 1$, $w_1 = 3(\cos 180^\circ + i\sin 180^\circ)$

when $k = 2$, $w_2 = 3(\cos 300^\circ + i\sin 300^\circ)$



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**25.** In trigonometric form, $64i = 64(\cos 90^\circ + i\sin 90^\circ)$

The 3 cube roots will be $w_k = 64^{1/3}\left(\text{cis}\dfrac{90^\circ + 360^\circ k}{3}\right) = 4\text{cis}\left(30^\circ + 120^\circ k\right)$ for $k = 0,1,2$

We replace $k$ with 0, 1, and 2:

when $k = 0$, $w_0 = 4(\cos 30^\circ + i\sin 30^\circ)$

when $k = 1$, $w_1 = 4(\cos 150^\circ + i\sin 150^\circ)$

when $k = 2$, $w_2 = 4(\cos 270^\circ + i\sin 270^\circ)$

**27.** $x^3 + 8 = 0$

$x^3 = -8$ The solutions are the 3 cube roots of $-8$.

In trigonometric form, $-8 = 8(\cos 180^\circ + i\sin 180^\circ)$

The 3 cube roots will be $w_k = 8^{1/3}\left(\text{cis}\dfrac{180^\circ + 360^\circ k}{3}\right)$ for $k = 0,1,2$

$= 2\text{cis}\left(60^\circ + 120^\circ k\right)$

We replace $k$ with 0, 1, and 2:

when $k = 0$, $w_0 = 2(\cos 60^\circ + i\sin 60^\circ) = 2\left(\dfrac{1}{2} + i\cdot\dfrac{\sqrt{3}}{2}\right) = 1 + i\sqrt{3}$

when $k = 1$, $w_1 = 2(\cos 180^\circ + i\sin 180^\circ) = 2\left(-1 + i\cdot 0\right) = -2$

when $k = 2$, $w_2 = 2(\cos 300^\circ + i\sin 300^\circ) = 2\left(\dfrac{1}{2} + i\cdot -\dfrac{\sqrt{3}}{2}\right) = 1 - i\sqrt{3}$

**29.** $x^4 + 81 = 0$

$x^4 = -81$ The solutions are the 4 fourth roots of $-81$.

In trigonometric form, $-81 = 81(\cos 180^\circ + i\sin 180^\circ)$

The 4 fourth roots will be $w_k = 81^{1/4}\left(\text{cis}\dfrac{180^\circ + 360^\circ k}{4}\right)$ for $k = 0,1,2,3$

$= 3\text{cis}\left(45^\circ + 90^\circ k\right)$

We replace $k$ with 0, 1, 2,and 3:

when $k = 0$, $w_0 = 3(\cos 45^\circ + i\sin 45^\circ) = 3\left(\dfrac{\sqrt{2}}{2} + i\cdot\dfrac{\sqrt{2}}{2}\right) = \dfrac{3\sqrt{2}}{2} + \dfrac{3\sqrt{3}}{2}i$

when $k = 1$, $w_1 = 3(\cos 135^\circ + i\sin 135^\circ) = 3\left(-\dfrac{\sqrt{2}}{2} + i\cdot\dfrac{\sqrt{2}}{2}\right) = -\dfrac{3\sqrt{2}}{2} + \dfrac{3\sqrt{2}}{2}i$

when $k = 2$, $w_2 = 3(\cos 225^\circ + i\sin 225^\circ) = 3\left(-\dfrac{\sqrt{2}}{2} + i\cdot -\dfrac{\sqrt{2}}{2}\right) = -\dfrac{3\sqrt{2}}{2} - \dfrac{3\sqrt{2}}{2}i$

when $k = 3$, $w_3 = 3(\cos 315^\circ + i\sin 315^\circ) = 3\left(\dfrac{\sqrt{2}}{2} + i\cdot -\dfrac{\sqrt{2}}{2}\right) = \dfrac{3\sqrt{2}}{2} - \dfrac{3\sqrt{2}}{2}i$



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**31.** The 4 fourth roots will be $w_k = 16^{1/4}\left(\text{cis}\dfrac{\frac{2\pi}{3}+2k\pi}{4}\right) = 2\text{cis}\left(\dfrac{\pi}{6}+\dfrac{\pi}{2}k\right)$ for $k = 0,1,2,3$

We replace $k$ with 0, 1, 2,and 3:

when $k=0,\ w_0 = 2\left(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right) = 2\left(\dfrac{\sqrt{3}}{2}+i\cdot\dfrac{1}{2}\right) = \sqrt{3}+i$

when $k=1,\ w_1 = 2\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right) = 2\left(-\dfrac{1}{2}+i\cdot\dfrac{\sqrt{3}}{2}\right) = -1+i\sqrt{3}$

when $k=2,\ w_2 = 2\left(\cos\dfrac{7\pi}{6}+i\sin\dfrac{7\pi}{6}\right) = 2\left(-\dfrac{\sqrt{3}}{2}+i\cdot-\dfrac{1}{2}\right) = -\sqrt{3}-i$

when $k=3,\ w_3 = 2\left(\cos\dfrac{5\pi}{3}+i\sin\dfrac{5\pi}{3}\right) = 2\left(\dfrac{1}{2}+i\cdot-\dfrac{\sqrt{3}}{2}\right) = 1-i\sqrt{3}$

**33.** The 5 fifth roots will be $w_k = \left(10^5\right)^{1/5}\left(\text{cis}\dfrac{15^\circ+360^\circ k}{5}\right)$ for $k = 0,1,2,3,4$

$$= 10\text{cis}\left(3^\circ+72^\circ k\right)$$

We replace $k$ with 0, 1, 2, 3, and 4:

when $k=0,\ w_0 = 10(\cos 3^\circ + i\sin 3^\circ) = 10\left[0.999+i(0.052)\right] = 9.99+0.52i$

when $k=1,\ w_1 = 10(\cos 75^\circ + i\sin 75^\circ) = 10\left[0.259+i\left(0.966\right)\right] = 2.59+9.66i$

when $k=2,\ w_2 = 10(\cos 147^\circ + i\sin 147^\circ) = 10\left[-0.839+i(0.545)\right] = -8.39+5.45i$

when $k=3,\ w_3 = 10(\cos 219^\circ + i\sin 219^\circ) = 10\left[-0.777+i(-0.629)\right] = -7.77-6.29i$

when $k=4,\ w_4 = 10(\cos 291^\circ + i\sin 291^\circ) = 10\left[0.358+i(-0.934)\right] = 3.58-9.34i$

**35.** In trigonometric form, $-1 = 1(\cos 180^\circ + i\sin 180^\circ)$

The 6 sixth roots will be $w_k = 1^{1/6}\left(\text{cis}\dfrac{180^\circ+360^\circ k}{6}\right)$ for $k = 0,1,2,3,4,5$

$$= 1\text{cis}\left(30^\circ+60^\circ k\right)$$

We replace $k$ with 0, 1, 2, 3, 4, and 5:

when $k=0,\ w_0 = \cos 30^\circ + i\sin 30^\circ$

when $k=1,\ w_1 = \cos 90^\circ + i\sin 90^\circ$

when $k=2,\ w_2 = \cos 150^\circ + i\sin 150^\circ$

when $k=3,\ w_3 = \cos 210^\circ + i\sin 210^\circ$

when $k=4,\ w_4 = \cos 270^\circ + i\sin 270^\circ$

when $k=5,\ w_5 = \cos 330^\circ + i\sin 330^\circ$



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**37.** Applying the quadratic formula, we have $x^2 = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = \dfrac{2 \pm \sqrt{-12}}{2} = \dfrac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$

First, we find the 2 square roots of $1 + i\sqrt{3}$:

In trigonometric form, $1 + i\sqrt{3} = 2(\cos 60^\circ + i \sin 60^\circ)$

The 2 square roots will be $w_k = 2^{1/2}\left(\text{cis}\dfrac{60^\circ + 360^\circ k}{2}\right) = \sqrt{2}\text{cis}\left(30^\circ + 180^\circ k\right)$ for $k = 0,1$

We replace $k$ with 0 and 1:

when $k = 0$, $w_0 = \sqrt{2}(\cos 30^\circ + i \sin 30^\circ)$

when $k = 1$, $w_1 = \sqrt{2}(\cos 210^\circ + i \sin 210^\circ)$

Next, we find the 2 square roots of $1 - i\sqrt{3}$:

In trigonometric form, $1 - i\sqrt{3} = 2(\cos 300^\circ + i \sin 300^\circ)$

The 2 square roots will be $w_k = 2^{1/2}\left(\text{cis}\dfrac{300^\circ + 360^\circ k}{2}\right) = \sqrt{2}\text{cis}\left(150^\circ + 180^\circ k\right)$ for $k = 0,1$

We replace $k$ with 0 and 1:

when $k = 0$, $w_0 = \sqrt{2}(\cos 150^\circ + i \sin 150^\circ)$

when $k = 1$, $w_1 = \sqrt{2}(\cos 330^\circ + i \sin 330^\circ)$

**39.** Applying the quadratic formula, we have $x^2 = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)} = \dfrac{-2 \pm \sqrt{-4}}{2} = \dfrac{-2 \pm 2i}{2} = -1 \pm i$

First, we find the 2 square roots of $-1 + i$:

In trigonometric form, $-1 + i = \sqrt{2}(\cos 135^\circ + i \sin 135^\circ)$

The 2 square roots will be $w_k = \left(\sqrt{2}\right)^{1/2}\left(\text{cis}\dfrac{135^\circ + 360^\circ k}{2}\right) = \sqrt[4]{2}\text{cis}\left(67.5^\circ + 180^\circ k\right)$ for $k = 0,1$

We replace $k$ with 0 and 1:

when $k = 0$, $w_0 = \sqrt[4]{2}(\cos 67.5^\circ + i \sin 67.5^\circ)$

when $k = 1$, $w_1 = \sqrt[4]{2}(\cos 247.5^\circ + i \sin 247.5^\circ)$

Next, we find the 2 square roots of $-1 - i$:

In trigonometric form, $-1 - i = \sqrt{2}(\cos 225^\circ + i \sin 225^\circ)$

The 2 square roots will be $w_k = \left(\sqrt{2}\right)^{1/2}\left(\text{cis}\dfrac{225^\circ + 360^\circ k}{2}\right) = \sqrt[4]{2}\text{cis}\left(112.5^\circ + 180^\circ k\right)$ for $k = 0,1$

We replace $k$ with 0 and 1:

when $k = 0$, $w_0 = \sqrt[4]{2}(\cos 112.5^\circ + i \sin 112.5^\circ)$

when $k = 1$, $w_1 = \sqrt[4]{2}(\cos 292.5^\circ + i \sin 292.5^\circ)$

**43.** $y = -2\sin(-3x) = 2\sin 3x$ because sine is an odd function.

The graph is a sine curve with amplitude = 2 and period = $\dfrac{2\pi}{3}$.

**45.** The graph is a cosine curve that has been reflected about the $x$-axis with:

Amplitude = 1

Period = $\dfrac{2\pi}{2} = \pi$

Horizontal shift = $\dfrac{-\pi/2}{2} = -\dfrac{\pi}{4}$



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**47.** $S = \frac{1}{2}bc\sin A$ Formula for area of a triangle

$= \frac{1}{2}(2.65)(3.84)\sin 56.2^\circ$ Substitute known values

$= 4.23\,\text{cm}^2$ Simplify and round to 3 significant digits

**49.** $s = \frac{1}{2}(a+b+c)$ Formula for half-perimeter

$= \frac{1}{2}(2.3+3.4+4.5) = 5.1$ Substitute known values and simplify

$S = \sqrt{s(s-a)(s-b)(s-c)}$ Formula for area of a triangle

$= \sqrt{5.1(5.1-2.3)(5.1-3.4)(5.1-4.5)}$ Substitute known values

$= \sqrt{14.5656} = 3.8\ \text{ft}^2$ Simplify and round to 2 significant digits

## 8.5 Polar Coordinates

**EVEN SOLUTIONS**

**2**. In polar coordinates, $r$ is the directed distance on the terminal side of an angle $\theta$ whose vertex is at the pole and whose initial side lies along the polar axis.

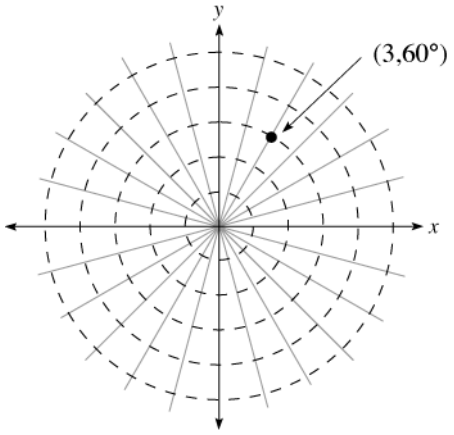
**4**. To graph a point $(r,\theta)$ with negative $r$, plot a point $r$ units away along a ray in the opposite direction from the terminal side of $\theta$.

**6**. To convert an ordered pair from a polar to rectangular coordinates, use the relationships $x = r\cos\theta$ and $y = r\sin\theta$.

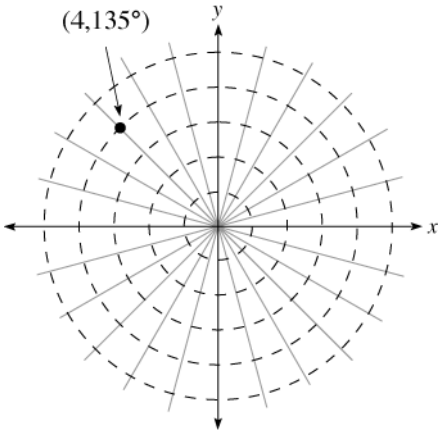
To convert an ordered pair from rectangular to polar coordinates, use the relationships $r = \sqrt{x^2+y^2}$ and $\tan\theta = \frac{y}{x}$, keeping in mind the quadrant the point lies in.

**8**. This statement is false.

**10**. Graphing the ordered pair:




**12**. Graphing the ordered pair:



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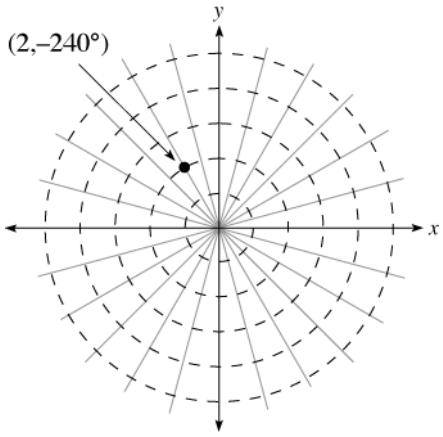
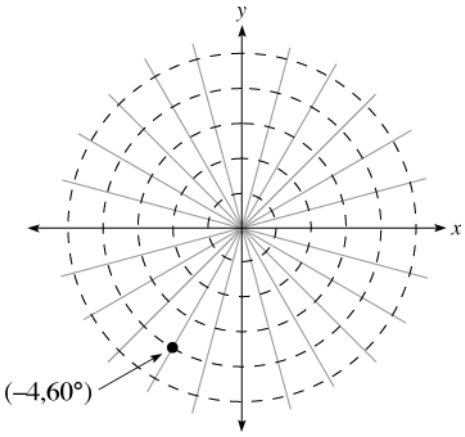
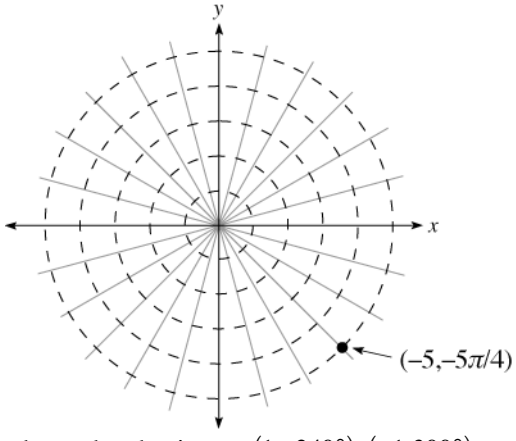
**14**. Graphing the ordered pair:



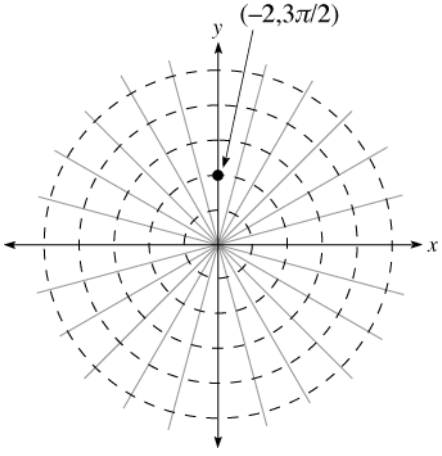

**16**. Graphing the ordered pair:




**18**. Graphing the ordered pair:




**20**. Graphing the ordered pair:




**22**. Three other ordered pairs are $(1,-240°), (-1,300°)$, and $(-1,-60°)$.

**24**. Three other ordered pairs are $(3,330°), (-3,-210°)$, and $(-3,150°)$.

**26**. Three other ordered pairs are $(-2,-315°), (2,225°)$, and $(2,-135°)$.

**28**. Three other ordered pairs are $(-6,120°), (6,300°)$, and $(6,-60°)$.

**30**. Here $r = -2$ and $\theta = 60°$. Converting to rectangular coordinates:

$$x = r\cos\theta = -2\cos 60° = -2 \cdot \frac{1}{2} = -1 \qquad y = r\sin\theta = -2\sin 60° = -2 \cdot \frac{\sqrt{3}}{2} = -\sqrt{3}$$

The rectangular coordinates are $\left(-1,-\sqrt{3}\right)$.

**32**. Here $r = 1$ and $\theta = \pi$. Converting to rectangular coordinates:

$$x = r\cos\theta = 1\cos\pi = 1 \cdot (-1) = -1 \qquad y = r\sin\theta = 1\sin\pi = 1 \cdot 0 = 0$$

The rectangular coordinates are $(-1,0)$.

**34**. Here $r = \sqrt{2}$ and $\theta = -225°$. Converting to rectangular coordinates:

$$x = r\cos\theta = \sqrt{2}\cos(-225°) = \sqrt{2} \cdot \left(-\frac{1}{\sqrt{2}}\right) = -1 \qquad y = r\sin\theta = \sqrt{2}\sin(-225°) = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1$$

The rectangular coordinates are $(-1,1)$.



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**36**. Here $r = 4\sqrt{3}$ and $\theta = -\dfrac{\pi}{6}$. Converting to rectangular coordinates:

$$x = r\cos\theta = 4\sqrt{3}\cos\left(-\frac{\pi}{6}\right) = 4\sqrt{3}\bullet\left(\frac{\sqrt{3}}{2}\right) = 6 \qquad y = r\sin\theta = 4\sqrt{3}\sin\left(-\frac{\pi}{6}\right) = 4\sqrt{3}\bullet\left(-\frac{1}{2}\right) = -2\sqrt{3}$$

The rectangular coordinates are $\left(6,-2\sqrt{3}\right)$.

**38**. Here $r = 3$ and $\theta = 124°$. Converting to rectangular coordinates:

$x = r\cos\theta = 3\cos 124° \approx -1.678$ $\qquad$ $y = r\sin\theta = 3\sin 124° \approx 2.487$

The rectangular coordinates are (–1.678, 2.487).

**40**. Here $r = -4$ and $\theta = 261°$. Converting to rectangular coordinates:

$x = r\cos\theta = -4\cos 261° \approx 0.6257$ $\qquad$ $y = r\sin\theta = -4\sin 261° \approx 3.951$

The rectangular coordinates are (0.6257, 3.951).

**42**. Compute $r = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$. Also $\tan\theta = \dfrac{-3}{-3} = 1$ with $\theta$ in quadrant III, so $\theta = 225°$.

The polar coordinates are $\left(3\sqrt{2},225°\right)$.

**44**. Compute $r = \sqrt{\left(-2\sqrt{3}\right)^2 + 2^2} = \sqrt{12+4} = \sqrt{16} = 4$. Also $\tan\theta = \dfrac{2}{-2\sqrt{3}} = -\dfrac{1}{\sqrt{3}}$ with $\theta$ in quadrant II, so $\theta = 150°$.

The polar coordinates are $\left(4,150°\right)$.

**46**. Compute $r = \sqrt{(-2)^2 + 0^2} = \sqrt{4+0} = \sqrt{4} = 2$. Also $\tan\theta = \dfrac{0}{-2} = 0$, so $\theta = \pi$. The polar coordinates are $\left(2,\pi\right)$.

**48**. Compute $r = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$. Also $\tan\theta = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$ with $\theta$ in quadrant III, so $\theta = \dfrac{4\pi}{3}$.

The polar coordinates are $\left(2,\dfrac{4\pi}{3}\right)$.

**50**. Compute $r = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$. Also $\tan\theta = \dfrac{3}{4}$ with $\theta$ in quadrant I, so $\theta \approx 36.9°$.

The polar coordinates are $\left(5,36.9°\right)$.

**52**. Compute $r = \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$. Also $\tan\theta = \dfrac{-2}{1} = -2$ with $\theta$ in quadrant IV, so $\theta \approx 296.6°$.

The polar coordinates are $\left(\sqrt{5},296.6°\right)$.

**54**. Compute $r = \sqrt{(-3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$. Also $\tan\theta = \dfrac{-2}{-3} = \dfrac{2}{3}$ with $\theta$ in quadrant III, so $\theta \approx 213.7°$.

The polar coordinates are $\left(\sqrt{13},213.7°\right)$.

**56**. Compute $r = \sqrt{(-7)^2 + 1^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$. Also $\tan\theta = \dfrac{1}{-7} = -\dfrac{1}{7}$ with $\theta$ in quadrant II, so $\theta \approx 171.9°$.

The polar coordinates are $\left(5\sqrt{2},171.9°\right)$.

**58**. The polar coordinates are $\left(9.220,102.5°\right)$.

**60**. The polar coordinates are $\left(7.616,-23.2°\right)$. This could also be written as $\left(7.616,336.8°\right)$.

**62**. Since $r^2 = x^2 + y^2$, the equation is $x^2 + y^2 = 4$.



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**64.** Since $y = r\sin\theta$, $\sin\theta = \dfrac{y}{r}$. Therefore:

$$\begin{aligned} r &= 6\sin\theta \\ r &= \frac{6y}{r} \\ r^2 &= 6y \\ x^2 + y^2 &= 6y \end{aligned}$$

**66.** Using the double-angle formula for sine:

$$\begin{aligned} r^2 &= 4\sin 2\theta \\ r^2 &= 8\sin\theta\cos\theta \\ r^2 &= 8\left(\frac{y}{r}\right)\left(\frac{x}{r}\right) \\ r^4 &= 8xy \\ \left(x^2 + y^2\right)^2 &= 8xy \end{aligned}$$

**68.** Since $x = r\cos\theta$ and $y = r\sin\theta$:

$$\begin{aligned} r(\cos\theta - \sin\theta) &= 2 \\ r\cos\theta - r\sin\theta &= 2 \\ x - y &= 2 \end{aligned}$$

**70.** Substituting $x = r\cos\theta$ and $y = r\sin\theta$:

$$\begin{aligned} x - y &= 5 \\ r\cos\theta - r\sin\theta &= 5 \\ r(\cos\theta - \sin\theta) &= 5 \\ r &= \frac{5}{\cos\theta - \sin\theta} \end{aligned}$$

**72.** Since $r^2 = x^2 + y^2$, the equation is $r^2 = 9$, or $r = 3$.

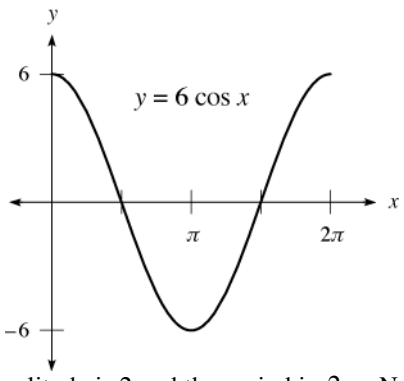
**74.** Since $x^2 + y^2 = r^2$ and $x = r\cos\theta$:

$$\begin{aligned} x^2 + y^2 &= 4x \\ r^2 &= 4r\cos\theta \\ r &= 4\cos\theta \end{aligned}$$

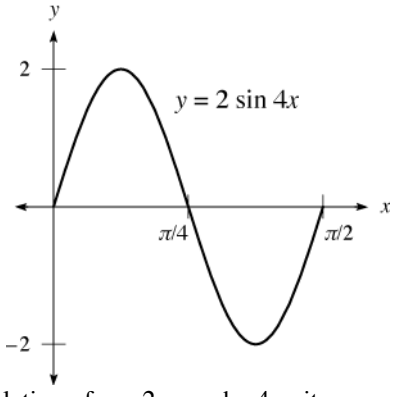
**76.** Since $x = r\cos\theta$ and $y = r\sin\theta$:

$$\begin{aligned} y &= -x \\ r\sin\theta &= -r\cos\theta \\ \frac{\sin\theta}{\cos\theta} &= -1 \\ \tan\theta &= -1 \\ \theta &= 135° \end{aligned}$$

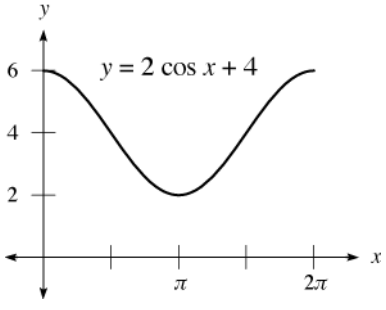
**78.** The amplitude is 6 and the period is $2\pi$:




**80.** The amplitude is 2 and the period is $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$:




**82.** The amplitude is 2 and the period is $2\pi$. Note this is a vertical translation of $y = 2\cos x$ by 4 units:



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**84**. Here $r=-2$ and $\theta=\dfrac{5\pi}{6}$. Converting to rectangular coordinates:

$$x=r\cos\theta=-2\cos\left(\frac{5\pi}{6}\right)=-2\bullet\left(-\frac{\sqrt{3}}{2}\right)=\sqrt{3} \qquad y=r\sin\theta=-2\sin\left(\frac{5\pi}{6}\right)=-2\bullet\left(\frac{1}{2}\right)=-1$$

The rectangular coordinates are $\left(\sqrt{3},-1\right)$. The correct answer is d.

**86**. Substituting $x=r\cos\theta$ and $y=r\sin\theta$:

$$\begin{aligned}(r\cos\theta)^2+(r\cos\theta)(r\sin\theta)+(r\sin\theta)^2&=1\\ r^2\left(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta\right)&=1\\ r^2\left(1+\cos\theta\sin\theta\right)&=1\\ r^2&=\frac{1}{1+\cos\theta\sin\theta}\\ r&=\frac{1}{\sqrt{1+\cos\theta\sin\theta}}\end{aligned}$$

The correct answer is c.

**ODD SOLUTIONS**

**1.** pole, polar axis
**3.** counterclockwise, polar
**5.** origin, positive $x$
**7.** true

For problems 9 through 19, see textbook answer section for graphs.

**21.** All points of the form $(2,60^\circ+360^\circ k)$, where $k$ is an integer, will name the point $(2,60^\circ)$.

For example, if $k=-1$, we have $(2,-300^\circ)$.

Also, all points of the form $(-2,240^\circ+360^\circ k)$, where $k$ is an integer, will name the point $(2,60^\circ)$.

For example, if $k=0$, we have $(-2,240^\circ)$ and if $k=-1$, we have $(-2,-120^\circ)$.

**23.** All points of the form $(5,-135^\circ+360^\circ k)$, where $k$ is an integer, will name the point $(5,-135^\circ)$.

For example, if $k=1$, we have $(5,225^\circ)$.

Also, all points of the form $(-5,45^\circ+360^\circ k)$, where $k$ is an integer, will name the point $(5,-135^\circ)$.

For example, if $k=0$, we have $(-5,45^\circ)$ and if $k=-1$, we have $(-5,-315^\circ)$.

**25.** All points of the form $(-3,330^\circ+360^\circ k)$, where $k$ is an integer, will name the point $(-3,330^\circ)$.

For example, if $k=-1$, we have $(-3,-30^\circ)$.

Also, all points of the form $(3,150^\circ+360^\circ k)$, where $k$ is an integer, will name the point $(-3,330^\circ)$.

For example, if $k=0$, we have $(3,150^\circ)$ and if $k=-1$, we have $(3,-210^\circ)$.

**27.** All points of the form $(-4,-60^\circ+360^\circ k)$, where $k$ is an integer, will name the point $(-4,-60^\circ)$.

For example, if $k=1$, we have $(-4,300^\circ)$.

Also, all points of the form $(4,120^\circ+360^\circ k)$, where $k$ is an integer, will name the point $(-4,-60^\circ)$.

For example, if $k=0$, we have $(4,120^\circ)$ and if $k=-1$, we have $(4,-240^\circ)$.

**29.**

$x=r\cos\theta$ and $y=r\sin\theta$

$=2\cos60^\circ$ $=2\sin60^\circ$

$=2\left(\dfrac{1}{2}\right)=1$ $=2\left(\dfrac{\sqrt{3}}{2}\right)=\sqrt{3}$

$(2,60^\circ)=\left(1,\sqrt{3}\right)$

**31.**

$x=r\cos\theta$ and $y=r\sin\theta$

$=3\cos\dfrac{3\pi}{2}$ $=3\sin\dfrac{3\pi}{2}$

$=3(0)=0$ $=3(-1)=-3$

$\left(3,\dfrac{3\pi}{2}\right)=(0,-3)$



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**33.** $x = r\cos\theta$ and $y = r\sin\theta$

$= \sqrt{2}\cos(-135^\circ)$ $= \sqrt{2}\sin(-135^\circ)$

$= \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right) = -1$ $= \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right) = -1$

$(\sqrt{2}, -135^\circ) = (-1,-1)$

**35.** $x = r\cos\theta$ and $y = r\sin\theta$

$= -4\sqrt{3}\cos\frac{\pi}{6}$ $= -4\sqrt{3}\sin\frac{\pi}{6}$

$= -4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$ $= -4\sqrt{3}\left(\frac{1}{2}\right)$

$= -6$ $= -2\sqrt{3}$

$\left(-4\sqrt{3}, \frac{\pi}{6}\right) = \left(-6, -2\sqrt{3}\right)$

**41.** $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

$= \sqrt{(-3)^2 + 3^2}$ $= \frac{3}{-3}$

$= \sqrt{18} = 3\sqrt{2}$ $= -1$

$\theta = 135^\circ$ or $315^\circ$

Since $(-3,3)$ is in QII, one solution is $\left(3\sqrt{2}, 135^\circ\right)$.

**43.** $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

$= \sqrt{2^2 + \left(-2\sqrt{3}\right)^2}$ $= \frac{-2\sqrt{3}}{2}$

$= \sqrt{16} = 4$ $= -\sqrt{3}$

$\theta = 120^\circ$ or $300^\circ$

Since $(2, -2\sqrt{3})$ is in QIV, one solution is $\left(4, 300^\circ\right)$.

**45.** $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

$= \sqrt{2^2 + 0^2}$ $= \frac{0}{2}$

$= \sqrt{4} = 2$ $= 0$

$\theta = 0$

One solution is $(2,0)$.

**47.** $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

$= \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}$ $= \frac{-1}{-\sqrt{3}}$

$= \sqrt{4} = 2$ $= \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ or $\frac{7\pi}{6}$

Since $\left(-\sqrt{3}, -1\right)$ is in QIII, one solution is $\left(2, \frac{7\pi}{6}\right)$.

**49.** $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

$= \sqrt{3^2 + 4^2}$ $= \frac{4}{3}$

$= \sqrt{25} = 5$ $\theta = 53.1^\circ$ or $233.1^\circ$

Since (3, 4) is in QI, one solution is $\left(5, 53.1^\circ\right)$.

**51.** $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

$= \sqrt{(-1)^2 + 2^2}$ $= \frac{2}{-1} = -2$

$= \sqrt{5}$ $\theta = 116.6^\circ$ or $296.6^\circ$

Since $(-1,2)$ is in QII, one solution is $\left(\sqrt{5}, 116.6^\circ\right)$.

**53.** $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

$= \sqrt{(-2)^2 + (-3)^2}$ $= \frac{-3}{-2} = 1.5$

$= \sqrt{13}$ $\theta = 56.3^\circ$ or $236.3^\circ$

Since $(-2,-3)$ is in QIII, one solution is $\left(\sqrt{13}, 236.3^\circ\right)$.

**55.** $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

$= \sqrt{(7)^2 + (-1)^2}$ $= \frac{-1}{7}$

$= \sqrt{50} = 5\sqrt{2}$ $\hat{\theta} = 8.13^\circ$

Since $(7,-1)$ is in QIV, one solution is $\left(5\sqrt{2}, 351.9^\circ\right)$

**61.** $r^2 = 9$

$x^2 + y^2 = 9$ Substitution

**63.** $r = 6\cos\theta$

$r^2 = 6r\cos\theta$ Multiply both sides by $r$

$x^2 + y^2 = 6x$ Substitution



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**65.**

$$r^2 = 4\cos 2\theta$$
$$= 4\left(\cos^2\theta - \sin^2\theta\right) \quad \text{Double-angle identity}$$
$$= 4\left[\left(\frac{x}{r}\right)^2 - \left(\frac{y}{r}\right)^2\right] \quad \text{Substitution}$$
$$= \frac{4\left(x^2 - y^2\right)}{r^2} \quad \text{Expand and combine}$$
$$r^4 = 4\left(x^2 - y^2\right) \quad \text{Multiply both sides by } r^2$$
$$\left(x^2 + y^2\right)^2 = 4\left(x^2 - y^2\right) \quad \text{Substitution}$$

**67.**

$$r\left(\cos\theta + \sin\theta\right) = 3$$
$$r\cos\theta + r\sin\theta = 3 \quad \text{Distributive property}$$
$$x + y = 3 \quad \text{Substitution}$$

**69.**

$$x - y = 5$$
$$r\cos\theta - r\sin\theta = 5 \quad \text{Substitution}$$
$$r\left(\cos\theta - \sin\theta\right) = 5 \quad \text{Factor}$$
$$r = \frac{5}{\cos\theta - \sin\theta} \quad \text{Divide}$$

**71.**

$$x^2 + y^2 = 4$$
$$r^2 = 4 \quad \text{Substitution}$$
$$r = 2 \quad \text{Solve for } r$$

**73.**

$$x^2 + y^2 = 6x$$
$$r^2 = 6r\cos\theta \quad \text{Substitution}$$
$$r = 6\cos\theta \quad \text{Divide both sides by r}$$

**75.**

$$y = x$$
$$r\sin\theta = r\cos\theta \quad \text{Substitution}$$
$$\sin\theta = \cos\theta \text{ or} \quad \text{Divide both sides by } r$$
$$\theta = 45^\circ$$

**77.** The graph is a sine curve with amplitude of 6.

**79.** The graph is a sine curve with amplitude of 4 and period of $\pi$ .

**81.** The graph is a sine curve with amplitude of 2, period of $2\pi$ and a vertical translation of 4.



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# 8.6 Equations in Polar Coordinates and Their Graphs

**EVEN SOLUTIONS**

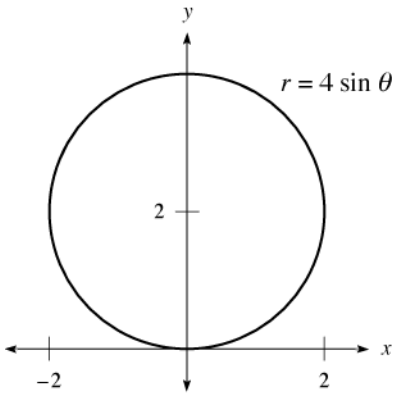
**2**. Another way to graph a polar equation is to convert it to rectangular coordinates and see if the resulting equation has a recognizable graph.

**4**. **a**. This is a circle (ii). **b**. This is a line (v).
**c**. This is a 5-leafed rose (vii). **d**. This is a 6-leafed rose (vi).
**e**. This is a lemniscate (iii). **f**. This is a cardiod (i).
**g**. This is a limacon (iv).

**6**. Making a table of values:

| $\theta$ | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| $r = 4\sin\theta$ | 0 | $2\sqrt{2}$ | 4 | $2\sqrt{2}$ | 0 | $-2\sqrt{2}$ | $-4$ | $-2\sqrt{2}$ | 0 |

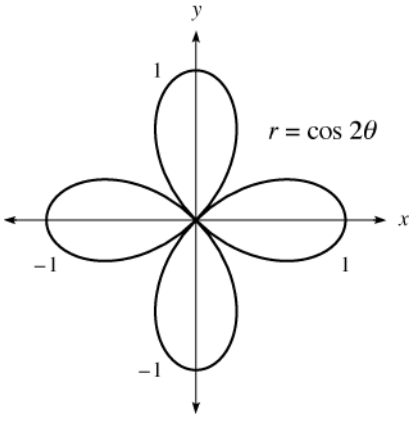
Sketching the graph:




**8**. Making a table of values:

| $\theta$ | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| $r = \cos 2\theta$ | 1 | 0 | $-1$ | 0 | 1 | 0 | $-1$ | 0 | 1 |

Sketching the graph:



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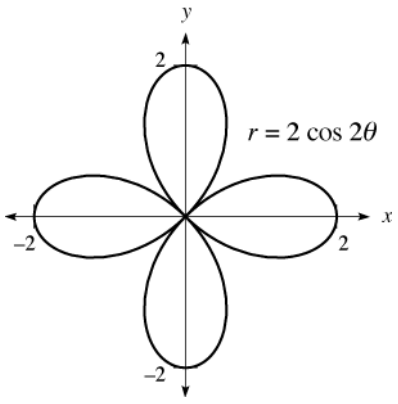
**10**. Making a table of values:

| $\theta$ | 0° | 15° | 30° | 45° | 60° | 75° | 90° | 105° | 120° | 135° | 150° | 165° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r = 2\cos 2\theta$ | 1 | $\sqrt{3}$ | 1 | 0 | −1 | $-\sqrt{3}$ | −1 | $-\sqrt{3}$ | −1 | 0 | 1 | $\sqrt{3}$ | 1 |

Sketching the graph:

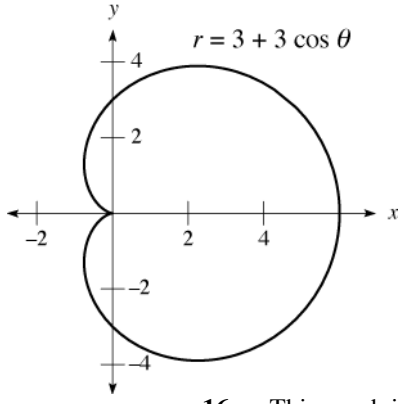



**12**. Making a table of values:

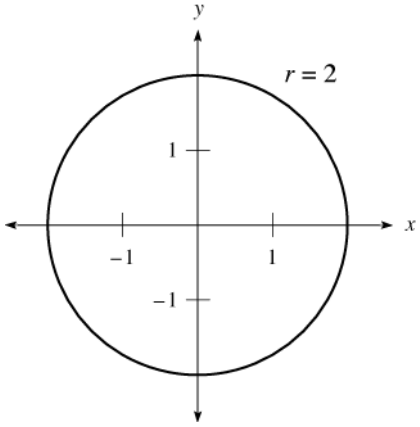
| $\theta$ | 0° | 15° | 30° | 45° | 60° | 75° | 90° | 105° | 120° | 135° | 150° | 165° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r = 3 + 3\cos\theta$ | 6 | 5.9 | 5.6 | 5.1 | 4.5 | 3.8 | 3 | 2.2 | 1.5 | 0.9 | 0.4 | 0.1 | 0 |

| $\theta$ | 180° | 195° | 210° | 225° | 240° | 255° | 270° | 285° | 300° | 315° | 330° | 345° | 360° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r = 3 + 3\cos\theta$ | 0 | 0.1 | 0.4 | 0.9 | 1.5 | 2.2 | 3 | 3.8 | 4.5 | 5.1 | 5.6 | 5.9 | 6 |

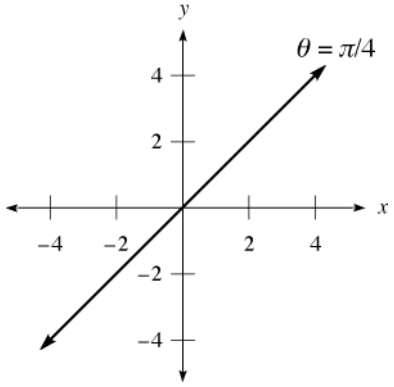
Sketching the graph:




**14**. This graph is a lemniscate.
**16**. This graph is a circle.
**18**. This graph is a limacon.
**20**. This graph is a rose.
**22**. This graph is a cardioid.
**24**. This graph is a line.
**26**. Graphing the equation:




**28**. Graphing the equation:



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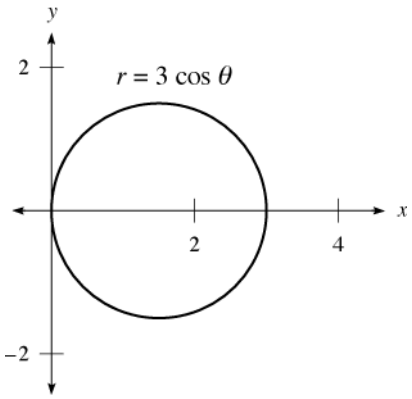
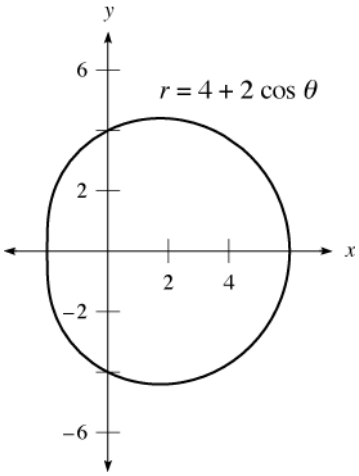
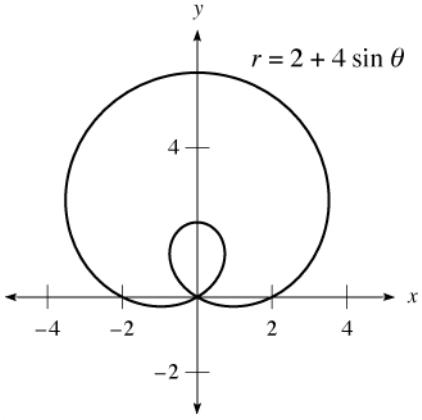
**30**. Graphing the equation:



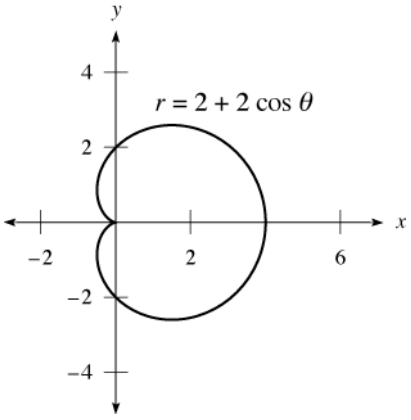

**32**. Graphing the equation:



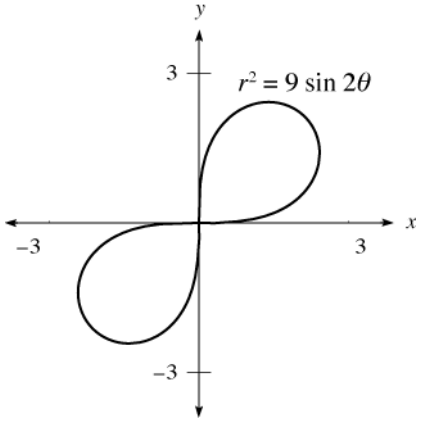

**34**. Graphing the equation:



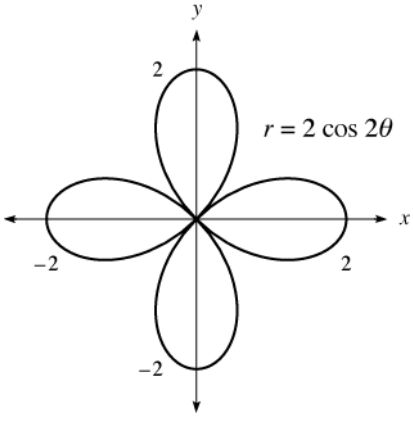

**36**. Graphing the equation:




**38**. Graphing the equation:




**40**. Graphing the equation:



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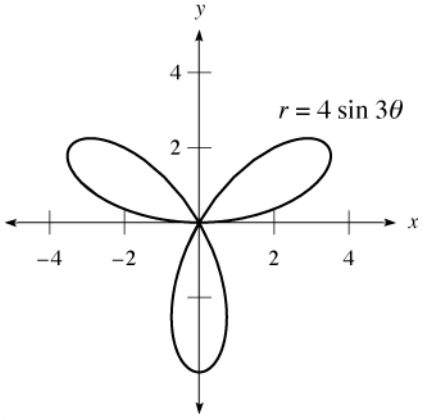
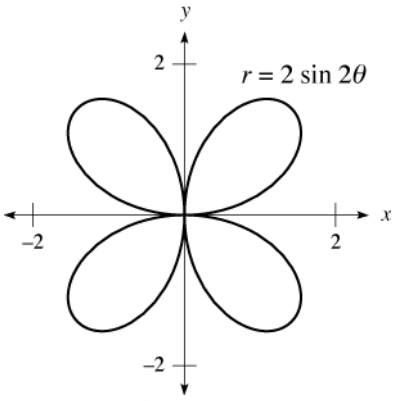
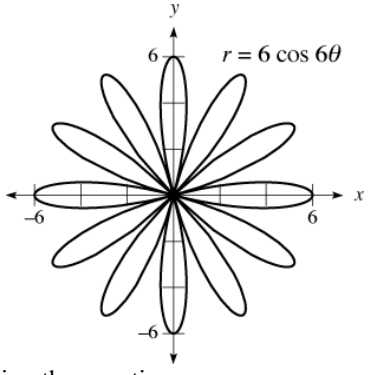
**42**. Graphing the equation:



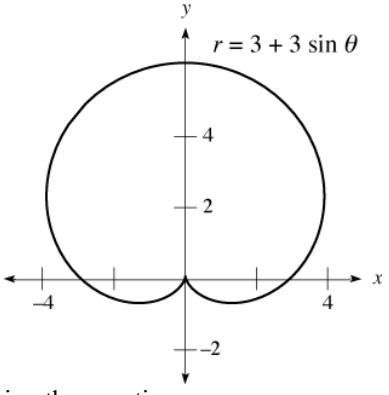

**44**. Graphing the equation:



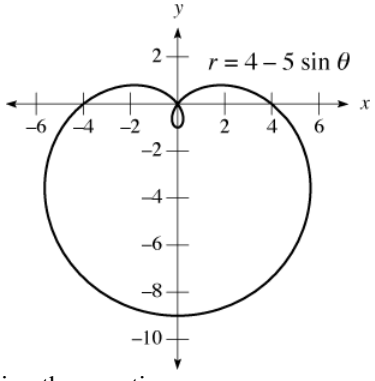

**46**. Graphing the equation:



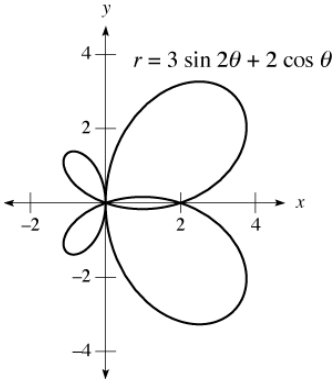

**48**. Graphing the equation:



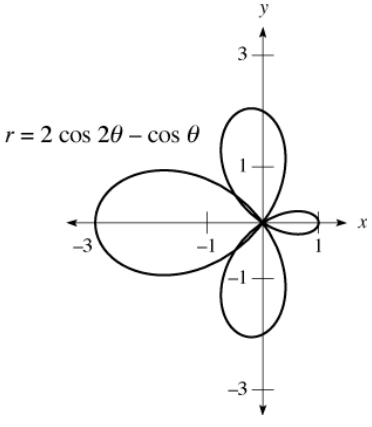

**50**. Graphing the equation:




**52**. Graphing the equation:




**54**. Graphing the equation:



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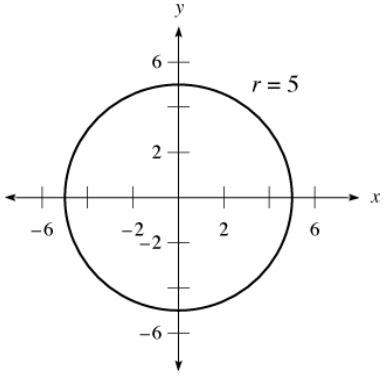
**56**. Converting to polar coordinates:

$$x^2 + y^2 = 25$$
$$r^2 = 25$$
$$r = 5$$

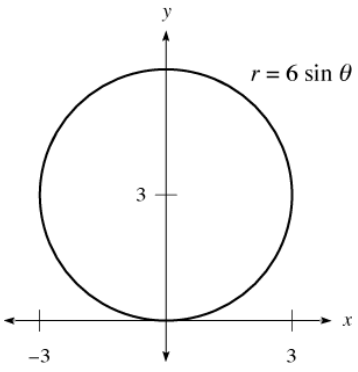
Sketching the graph:




**58**. Converting to polar coordinates:

$$x^2 + y^2 = 6y$$
$$r^2 = 6r\sin\theta$$
$$r = 6\sin\theta$$

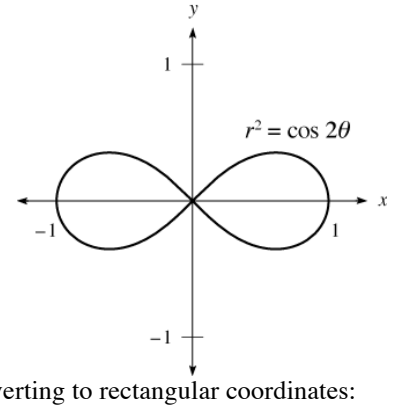
Sketching the graph:




**60**. Converting to polar coordinates:

$$\left(x^2 + y^2\right)^2 = x^2 - y^2$$
$$\left(r^2\right)^2 = r^2\cos^2\theta - r^2\sin^2\theta$$
$$r^4 = r^2\left(\cos^2\theta - \sin^2\theta\right)$$
$$r^2 = \cos 2\theta$$

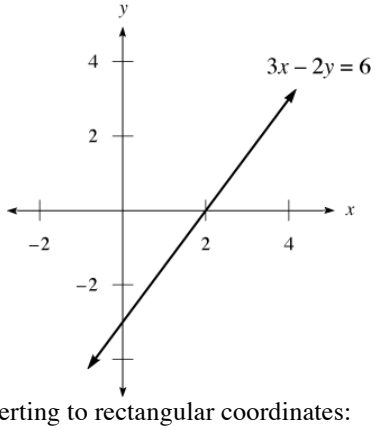
Sketching the graph:




**62**. Converting to rectangular coordinates:

$$r\left(3\cos\theta - 2\sin\theta\right) = 6$$
$$3r\cos\theta - 2r\sin\theta = 6$$
$$3x - 2y = 6$$

Sketching the graph:




**64**. Converting to rectangular coordinates:

$$r\left(1 - \sin\theta\right) = 1$$
$$r - r\sin\theta = 1$$
$$r = 1 + r\sin\theta$$
$$\sqrt{x^2 + y^2} = 1 + y$$
$$x^2 + y^2 = \left(1 + y\right)^2$$
$$x^2 + y^2 = 1 + 2y + y^2$$
$$x^2 = 2y + 1$$

**66**. Converting to rectangular coordinates:

$$r = 6\cos\theta$$
$$r^2 = 6r\cos\theta$$
$$x^2 + y^2 = 6x$$
$$x^2 - 6x + y^2 = 0$$
$$x^2 - 6x + 9 + y^2 = 9$$
$$\left(x - 3\right)^2 + y^2 = 9$$



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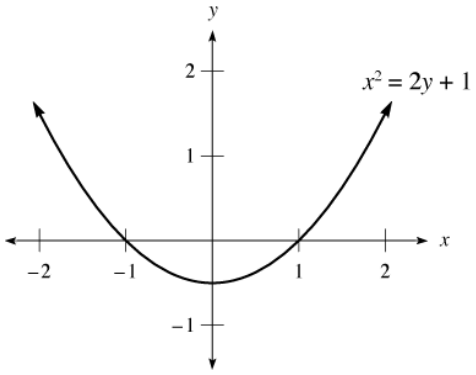
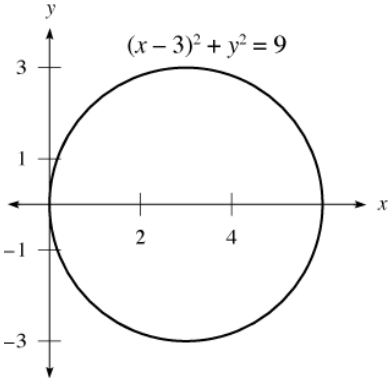
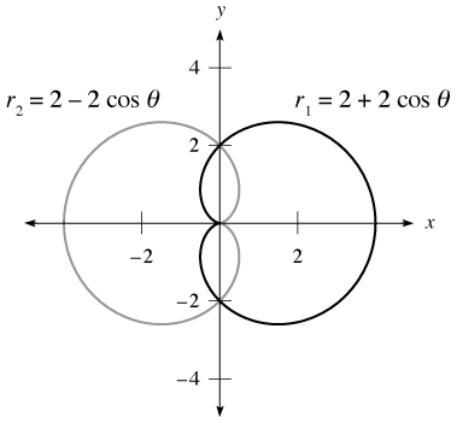
Sketching the graph:




Sketching the graph:

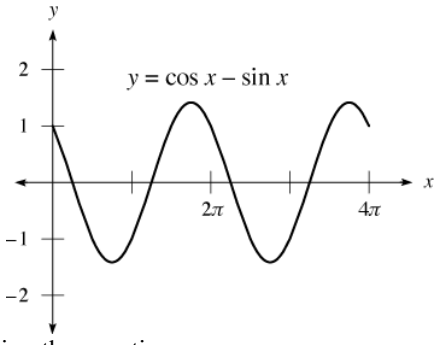



**68**. Graphing the two curves:

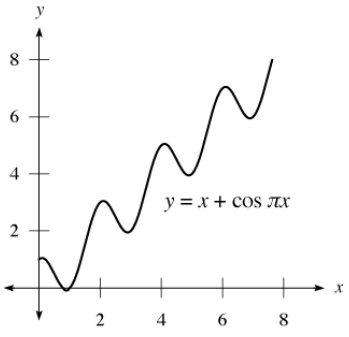



Two points they have in common are $(0,2) = (2,90°)$ and $(0,-2) = (2,270°)$. Note that although the rectangular point $(0,0)$ lies on both curves, and thus is a common point, the polar point $(0,0°)$ is not a point in common, since it does not satisfy both equations.

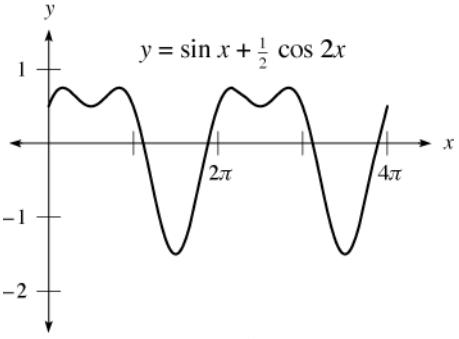
**70**. Graphing the equation:




**72**. Graphing the equation:




**74**. Graphing the equation:




**76**. The correct graph is d.

**78**. The correct equation is b.



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**ODD SOLUTIONS**

**1.** $\theta, r$

**3.** rectangular

**5.**

| $\theta$ | $r = 6\cos\theta$ | $(r,\theta)$ |
|---|---|---|
| $0^\circ$ | $r = 6\cos 0^\circ = 6$ | $(6,0^\circ)$ |
| $45^\circ$ | $r = 6\cos 45^\circ = 3\sqrt{2}$ | $(3\sqrt{2},45^\circ)$ |
| $90^\circ$ | $r = 6\cos 90^\circ = 0$ | $(0,90^\circ)$ |
| $135^\circ$ | $r = 6\cos 135^\circ = -3\sqrt{2}$ | $(-3\sqrt{2},135^\circ)$ |
| $180^\circ$ | $r = 6\cos 180^\circ = -6$ | $(-6,180^\circ)$ |
| $225^\circ$ | $r = 6\cos 225^\circ = -3\sqrt{2}$ | $(-3\sqrt{2},225^\circ)$ |
| $270^\circ$ | $r = 6\cos 270^\circ = 0$ | $(0,270^\circ)$ |
| $315^\circ$ | $r = 6\cos 315^\circ = 3\sqrt{2}$ | $(3\sqrt{2},315^\circ)$ |

**7.**

| $\theta$ | $r = \sin 2\theta$ | $(r,\theta)$ |
|---|---|---|
| $0^\circ$ | $r = \sin 2(0^\circ) = 0$ | $(0,0^\circ)$ |
| $45^\circ$ | $r = \sin 2(45^\circ) = 1$ | $(1,45^\circ)$ |
| $90^\circ$ | $r = \sin 2(90^\circ) = 0$ | $(0,90^\circ)$ |
| $135^\circ$ | $r = \sin 2(135^\circ) = -1$ | $(-1,135^\circ)$ |
| $180^\circ$ | $r = \sin 2(180^\circ) = 0$ | $(0,180^\circ)$ |
| $225^\circ$ | $r = \sin 2(225^\circ) = 1$ | $(1,225^\circ)$ |
| $270^\circ$ | $r = \sin 2(270^\circ) = 0$ | $(0,270^\circ)$ |
| $315^\circ$ | $r = \sin 2(315^\circ) = -1$ | $(-1,315^\circ)$ |

**25.** $r = 3$

| | |
|---|---|
| $\pm\sqrt{x^2 + y^2} = 3$ | Substitution |
| $x^2 + y^2 = 9$ | Square both sides |

The graph is a circle with center at the origin or pole and radius of 3.

**27.** $\theta = \dfrac{3\pi}{4}$

| | |
|---|---|
| $\tan\theta = -1$ | Find tangent of $\dfrac{3\pi}{4}$ |
| $\dfrac{y}{x} = -1$ | Substitution |
| $y = -x$ | Multiply both sides by $x$ |

The graph is a line through the origin with a slope of $-1$.

**29.** To graph by hand, first we sketch $y = 3\sin x$. We note the relationship between the variations in $x$ or $\theta$ and the corresponding variations in $y$ or $r$.

| Variations in $x$ or $\theta$ | Corresponding Variations in $y$ or $r$ |
|---|---|
| $0^\circ$ to $90^\circ$ | Increases from 0 to 3 |
| $90^\circ$ to $180^\circ$ | Decreases from 3 to 0 |
| $180^\circ$ to $270^\circ$ | Decreases from 0 to –3 |
| $270^\circ$ to $360^\circ$ | Increases from –3 to 0 |

Then we sketch the graph using this relationship.



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**31.** To graph by hand, first we sketch $y = 4 + 2\sin x$. We note the relationship between the variations in $x$ or $\theta$ and the corresponding variations in $y$ or $r$.

| Variations in $x$ or $\theta$ | Corresponding Variations in $y$ or $r$ |
|---|---|
| $0^\circ$ to $90^\circ$ | Increases from 4 to 6 |
| $90^\circ$ to $180^\circ$ | Decreases from 6 to 4 |
| $180^\circ$ to $270^\circ$ | Decreases from 4 to 2 |
| $270^\circ$ to $360^\circ$ | Increases from 2 to 4 |

Then we sketch the graph using this relationship.

**33.** To graph by hand, first we sketch $y = 2 + 4\cos x$. We note the relationship between the variations in $x$ or $\theta$ and the corresponding variations in $y$ or $r$.

| Variations in $x$ or $\theta$ | Corresponding Variations in $y$ or $r$ |
|---|---|
| $0^\circ$ to $90^\circ$ | Decreases from 6 to 2 |
| $90^\circ$ to $180^\circ$ | Decreases from 2 to –2 |
| $180^\circ$ to $270^\circ$ | Increases from –2 to 2 |
| $270^\circ$ to $360^\circ$ | Increases from 2 to 6 |

Then we sketch the graph using this relationship.

**35.** To graph by hand, first we sketch $y = 2 + 2\sin x$. We note the relationship between the variations in $x$ or $\theta$ and the corresponding variations in $y$ or $r$.

| Variations in $x$ or $\theta$ | Corresponding Variations in $y$ or $r$ |
|---|---|
| $0^\circ$ to $90^\circ$ | Increases from 2 to 4 |
| $90^\circ$ to $180^\circ$ | Decreases from 4 to 2 |
| $180^\circ$ to $270^\circ$ | Decreases from 2 to 0 |
| $270^\circ$ to $360^\circ$ | Increases from 0 to 2 |

Then we sketch the graph using this relationship.

**37.** $r^2 = 4\cos 2\theta$ is a lemniscate or two-leaved rose where $a = 4$. (See Figure 19 in textbook.) The endpoints of the leaves are $\left(\sqrt{a}, 0^\circ\right) = \left(2, 0^\circ\right)$ and $\left(-\sqrt{a}, 0^\circ\right) = \left(-2, 0^\circ\right)$.

**39.** $r = 2\sin 2\theta$ is a four-leaved rose where $a = 2$. (See Figure 17 in textbook.) The endpoints of the leaves are $\left(2, 45^\circ\right), \left(2, 225^\circ\right), \left(-2, 135^\circ\right)$, and $\left(-2, 315^\circ\right)$.

**41.** $r = 4\cos 3\theta$ is a three-leaved rose where $a = 4$. (See Figure 18 in textbook.) The endpoints of the leaves are $\left(4, 0^\circ\right), \left(4, 120^\circ\right)$ and $\left(-4, 60^\circ\right)$.

**55.** $x^2 + y^2 = 16$

$r^2 = 16$

$r = 4$

The graph is a circle with center at the pole and radius of 4.

**57.** $x^2 + y^2 = 6x$

$r^2 = 6r\cos\theta$

$r = 6\cos\theta$

The graph is a circle with center at $(3, 0^\circ)$ and radius of 3.

**59.** $\left(x^2 + y^2\right)^2 = 2xy$

$\left(r^2\right)^2 = 2\left(r\cos\theta\right)\left(r\sin\theta\right)$ Substitution

$r^4 = r^2\left(2\sin\theta\cos\theta\right)$ Simplify

$r^2 = \sin 2\theta$ Divide both sides by $r^2$ and use the double-angle identity

The graph is a lemniscate with $a = 1$. (See Figure 19 in textbook). The endpoints of the leaves are at $\left(1, 45^\circ\right)$ and $\left(-1, 45^\circ\right)$.



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**61.** $r(2\cos\theta + 3\sin\theta) = 6$

$2r\cos\theta + 3r\sin\theta = 6$ Multiply

$2x + 3y = 6$ Substitution

The graph is a line through (0, 2) and (3, 0) in rectangular coordinates.

**63.** $r(1 - \cos\theta) = 1$

$r - r\cos\theta = 1$ Multiply

$r - x = 1$ Substitution

$r = x + 1$ Add $x$ to both sides

$r^2 = (x+1)^2$ Square both sides

$x^2 + y^2 = x^2 + 2x + 1$ Substitution on left side and expand right side

$y^2 = 2x + 1$ Subtract $x^2$ from both sides

$x = \frac{1}{2}y^2 - \frac{1}{2}$ Solve for $x$

The graph is a parabola that opens to the right with vertex at ( – ½, 0) in rectangular coordinates.

**65.** $r = 4\sin\theta$

$r^2 = 4r\sin\theta$ Multiply both sides by $r$

$x^2 + y^2 = 4y$ Substitution

$x^2 + (y-2)^2 = 4$ Complete the square and put in standard form

The graph is a circle with center at (0, 2) and radius of 2 in rectangular coordinates.

**67.** We graph the two equations. The equation $r_1 = 2\sin\theta$ is a circle with center at (0, 1) and radius of 1. The equation $r_2 = 2\cos\theta$ is a circle with center at (1, 0) and radius of 1.

Next, we solve the equations algebraically by substitution:

$2\sin\theta = 2\cos\theta$ $r = r$

$\frac{2\sin\theta}{2\cos\theta} = 1$ Divide both sides by $2\cos\theta$

$\tan\theta = 1$ Ratio identity

$\theta = 45^\circ$ or $\theta = 225^\circ$

Then we solve for $r$ in each case:

$\theta = 45^\circ$, $r = 2\sin 45^\circ$

$= 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

$\theta = 225^\circ$, $r = 2\sin 225^\circ$

$= 2\left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

Therefore, the point of intersection is $(1,1)$ or $\left(\sqrt{2}, 45^\circ\right)$ and $\left(-\sqrt{2}, 225^\circ\right)$. The graphs also have the point (0, 0) in common. On one graph it is the point $\left(0, 0^\circ\right)$ and on the other it is $\left(0, 90^\circ\right)$.

**69.** Let $y_1 = \sin x$ (the basic sine curve) and $y_2 = -\cos x$ (a cosine curve reflected about the $x$-axis). Graph $y_1, y_2$, and $y = y_1 + y_2$ on the same coordinate axes.

**71.** Let $y_1 = x$ (a line through the origin with a slope of 1) and $y_2 = \sin \pi x$ (a sine curve with period $\frac{2\pi}{\pi} = 2$). Graph $y_1, y_2$, and $y = y_1 + y_2$ on the same coordinate axes.

**73.** Let $y_1 = 3\sin x$ (a sine curve with amplitude of 3) and $y_2 = \cos 2x$ (a cosine curve with period of $\frac{2\pi}{2} = \pi$). Graph $y_1, y_2$, and $y = y_1 + y_2$ on the same coordinate axes.



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# Chapter 8 Test

**1**. Writing in terms of $i$: $\sqrt{-12} = i\sqrt{12} = 2i\sqrt{3}$

**2**. Setting the real parts equal:

$$x^2 - 3x = 10$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$$x = -2, 5$$

Setting the imaginary parts equal:

$$16 = 8y$$
$$y = 2$$

**3**. Combining the numbers: $(6-3i)+[(4-2i)-(3+i)] = (6-3i)+(4-2i-3-i) = 6-3i+1-3i = 7-6i$

**4**. Computing the power: $i^{64} = i^{64} \bullet i^3 = \left(i^4\right)^{16} \bullet i^2 \bullet i = 1 \bullet (-1) \bullet i = -i$

**5**. Computing the product: $(8+5i)(8-5i) = 64 - 40i + 40i - 25i^2 = 64 + 25 = 89$

**6**. Computing the product: $(3-4i)^2 = (3-4i)(3-4i) = 9 - 12i - 12i + 16i^2 = 9 - 24i - 16 = -7 - 24i$

**7**. Finding the quotient: $\dfrac{6+5i}{6-5i} \bullet \dfrac{6+5i}{6+5i} = \dfrac{36+60i+25i^2}{36-25i^2} = \dfrac{36+60i-25}{36+25} = \dfrac{11+60i}{61} = \dfrac{11}{61} + \dfrac{60}{61}i$

**8**. **a**. The absolute value is: $|3-4i| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

**b**. The opposite is: $-(3-4i) = -3+4i$

**c**. The conjugate is: $3+4i$

**9**. **a**. The absolute value is: $|8i| = \sqrt{0^2 + 8^2} = \sqrt{0+64} = \sqrt{64} = 8$

**b**. The opposite is: $-(8i) = -8i$

**c**. The conjugate is: $-8i$

**10**. Writing in standard form: $8(\cos 330° + i \sin 330°) = 8\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right) = 4\sqrt{3} - 4i$

**11**. Writing in standard form: $2 \operatorname{cis} \dfrac{3\pi}{4} = 2\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right) = 2\left(-\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right) = -\sqrt{2} + i\sqrt{2}$

**12**. Here $x = -\sqrt{3}$, $y = 1$, and $r = \sqrt{\left(-\sqrt{3}\right)^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$. Since $\tan\theta = -\dfrac{1}{\sqrt{3}}$ in quadrant II, $\theta = 150°$.

So the trigonometric form is $-\sqrt{3} + i = 2(\cos 150° + i \sin 150°)$.

**13**. Here $x = 0$, $y = -5$, and $r = \sqrt{0^2 + (-5)^2} = \sqrt{0+25} = \sqrt{25} = 5$. Since $\tan\theta$ is undefined and $y$ is negative, $\theta = 270°$.

So the trigonometric form is $-5i = 5(\cos 270° + i \sin 270°)$.

**14**. Multiplying in trigonometric form:

$$5(\cos 25° + i \sin 25°) \bullet 3(\cos 80° + i \sin 80°) = 15[\cos(25° + 80°) + i \sin(25° + 80°)] = 15(\cos 125° + i \sin 125°)$$

**15**. Dividing in trigonometric form: $\dfrac{10(\cos 50° + i \sin 50°)}{2(\cos 20° + i \sin 20°)} = \dfrac{10}{2}[\cos(50° - 20°) + i \sin(50° - 20°)] = 5(\cos 30° + i \sin 30°)$

**16**. Using deMoivre's Theorem:

$$(3 \text{ cis } 80°)^4 = [3(\cos 80° + i \sin 80°)]^4 = 3^4[\cos(4 \bullet 80°) + i \sin(4 \bullet 80°)] = 81(\cos 320° + i \sin 320°) = 81 \operatorname{cis} 320°$$



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**17**. The square roots are given by:

$$w_k = 49^{1/2}\left[\cos\frac{50°+360°k}{2}+i\sin\frac{50°+360°k}{2}\right]=7\left[\cos\left(25°+180°k\right)+i\sin\left(25°+180°k\right)\right]$$

Substituting $k = 0,1$:

$$w_0 = 7\left(\cos 25°+i\sin 25°\right)$$

$$w_1 = 7\left(\cos 205°+i\sin 205°\right)$$

**18**. First write the number in trigonometric form: $2+2i\sqrt{3}=4\left(\cos 60°+i\sin 60°\right)$

The fourth roots are given by:

$$w_k = 4^{1/4}\left[\cos\frac{60°+360°k}{4}+i\sin\frac{60°+360°k}{4}\right]=\sqrt{2}\left[\cos\left(15°+90°k\right)+i\sin\left(15°+90°k\right)\right]$$

Substituting $k = 0,1,2,3$:

$$w_0 = \sqrt{2}\left(\cos 15°+i\sin 15°\right)$$

$$w_1 = \sqrt{2}\left(\cos 105°+i\sin 105°\right)$$

$$w_2 = \sqrt{2}\left(\cos 195°+i\sin 195°\right)$$

$$w_3 = \sqrt{2}\left(\cos 285°+i\sin 285°\right)$$

**19**. Using the quadratic formula with $a = 1$, $b = -2\sqrt{3}$, and $c = 4$:

$$x^2=\frac{-\left(-2\sqrt{3}\right)\pm\sqrt{\left(-2\sqrt{3}\right)^2-4(1)(4)}}{2(1)}=\frac{2\sqrt{3}\pm\sqrt{12-16}}{2}=\frac{2\sqrt{3}\pm\sqrt{-4}}{2}=\frac{2\sqrt{3}\pm 2i}{2}=\sqrt{3}\pm i$$

Thus either $x^2=\sqrt{3}+i$ or $x^2=\sqrt{3}-i$. Writing $\sqrt{3}+i$ in trigonometric form: $\sqrt{3}+i=2\left(\cos 30°+i\sin 30°\right)$

So its square roots are given by:

$$w_k = 2^{1/2}\left[\cos\frac{30°+360°k}{2}+i\sin\frac{30°+360°k}{2}\right]=\sqrt{2}\left[\cos\left(15°+180°k\right)+i\sin\left(15°+180°k\right)\right]$$

Substituting $k = 0,1$:

$$w_0 = \sqrt{2}\left(\cos 15°+i\sin 15°\right)$$

$$w_1 = \sqrt{2}\left(\cos 195°+i\sin 195°\right)$$

Writing $\sqrt{3}+i$ in trigonometric form: $\sqrt{3}-i=2\left(\cos 330°+i\sin 330°\right)$

So its square roots are given by:

$$w_k = 2^{1/2}\left[\cos\frac{330°+360°k}{2}+i\sin\frac{330°+360°k}{2}\right]=\sqrt{2}\left[\cos\left(165°+180°k\right)+i\sin\left(165°+180°k\right)\right]$$

Substituting $k = 0,1$:

$$w_0 = \sqrt{2}\left(\cos 165°+i\sin 165°\right)$$

$$w_1 = \sqrt{2}\left(\cos 345°+i\sin 345°\right)$$

Summarizing, the solutions are $\sqrt{2}\left(\cos\theta+i\sin\theta\right)$, where $\theta = 15°,165°,195°,345°$.

**20**. The solutions will be the cube roots of –1. Since $-1=\cos 180°+i\sin 180°$, the cube roots are given by:

$$w_k = 1^{1/3}\left[\cos\frac{180°+360°k}{3}+i\sin\frac{180°+360°k}{3}\right]=\cos\left(60°+120°k\right)+i\sin\left(60°+120°k\right)$$

Substituting $k = 0,1,2$:

$$w_0 = \cos 60°+i\sin 60°$$

$$w_1 = \cos 180°+i\sin 180°$$

$$w_2 = \cos 300°+i\sin 300°$$



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**21**. Two other points are $(6,-120°)$ and $(6,240°)$. The rectangular coordinates are given by:

$$x = r\cos\theta = -6\cos 60° = -6 \cdot \frac{1}{2} = -3 \qquad y = r\sin\theta = -6\sin 60° = -6 \cdot \frac{\sqrt{3}}{2} = -3\sqrt{3}$$

The rectangular coordinates are $\left(-3,-3\sqrt{3}\right)$.

**22**. Compute $r = \sqrt{(-3)^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$. Also $\tan\theta = \frac{3}{-3} = -1$ with $\theta$ in quadrant II, so $\theta = 135°$.

The polar coordinates are $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$.

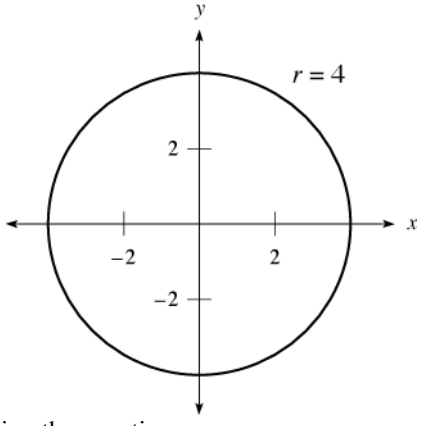
**23**. Converting to rectangular coordinates:

$$r = 5\cos\theta$$
$$r = 5\left(\frac{x}{r}\right)$$
$$r^2 = 5x$$
$$x^2 + y^2 = 5x$$

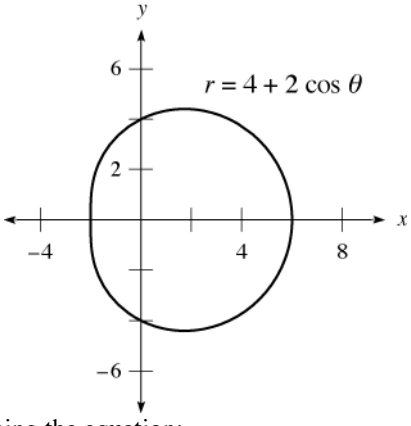
**24**. Since $x^2 + y^2 = r^2$ and $y = r\sin\theta$:

$$x^2 + y^2 = 8y$$
$$r^2 = 8r\sin\theta$$
$$r = 8\sin\theta$$

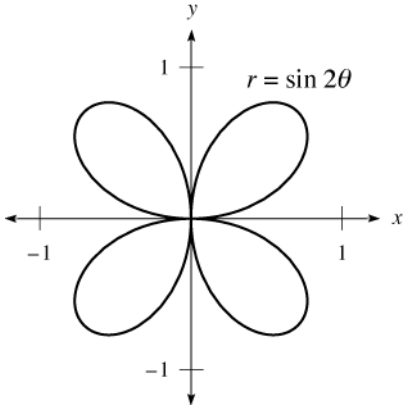
**25**. Graphing the equation:



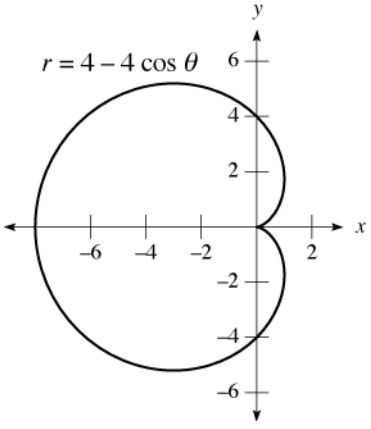

**26**. Graphing the equation:




**27**. Graphing the equation:




**28**. Graphing the equation:



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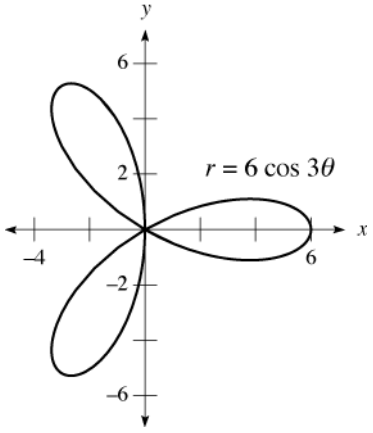
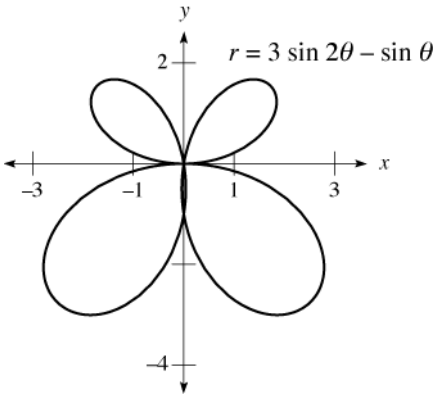
**29**. Graphing the equation:




**30**. Graphing the equation:




## Cumulative Test Chapters 1–8

**1**. If $\cos\theta = \frac{3}{5}$, we can choose $x = 3$ and $r = 5$. To find $y$, use $x^2 + y^2 = r^2$:

$$3^2 + y^2 = 5^2$$
$$9 + y^2 = 25$$
$$y^2 = 16$$
$$y = \pm 4$$

Since $\theta$ terminates in quadrant IV, $y < 0$ and thus $y = -4$. Using $x = 3$, $y = -4$, and $r = 5$ in the definitions:

$$\sin\theta = \frac{y}{r} = -\frac{4}{5} \qquad \sec\theta = \frac{r}{x} = \frac{5}{3} \qquad \csc\theta = \frac{r}{y} = -\frac{5}{4}$$

$$\tan\theta = \frac{y}{x} = -\frac{4}{3} \qquad \cot\theta = \frac{x}{y} = -\frac{3}{4}$$

**2**. Using a calculator: $\theta = \cos^{-1}(0.7309) \approx 43.0°$

**3**. The reference angle is given by $\hat{\theta} = \tan^{-1}\left(\frac{1}{1.5450}\right) \approx 32.9°$. Since $\theta$ terminates in quadrant III,

$\theta = 180° + 32.9° = 212.9°$.

**4**. Let $x$ represent the original distance from the tree, and $h$ the height of the tree. Therefore:

$$\tan 79.0° = \frac{h}{x} \qquad \tan 72.6° = \frac{h}{x+40}$$
$$h = x\tan 79.0° \qquad h = (x+40)\tan 72.6°$$
$$x = \frac{h}{\tan 79.0°} \qquad x + 40 = \frac{h}{\tan 72.6°}$$
$$x = h\cot 79.0° \qquad x = h\cot 72.6° - 40$$

Setting the two expressions equal:

$$h\cot 79.0° = h\cot 72.6° - 40$$
$$40 = h\cot 72.6° - h\cot 79.0°$$
$$40 = h(\cot 72.6° - \cot 79.0°)$$
$$h = \frac{40}{\cot 72.6° - \cot 79.0°} \approx 336 \text{ ft}$$

The tree is approximately 336 feet tall.



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**5**. First find side $c$ using the Pythagorean Theorem:

$$20.5^2 + 31.4^2 = c^2$$

$$420.25 + 985.96 = c^2$$

$$c^2 = 1{,}406.21$$

$$c \approx 37.5$$

Now finding angle $A$:

$$\sin B = \frac{b}{c}$$

$$\sin B = \frac{31.4}{37.5}$$

$$B = \sin^{-1}\left(\frac{31.4}{37.5}\right) \approx 56.9°$$

$$A = 90° - 56.9° = 33.1°$$

**6**. Converting to degrees: $\frac{4\pi}{9} \bullet \frac{180°}{\pi} = 80°$

**7**. Using the arc length formula $s = r\theta$:

$$s = r\theta$$

$$\frac{\pi}{4} = r \bullet \frac{2\pi}{3}$$

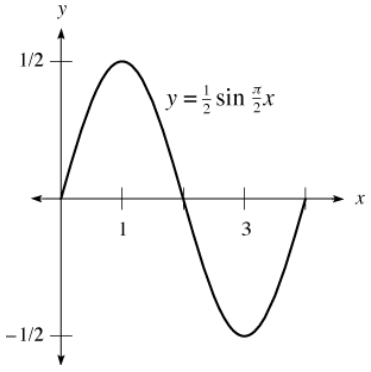
$$r = \frac{3}{8} = 0.375 \text{ cm}$$

**8**. Finding the angular velocity: $\omega = 10 \frac{\text{rev}}{\text{min}} \bullet 2\pi \frac{\text{rad}}{\text{rev}} = 20\pi \frac{\text{rad}}{\text{min}}$

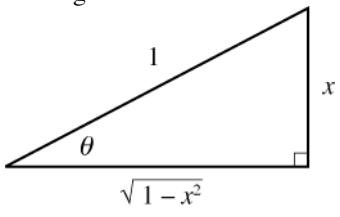
Finding the linear velocity: $v = r\omega = 1 \text{ foot} \bullet 20\pi \frac{\text{rad}}{\text{min}} = 20\pi \frac{\text{feet}}{\text{min}}$

**9**. The secant values are undefined when the cosine values are 0. These occur when $x = -\frac{7\pi}{2}, -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$.

**10**. Sketching the graph:




**11**. Let $\theta = \sin^{-1} x$, so $\sin\theta = x$. Drawing the triangle:




Therefore: $\tan\left(\sin^{-1} x\right) = \tan\theta = \frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{1-x^2}}{1-x^2}$



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**12**. Working from the right side:

$$\begin{aligned}\sec^2 x + \csc^2 x &= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \\ &= \frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \\ &= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} \\ &= \frac{1}{\sin^2 x \cos^2 x} \\ &= \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x} \\ &= \sec^2 x \csc^2 x\end{aligned}$$

**13**. Using the difference formula for cosine:

$$\cos 15° = \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30° = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{1}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

**14**. Using the difference formula for tangent: $\tan \frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{4}} = \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

Therefore: $\cot \frac{\pi}{12} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{3 + 2\sqrt{3} + 1}{3 - 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$

**15**. Using the double-angle formula:

$$\begin{aligned}3\cos 2\theta + 5\sin\theta &= 0 \\ 3\left(1 - 2\sin^2\theta\right) + 5\sin\theta &= 0 \\ 3 - 6\sin^2\theta + 5\sin\theta &= 0 \\ 6\sin^2\theta - 5\sin\theta - 3 &= 0\end{aligned}$$

Using the quadratic formula: $\sin\theta = \frac{5 \pm \sqrt{(-5)^2 - 4(6)(-3)}}{2(6)} = \frac{5 \pm \sqrt{25 + 72}}{12} = \frac{5 \pm \sqrt{97}}{12} \approx -0.4041, 1.2374$

Since $\sin\theta = 1.2374$ is impossible, $\sin\theta = -0.4041$. The reference angle is given by $\hat{\theta} = \sin^{-1}(0.4041) \approx 23.8°$.

Since $\theta$ terminates in quadrants III or IV, $\theta = 180° + 23.8° = 203.8°$ or $\theta = 360° - 23.8° = 336.2°$.

**16**. Using the law of sines:

$$\begin{aligned}\frac{b}{\sin B} &= \frac{c}{\sin C} \\ \frac{b}{\sin 108°} &= \frac{2.9}{\sin 37°} \\ b &= \frac{2.9 \sin 108°}{\sin 37°} \approx 4.6 \text{ in.}\end{aligned}$$

**17**. Use the law of sines to find $B$:

$$\begin{aligned}\frac{\sin B}{b} &= \frac{\sin A}{a} \\ \frac{\sin B}{9.4} &= \frac{\sin 26°}{4.8} \\ \sin B &= \frac{9.4 \sin 26°}{4.8} \approx 0.8585 \\ B &\approx 59° \\ B' &\approx 121°\end{aligned}$$



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If $B = 59°$, $C = 180° - (59° + 26°) = 95°$. Use the law of sines to find $c$:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 26°}{4.8} = \frac{\sin 95°}{c}$$
$$c = \frac{4.8 \sin 95°}{\sin 26°} \approx 11 \text{ ft}$$

If $B' = 121°$, $C' = 180° - (121° + 26°) = 33°$. Use the law of sines to find $c'$:

$$\frac{\sin A}{a} = \frac{\sin C'}{c'}$$
$$\frac{\sin 26°}{4.8} = \frac{\sin 33°}{c'}$$
$$c' = \frac{4.8 \sin 33°}{\sin 26°} \approx 6.0 \text{ ft}$$

**18**. Using the law of cosines:

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$12^2 = 10^2 + 17^2 - 2(10)(17)\cos B$$
$$144 = 100 + 289 - 340\cos B$$
$$-245 = -340\cos B$$
$$\cos B \approx 0.7206$$
$$B \approx 43.9°$$

**19**. Using the area formula: $S = \frac{1}{2}ac\sin B = \frac{1}{2}(10)(17)\sin 43.9° \approx 59 \text{ km}^2$

**20**. Finding the vector: $3\mathbf{U} - 5\mathbf{V} = 3(5\mathbf{i} + 12\mathbf{j}) - 5(-4\mathbf{i} + \mathbf{j}) = 15\mathbf{i} + 36\mathbf{j} + 20\mathbf{i} - 5\mathbf{j} = 35\mathbf{i} + 31\mathbf{j}$

**21**. Using $\cos\theta = \frac{\mathbf{U} \bullet \mathbf{V}}{|\mathbf{U}||\mathbf{V}|}$ to find the angle:

$$\cos\theta = \frac{(5)(-4) + (12)(1)}{\sqrt{5^2 + 12^2}\sqrt{(-4)^2 + 1^2}} = \frac{-20 + 12}{\sqrt{169}\sqrt{17}} = \frac{-8}{13\sqrt{17}} \approx -0.1493$$
$$\theta = 98.6°$$

**22**. Simplifying: $(7 + 3i) - [(2 + i) - (3 - 4i)] = (7 + 3i) - (2 + i - 3 + 4i) = (7 + 3i) - (-1 + 5i) = 7 + 3i + 1 - 5i = 8 - 2i$

**23**. Computing the power: $i^{16} = (i^4)^4 = 1$

**24**. **a**. The absolute value is given by: $|3 + 4i| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

**b**. The opposite is $-3 - 4i$.

**c**. The conjugate is $3 - 4i$.

**25**. Here $x = 2$, $y = 2$, and $r = \sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$. Since $\tan\theta = \frac{2}{2} = 1$ in quadrant I, $\theta = 45°$.

So the trigonometric form is $2 + 2i = 2\sqrt{2}(\cos 45° + i\sin 45°) = 2\sqrt{2} \text{ cis } 45°$.

**26**. Using deMoivre's Theorem:

$$[2(\cos 10° + i\sin 10°)]^5 = 2^5[\cos(5 \bullet 10°) + i\sin(5 \bullet 10°)] = 32(\cos 50° + i\sin 50°) = 32\,\text{cis}\,50°$$



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**27**. First write the number in trigonometric form: $4-4i\sqrt{3}=8\left(\cos 300°+i\sin 300°\right)$

The square roots are given by:

$$w_k=8^{1/2}\left[\cos\frac{300°+360°k}{2}+i\sin\frac{300°+360°k}{2}\right]=2\sqrt{2}\left[\cos\left(150°+180°k\right)+i\sin\left(150°+180°k\right)\right]$$

Substituting $k=0,1$:

$$w_0=2\sqrt{2}\left(\cos 150°+i\sin 150°\right)=2\sqrt{2}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=-\sqrt{6}+i\sqrt{2}$$

$$w_1=2\sqrt{2}\left(\cos 330°+i\sin 330°\right)=2\sqrt{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)=\sqrt{6}-i\sqrt{2}$$

**28**. Here $r=4$ and $\theta=225°$. Converting to rectangular coordinates:

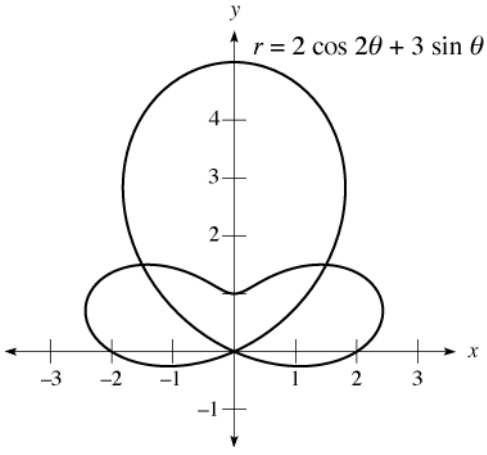
$$x=r\cos\theta=4\cos 225°=4\bullet\left(-\frac{\sqrt{2}}{2}\right)=-2\sqrt{2}\qquad y=r\sin\theta=4\sin 225°=4\bullet\left(-\frac{\sqrt{2}}{2}\right)=-2\sqrt{2}$$

The rectangular coordinates are $\left(-2\sqrt{2},-2\sqrt{2}\right)$. Two other polar coordinates are $\left(-4,45°\right)$ and $\left(4,-135°\right)$.

**29**. Substituting $x=r\cos\theta$ and $y=r\sin\theta$:

$$\begin{aligned}x+y&=2\\ r\cos\theta+r\sin\theta&=2\\ r\left(\cos\theta+\sin\theta\right)&=2\\ r&=\frac{2}{\cos\theta+\sin\theta}\end{aligned}$$

**30**. Graphing the equation:



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